Part A: Modeling Exponential Functions

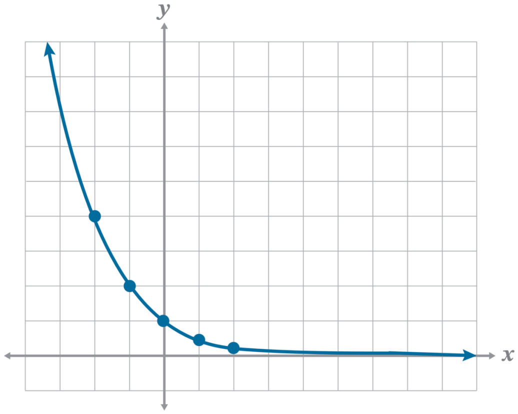
Practice 1

1.

$a = 1,\ b = \frac{1}{2},\ k = 0,\ (0, 1)$

x	$\left(\frac{1}{2}\right)^x$	***f(x)***
−2	$\left(\frac{1}{2}\right)^{-2} = 2^2$	4
−1	$\left(\frac{1}{2}\right)^{-1} = 2^1$	2
0	$\left(\frac{1}{2}\right)^0$	1
1	$\left(\frac{1}{2}\right)^1$	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2}$	$\frac{1}{4}$

2.



domain: all real numbers ($\mathscr{R}$)
range: $y > 0$

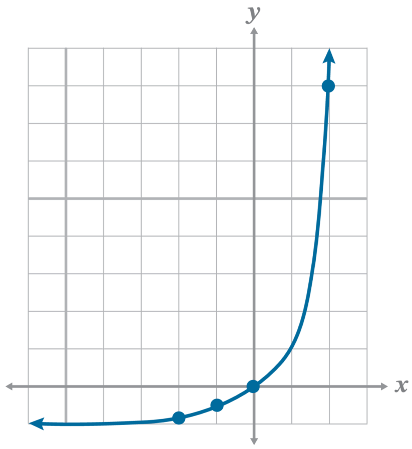
3.

$a = 1,\ b = 3,\ k = -1,\ (0,\ 0)$

x	$3^x - 1$	***y***
−2	$3^{-2} - 1 = \frac{1}{3^2} - 1 = \frac{1}{9} - 1$	$-\frac{8}{9}$
−1	$3^{-1} - 1 = \frac{1}{3^1} - 1 = \frac{1}{3} - 1$	$-\frac{2}{3}$

0	$3^0 - 1 = 1 - 1$	0
1	$3^1 - 1 = 3 - 1$	2
2	$3^2 - 1 = 9 - 1$	8

4.

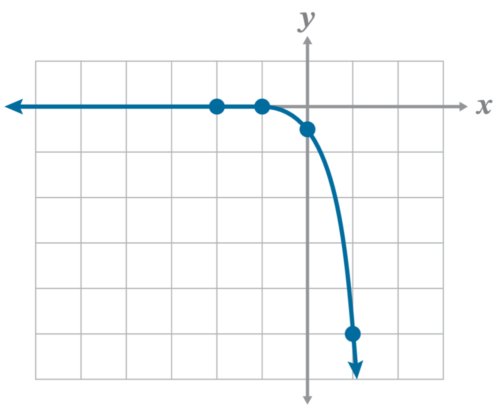


domain: all real numbers ($\mathcal{R}$)
range: $y > -1$

5.
$a = -\frac{1}{2},\ b = 10,\ k = 0,\ \left(0,\ -\frac{1}{2}\right)$

x	$-\frac{1}{2} \cdot 10^x$	***h(x)***
−2	$-\frac{1}{2} \cdot 10^{-2} = -\frac{1}{2} \cdot \frac{1}{10^2} = -\frac{1}{2} \cdot \frac{1}{100}$	$-\frac{1}{200}$
−1	$-\frac{1}{2} \cdot 10^{-1} = -\frac{1}{2} \cdot \frac{1}{10^1} = -\frac{1}{2} \cdot \frac{1}{10}$	$-\frac{1}{20}$
0	$-\frac{1}{2} \cdot 10^0 = -\frac{1}{2} \cdot 1$	$-\frac{1}{2}$
1	$-\frac{1}{2} \cdot 10^1 = -\frac{1}{2} \cdot 10$	-5
2	$-\frac{1}{2} \cdot 10^2 = -\frac{1}{2} \cdot 100$	-50

6.



domain: all real numbers ($\mathcal{R}$)
range: $y < 0$

7. B

8. C

9. A

10. D

11.
$m = \frac{2-1}{15-13} = \frac{1}{2}$
linear function, $m = \frac{1}{2}$

12.
$b = \frac{3}{9} = \frac{1}{3}$
exponential function, $b = \frac{1}{3}$

13.
$b = \frac{1{,}000}{100} = 10$
exponential function, $b = 10$

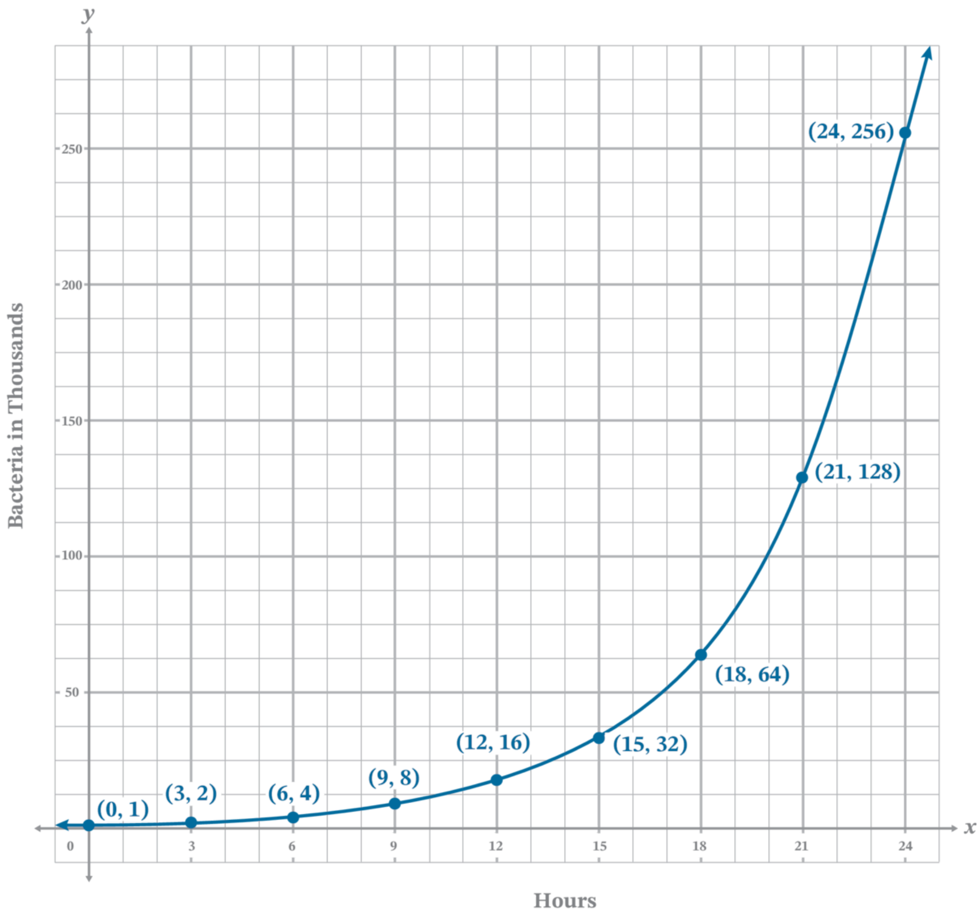
14.

t	***b*** (in thousands)
0	1
3	2
6	4
9	8

12	16
15	32
18	64
21	128
24	256

(hours, bacteria in thousands)

15.



Sample: After 27 hours (3 hours past 24), there will be 512,000 bacteria because this is double the previous number.

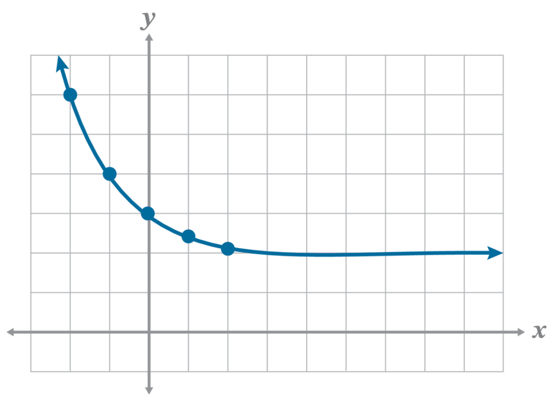
Practice 2

1.

$a = 1,\ b = \frac{1}{2},\ k = 2,\ (0, 3)$

x	$\left(\frac{1}{2}\right)^x + 2$	$f(x)$
−2	$\left(\frac{1}{2}\right)^{-2} + 2 = 2^2 + 2$	6
−1	$\left(\frac{1}{2}\right)^{-1} + 2 = 2^1 + 2$	4
0	$\left(\frac{1}{2}\right)^{0} + 2 = 1 + 2$	3
1	$\left(\frac{1}{2}\right)^{1} + 2 = \frac{1}{2} + 2$	$\frac{5}{2}$
2	$\left(\frac{1}{2}\right)^{1} + 2 = \frac{1}{2^2} + 2$	$\frac{9}{4}$

2.

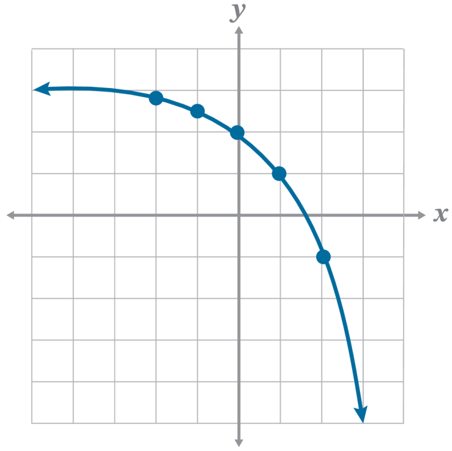


domain: all real numbers ($\mathcal{R}$)
range: $y > 2$

3.
$a = \ -1,\ b = 2,\ k = 3,\ (0, 2)$

x	$-2^x + 3$	$h(x)$
−2	$-2^{-2} + 3 = -\frac{1}{4} + 3$	$\frac{11}{4}$
−1	$-2^{-1} + 3 = -\frac{1}{2} + 3$	$\frac{5}{2}$
0	$-2^{0} + 3 = -1 + 3$	2
1	$-2^{1} + 3 = -2 + 3$	1
2	$-2^{2} + 3 = -4 + 3$	−1

4.

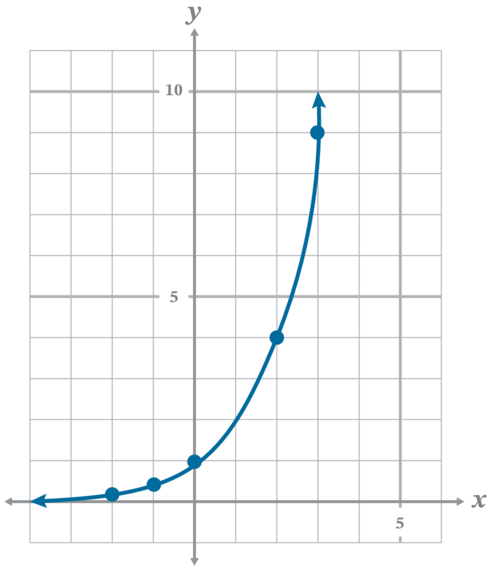


domain: all real numbers ($\mathcal{R}$)
range: $y < 3$

5.
$a = 1,\ b = 3,\ k = 0,\ (0,\ 1)$

x	3^x	$g(x)$
− 2	$3^{-2} = \frac{1}{3^2}$	$\frac{1}{9}$
− 1	$3^{-1} = \frac{1}{3^1}$	$\frac{1}{3}$
0	3^0	1
1	3^1	3
2	3^2	9

6.



domain: all real numbers ($\mathscr{R}$)
range: $y > 0$

7. D

8. B

9. A

10. C

11.
$b = \frac{64}{32} = 2$
Exponential function, $b = 2$

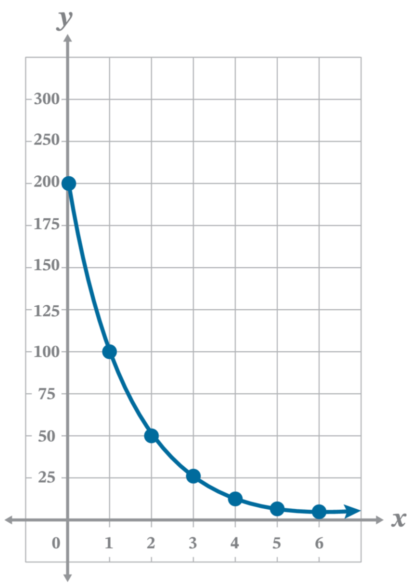
12.
$b = \frac{16}{64} = \frac{1}{4}$
Exponential function, $b = \frac{1}{4}$

13.
$m = \frac{4-2}{2-1} = 2$
Linear function, $m = 2$

14.
(x, m): (number of months, mass in grams)

x	m
0	200
1	100
2	50
3	25
4	12.5
5	6.25
6	3.125

15.



Sample: The graph will never reach $y = 0$, because the range is $y > 0$. Theoretically, the mass will continue to get smaller each month by half and never reach zero.

Part B: The Growth/Decay Formula

Practice 1

1.

$y = 35,000\,(1 + 0.027)^7$
$a = 35,000$
$t = 7$
$r = 2.7\%$
Growth, 1.027
This represents exponential growth because the common ratio is 1.027.

2.
$y = 125,500\,(1 - 0.084)^6$
$a = 125,500$
$t = 6$
$r = 8.4\%$
Decay, 0.916
This represents exponential decay because the common ratio is 0.916.

3.
$y = 632\,(1.26)^{14}$
$a = 632$
$t = 14$
$r = 26\%$
Growth, 1.26
This represents exponential growth because the common ratio is 1.26.

4.
$y = 8,590\,(0.985)^4$
$a = 8,590$
$t = 4$
$r = 1.5\%$
Decay, 0.985
This represents exponential decay because the common ratio is 0.985.

5.
Growth
$a = 3,250$
$t = 6$
$r = 4.1\% = 0.041$
$y = 3,250(1 + 0.041)^6$
$y = 4,136.07$
Jill has $4,136.07.

6.
Decay
$a = 25,450$
$t = 8$
$r = 12\% = 0.12$
$y = 25,450(1 - 0.12)^8$
$y = 9,152.70$
After 8 years, the value of the car will be $9,152.70.

7.
Decay
$t = 10$
$r = 1.2\% = 0.012$
$y = 12,345$
$a = ?$
$12,345 = a(1 - 0.012)^{10}$
$12,345 = a(0.988)^{10}$
$\frac{12,345}{(0.988)^{10}} = a$
$a = 13,929.05$
The population of Hometown 10 years ago was 13,929 people.

8.
Growth
$a = 500$
$t = 1$
$r = 250\% = 2.5$
$y = 500(1 + 2.5)^1$
$y = 1,750$
Generosity High School collected 1,750 cans this year.

9.
Decay
$a = 107,000$
$t = 2019 - 2004 = 15$
$r = 16.8\% = 0.168$
$y = 107,000(1 - 0.168)^{15}$
$y = 6,780.06$
There are 6,780 landlines in 2019.

$107,000 - 6,780$
$100,220$
There are 100, 220 fewer landlines.

10.
Decay
$a = 25,450$
$t = 6$
$r = 15\% = 0.15$
$y = 25,450(1 - 0.15)^{6}$
$y = 9,598.46$
The car is now valued at $9,598.46.

$25,450 - 9,598.46$
$15,851.54$
The car lost $15,851.54 in value.

11.
Growth
$t = 12$
$r = 0.5\% = 0.005$
$y = 983.11$
$a = ?$
$983.11 = a(1 + 0.005)^{12}$
$983.11 = a(1.005)^{12}$
$\frac{983.11}{(1.005)^{12}} = a$
$a = 926.00$
Beven started the savings account with $926.

12.
$a = 215,000$
$t = 25$
$r = 2.6\% = 0.026$
$y = 215,000(1 + 0.026)^{25}]$
$y = 408,434.56$
After 25 years, the value of the Guthrie home is $408,400.

Practice 2

1.
$y = 2,576\,(1 + 0.034)^{15}$
$a = 2,576$
$t = 15$
$r = 3.4\%$
Growth, 1.034
This represents exponential growth because the common ratio is 1.034.

2.
$y = 405\,(1.015)^8$
$a = 405$
$t = 8$
$r = 1.5\%$
Growth, 1.015
This represents exponential growth because the common ratio is 1.015.

3.
$y = 3,250\,(0.92)^5$
$a = 3,250$
$t = 5$
$r = 8\%$
Decay, 0.92
This represents exponential decay because the common ratio is 0.92.

4.
$y = 25,000\,(1 - 0.13)^9$
$a = 25,000$
$t = 9$
$r = 13\%$
Decay, 0.87
This represents exponential decay because the common ratio is 0.87.

5.
Decay
$a = 5,000$
$t = 2$
$r = 3.2\% = 0.032$
$y = 5,000(1 - 0.032)^2$
$y = 4,685.12$
The stock is now worth $4,685.12.

6.
Growth
$a = 1,500$
$t = 12$
$r = 6.5 = 0.065$
$y = 1,500(1 + 0.065)^{12}$
$y = 3,193.64$
After 12 years, the account contained $3,193.44.

7.
Growth
$t = 6$
$r = 15\% = 0.15$
$y = 520$
$a = ?$
$520 = a(1 + 0.15)^6$
$520 = a(1.15)^6$
$\frac{520}{(1.15)^6} = a$
$a = 224.81$
The Leighton family spent $225 on groceries six years ago.

$520 - 225$
295
The Leighton family spends $295 more now.

8.
Decay
$a = 526$
$t = 4$
$r = 5.3\% = 0.053$
$y = 526(1 - 0.053)^4$
$y = 423.04$
423 students will graduate from the program.

526 – 423
103
There were 103 students who did not graduate in the program.

9.
Growth
$a = 120,000$
$t = 30$
$r = 3.99\% = 0.0399$
$y = 120,000(1 + 0.0399)^{30}$
$y = 388,086.55$
In 30 years, Rylan will have $388,086.55

388,086.55 – 120,000
Rylan's investment increased by $268,086.55

10.
Decay
$a = 450$
$t = 15$
$r = 10\% = 0.10$
$y = 450(1 - 0.10)^{15}$
$y = 92.65$
This year Mrs. Neill printed 93 photos.

11.
Growth
$a = 50$
$t = 5$
$r = 1.2\% = 0.012$
$y = 50(1 + 0.012)^{5}$
$y = 53.07$

$t = 10$
$y = 50(1 + 0.012)^{10}$
$y = 56.33$

12.
50 + 15 = 65 miles of trail
$65 = 50(1 + 0.012)^{x}$
Try multiple values for x.
x = 22, so it will take 22 years to have 65 trails.

Targeted Review

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	28	28	29	29	29	29	26	27	27	29	29	29

1.
$\frac{2}{7a^4b}$

2.
$\frac{x^{\frac{3}{2}-1}}{y^{16-11}}$
$\frac{x^{\frac{1}{2}}}{y^5}$

3.
$a^{\frac{17}{3}}b^{\frac{33}{3}}c^{\frac{21}{3}}$
$a^5b^{11}c^7\sqrt[3]{a^2}$

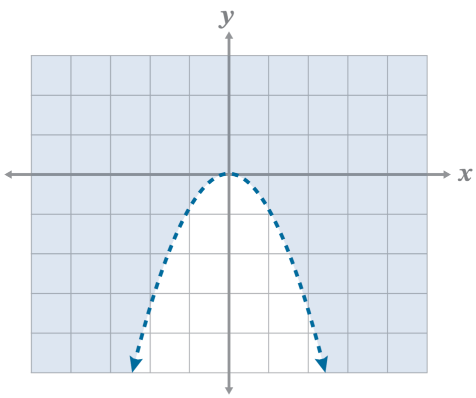
4.
$\sqrt{5^3} + 9\sqrt{5} - \sqrt{2^1 \cdot 5^2}$
$5^{\frac{3}{2}} + 9\sqrt{5} - 2^{\frac{1}{2}}5^{\frac{2}{2}}$
$5\sqrt{5} + 9\sqrt{5} - 5\sqrt{2}$
$14\sqrt{5} - 5\sqrt{2}$

5.
$3^4 = 3^{a-2}$
$4 = a - 2$
$a = 6$

6.
$\left(\sqrt{3x}\right)^2 = (12)^2$
$3x = 144$
$x = 48$

7.
$a = -2, b = 7, c = -4$
$x = 0.719, 2.781$

8.



9.

☐ vertical shift
☑ horizontal shift
☑ reflection
☐ dilation

Distractor Rationale:
$a = -1$, $h = 12$, $k = 0$
The graph will shift right 12 spaces and be reflected over the x-axis because $a = -1$.

10.

☑ $9xy^4\sqrt{x}$ $\sqrt{81x^3y^8} = \sqrt{9^2x^3y^8} = 9xy^4\sqrt{x}$

☑ $\sqrt{3^4x^3y^8}$ $\sqrt{81x^3y^8} = \sqrt{3^4x^3y^8}$

☐ $3xy\sqrt{xy^7}$

☑ $(81x^3y^8)^{\frac{1}{2}}$ $\sqrt{81x^3y^8} = (81x^3y^8)^{\frac{1}{2}}$

Distractor Rationale:
The third expression is not equivalent to the given expression.

11. C

A) $\sqrt[y]{5}$

B) -5^y

C) $\left(\frac{1}{5}\right)^y$ $5^{-y} = \frac{1}{5^y} = \left(\frac{1}{5}\right)^y$

D) $\frac{5}{y}$

Distractor rationale:
A) The denominator of an exponent represents the index, not the numerator, and the negative sign disappears in this answer.
B) A negative sign cannot transfer from the exponent to the base.
D) The exponent cannot be separated from the base to which it is assigned.

12. B

A) $\pm\sqrt{32}$

B) $\pm 4\sqrt{2}$

C) ± 8

D) no solution

$x^2 = 32$

$x = \pm\sqrt{32} = \pm\sqrt{2 \cdot 4 \cdot 4}$

$x = \pm 4\sqrt{2}$

Distractor Rationale:

A) This answer is not in simplified radical form.

C) This is the square root of 64, ignoring the coefficient −2.

D) This would be correct if there were only one negative sign in the problem.