Part A: Solving Quadratic Equations Not Equal to Zero

Practice 1

1.

$$x^2 - 10x + 21 = 0$$
$$(x - 7)(x - 3) = 0$$
$$x - 7 = 0,\ x - 3 = 0$$
$$x = 7,\ 3$$

2.

$$x^2 - 6x = 0$$
$$x(x - 6) = 0$$
$$x = 0,\ x - 6 = 0$$
$$x = 0,\ 6$$

3.

$$0 = x^2 - 3x - 10$$
$$0 = (x - 5)(x + 2)$$
$$x = 5,\ -2$$

4.

$$4x^2 - 19x - 30 = 0$$
$$(4x + 5)(x - 6) = 0$$
$$4x + 5 = 0,\ x - 6 = 0$$
$$x = -\frac{5}{4},\ 6$$

5.

$$2x^2 + 22x + 56 = 0$$
$$2(x^2 + 11x + 28) = 0$$
$$2(x + 7)(x + 4) = 0$$
$$x + 7 = 0,\ x + 4 = 0$$
$$x = -7,\ -4$$

6.

$$20x^2 - 80 = 0$$
$$20(x^2 - 4) = 0$$
$$20(x - 2)(x + 2) = 0$$
$$x = \pm 2$$

7.

$$9x^2 - 49 = 0$$
$$(3x - 7)(3x + 7) = 0$$
$$3x - 7 = 0,\ 3x + 7 = 0$$
$$x = \frac{7}{3},\ -\frac{7}{3}$$

The side length is 7 inches $\left(3\left(\frac{7}{3}\right) = \frac{21}{3} = 7\right)$.

The extraneous solution is $-\frac{7}{3}$ because a side length cannot be negative $\left(3\left(-\frac{7}{3}\right) = -\frac{21}{3} = -7\right)$.

8.

$$x^2 + 2x - 24 = 0$$
$$(x + 6)(x - 4) = 0$$
$$x + 6 = 0,\ x - 4 = 0$$
$$x = -6,\ 4$$

A side cannot be negative so $x = -6$ is not a possible solution.
When $x = 4$, the sides of the rectangle are 4 feet and 6 feet $(4 + 2 = 6)$.

9.

$$9x^2 + 12x + 4 = 9$$
$$9x^2 + 12x - 5 = 0$$
$$(3x - 1)(3x + 5) = 0$$
$$3x - 1 = 0,\ 3x + 5 = 0$$
$$x = \frac{1}{3},\ -\frac{5}{3}$$

$$\left(3\left(\frac{1}{3}\right) + 2\right)^2 = (3)^2$$
$$\left(3\left(-\frac{5}{3}\right) + 2\right)^2 = (-3)^2$$

When $x = \frac{1}{3}$, each side of the square will be 3 cm. The value $-\frac{5}{3}$ is extraneous because a side cannot be -3 cm.

10.

$$x^2 + x - 6 = 6$$
$$x^2 + x - 12 = 0$$
$$(x - 3)(x + 4) = 0$$
$$x = 3,\ -4$$

The side lengths are 1 unit $(3 - 2 = 1)$ and 6 units $(3 + 3 = 6)$. The solution $x = -4$ is extraneous.

11.

$4x^2 - 8x - 32 = 0$
$4(x^2 - 4x - 8) = 0$
$4(x + 2)(x - 4) = 0$
$x + 2 = 0,\ x - 4 = 0$
$x = -2, 4$

When $x = 4$, the sides of the rectangle are 16 inches $(4(4) = 16)$ and 2 inches $(4 - 2 = 2)$.
The extraneous solution is $x = -2$.

12.

$2x^2 - 7x + 3 = 18$
$2x^2 - 7x - 15 = 0$
$(2x + 3)(x - 5) = 0$
$x = -\frac{3}{2}, 5$

The sides of the rectangle are 9 feet $(2(5) - 1 = 9)$ and 2 feet $(5 - 3 = 2)$ long.
The value $-\frac{3}{2}$ is extraneous.

Practice 2

1.

$0 = x^2 + 10x - 24$
$0 = (x + 12)(x - 2)$
$x + 12 = 0,\ x - 2 = 0$
$x = -12, 2$

2.

$x^2 - 8x - 180 = 0$
$(x + 10)(x - 18) = 0$
$x + 10 = 0,\ x - 18 = 0$
$x = -10, 18$

3.

$2x^2 + x - 21 = 0$
$(2x + 7)(x - 3) = 0$
$2x + 7 = 0,\ x - 3 = 0$
$x = -\frac{7}{2}, 3$

4.

$3x^2 + 18x + 15 = 0$
$3(x^2 + 6x + 5) = 0$
$3(x + 5)(x + 1) = 0$

$x + 5 = 0,\ x + 1 = 0$
$x = -5,\ -1$

5.

$x^2 - 20x + 100 = 0$
$(x - 10)^2 = 0$
$x = 10$

6.

$3x^2 - 31x - 22 = 0$
$(3x + 2)(x - 11) = 0$
$3x + 2 = 0,\ x - 11 = 0$
$x = -\frac{2}{3},\ 11$

7.

$25x^2 - 20x + 4 = 16$
$25x^2 - 20x - 12 = 0$
$(5x + 2)(5x - 6) = 0$
$5x + 2 = 0,\ 5x - 6 = 0$
$x = -\frac{2}{5},\ \frac{6}{5}$

The side length of the square is 4 inches long $\left(5\left(\frac{6}{5}\right) - 2 = 4\right)$.
The value $-\frac{2}{5}$ is extraneous because a side length cannot be negative.

8.

$3x^2 + 5x - 12 = 0$
$(3x - 4)(x + 3) = 0$
$3x - 4 = 0,\ x + 3 = 0$
$x = \frac{4}{3},\ -3$

The side lengths of the rectangle are $\frac{4}{3}$ feet and 9 feet $\left(3\left(\frac{4}{3}\right) + 5 = 9\right)$.
The extraneous solution is -3.

9.

$x^2 - 9 = 0$
$(x - 3)(x + 3) = 0$
$x = 3,\ -3$

10.

$16x^2 + 8x + 1 = 8x + 10$
$16x^2 - 9 = 0$
$(4x - 3)(4x + 3) = 0$

$x = \frac{3}{4}, \ -\frac{3}{4}$

The side of the square is 4 units long $\left(4\left(\frac{3}{4}\right) + 1 = 4\right)$. The extraneous solution is $x = \ -\frac{3}{4}$.

11.

$$8x^2 - 2x - 3 = 63$$
$$8x^2 - 2x - 66 = 0$$
$$2(4x^2 - x - 33) = 0$$
$$2(x - 3)(4x + 11) = 0$$
$$x - 3 = 0, \ 4x + 11 = 0$$
$$x = 3, \ -\frac{11}{4}$$

The extraneous solution is $-\frac{11}{4}$.
The length is 9 units $(4(3) - 3 = 9)$. The width is 7 units $(2(3) + 1 = 7)$.

12.

$$6x^2 + x - 5 = 0$$
$$(6x - 5)(x + 1) = 0$$
$$6x - 5 = 0, \ x + 1 = 0$$
$$x = \frac{5}{6}, \ -1$$

The side lengths of the rectangle are $\frac{5}{6}$cm and 6 cm $\left(6\left(\frac{5}{6}\right) + 1 = 6\right)$.
The extraneous solution is –1.

Part B: Quadratic Applications

Practice 1

1. A: area, l: length, w: width

$$A = 52, \ l = 3w + 2, \ w = w$$
$$A = lw$$
$$52 = w(3w + 2)$$
$$52 = 3w^2 + 2w$$
$$0 = 3w^2 + 2w - 52$$

2. t: time, d: distance, v: velocity

$$d = 3, \ v = 15$$
$$d = 16t^2 + vt$$
$$3 = 16t^2 + 15t$$
$$0 = 16t^2 + 15t - 3$$

3. x: a number

$$x(5x + 4) = -273$$
$$5x^2 + 4x = -273$$
$$5x^2 + 4x + 273 = 0$$

4. A: area, w: width, l: length

$$A = 120$$
$$l = x + 2 + 1 + 1 = x + 4$$
$$w = x + 1 + 1 = x + 2$$
$$A = lw$$
$$120 = (x + 2)(x + 4)$$
$$120 = x^2 + 6x + 8$$
$$0 = x^2 + 6x - 112$$

5. b: base, h: height

$$b = b,\ h = 2b - 3$$
$$A = \frac{1}{2}bh$$
$$85 = \frac{1}{2}(b)(2b - 3)$$
$$85 = \frac{1}{2}b(2b - 3)$$
$$85 = b^2 - \frac{3}{2}b$$
$$0 = b^2 - \frac{3}{2}b - 85$$

6. x: first term, $x + 1$: second term, $x + 2$: third term

$$x(x + 1)(x + 2) = 336$$
$$x(x^2 + 3x + 2) = 336$$
$$x^3 + 3x^2 + 2x - 336 = 0$$

7. A: area, w: width, l: length

$$l = 2x + 3 + 4 + 4 = 2x + 11$$
$$w = x + 4 + 4 = x + 8$$
$$A = lw$$
$$348 = (2x + 11)(x + 8)$$
$$348 = 2x^2 + 27x + 88$$
$$0 = 2x^2 + 27x - 260$$
$$0 = (2x - 13)(x + 20)$$
$$2x - 13 = 0,\ x + 20 = 0$$
$$x = \frac{13}{2},\ -20$$

When $x = \frac{13}{2}$, $w = \frac{13}{2} = 6.5$

$l = 2\left(\frac{13}{2}\right) + 3 = 16$

A side length cannot be -20, making this solution extraneous. The dimensions of the playground are 6.5 feet by 16 feet.

8. n: a number, second number: $4n - 3$

$$n(4n - 3) = 10$$
$$4n^2 - 3n - 10 = 0$$
$$(4n + 5)(n - 2) = 0$$
$$4n + 5 = 0,\ n - 2 = 0$$
$$n = -\frac{5}{4},\ n = 2$$

$n = -\frac{5}{4}$: $-\frac{5}{4}\left(4\left(-\frac{5}{4}\right) - 3\right) = -\frac{5}{4}(-8) = 10$
$n = 2$: $2(4(2) - 3) = 2(8 - 3) = 2(5) = 10$
Both solutions make the equation true.

9. $l = 5w - 2, w = w, A = 39$

$$A = lw$$
$$39 = w(5w - 2)$$
$$39 = 5w^2 - 2w$$
$$0 = 5w^2 - 2w - 39$$
$$0 = (5w + 13)(w - 3)$$
$$5w + 13 = 0,\ w - 3 = 0$$
$$w = -\frac{13}{5},\ 3$$

The width cannot be a negative value, making $-\frac{13}{5}$ extraneous.
If the width is 3 yards, the length is 13 yards ($l = 5(3) - 2 = 13$).

10.

$$(x - 10)(x - 10) = 225$$
$$x^2 - 20x + 100 = 225$$
$$x^2 - 20x - 125 = 0$$
$$(x - 25)(x + 5) = 0$$
$$x - 25 = 0,\ x + 5 = 0$$
$$x = 25,\ -5$$

The side of the original sheet will be 25 inches. A negative side length is not possible; therefore, -5 is extraneous.

11. $d = 96,\ v = 16$

$$d = 16t^2 + vt$$
$$96 = 16t^2 + 16t$$
$$0 = 16t^2 + 16t - 96$$
$$0 = 16(t^2 + t - 6)$$
$$0 = 16(t + 3)(t - 2)$$
$$t + 3 = 0,\ t - 2 = 0$$
$$t = -3,\ 2$$

It cannot take -3 seconds for the object to hit the ground, so -3 is extraneous.

The object will take 2 seconds to hit the ground.

12. $d = 160,\ v = 24$

$$d = 16t^2 + vt$$
$$160 = 16t^2 + 24t$$
$$0 = 16t^2 + 24t - 160$$
$$0 = 8(2t^2 + 3t - 20)$$
$$0 = 8(2t - 5)(t + 4)$$
$$2t - 5 = 0,\ t + 4 = 0$$
$$t = \frac{5}{2},\ -4$$

It took the rock 2.5 seconds to hit the water. The number -4 is extraneous.

Practice 2

1. A: area, l: length, w: width

$$A = 63,\ l = 4w + 1,\ w = w$$
$$A = lw$$
$$63 = w(4w + 1)$$
$$63 = 4w^2 + w$$
$$0 = 4w^2 + w - 63$$

2. t: time, d: distance, v: velocity

$$d = 75,\ v = 8$$
$$d = 16t^2 + vt$$
$$75 = 16t^2 + 8t$$
$$0 = 16t^2 + 8t - 75$$

3. n: a number

$$87 = n(9n + 2)$$
$$87 = 9n^2 + 2n$$
$$0 = 9n^2 + 2n - 87$$

4. A: area, b: base, h: height

$$A = 37,\ b = 2h - 7,\ h = h$$
$$A = \frac{bh}{2}$$
$$37 = \frac{h(2h-7)}{2}$$
$$74 = 2h^2 - 7h$$
$$0 = 2h^2 - 7h - 74$$

5. A: area, l: length, w: width

$A = 500,\ l = 2x + 11,\ w = 2x$

$A = lw$

$500 = 2x(2x + 11)$

$500 = 4x^2 + 22x$

$0 = 4x^2 + 22x - 500$

6. x: first odd number, $(x + 2)$: next odd number

$x(x + 2) = 35$

$x^2 + 2x = 35$

$x^2 + 2x - 35 = 0$

7. $l = 3x - 1 + 3 + 3 = 3x + 5$

$w = x + 2 + 3 + 3 = x + 8$

$A = 170$

$A = lw$

$170 = (3x + 5)(x + 8)$

$170 = 3x^2 + 29x + 40$

$0 = 3x^2 + 29x - 130$

$0 = (3x - 10)(x + 13)$

$3x - 10 = 0,\ x + 13 = 0$

$x = \frac{10}{3} = 3\frac{1}{3},\ x = -13$

If –13 is used, the sides of the chicken coop will be negative. This is an extraneous solution.
The dimensions of the chicken coop are 9 feet by $5\frac{1}{3}$ feet.

$l = 3(\frac{10}{3}) - 1 = 9$

$w = (\frac{10}{3}) + 2 = 3\frac{1}{3} + 2 = 5\frac{1}{3}$

8. x: the number

$x(x + 1) = (x - 5)(3x + 7)$

$x^2 + x = 3x^2 - 8x - 35$

$0 = 2x^2 - 9x - 35$

$0 = (2x + 5)(x - 7)$

$2x + 5 = 0,\ x - 7 = 0$

$x = -\frac{5}{2},\ 7$

When $x = -\frac{5}{2}$, $x + 1 = -\frac{3}{2}$. The product of $-\frac{5}{2}$ and $-\frac{3}{2}$ is $\frac{15}{4}$.
Since $(-\frac{5}{2} - 5)(3(-\frac{5}{2}) + 7) = \frac{15}{4}$, $-\frac{5}{2}$ is a solution.
When $x = 7,\ x + 1 = 8$. This product is 56, and $(7 - 5)(3(7) + 7) = 56$ making 7 a solution.

9. $A = 15,\ l = 2w + 1,\ w = w,$

$$A = lw$$
$$15 = w(2w + 1)$$
$$15 = 2w^2 + w$$
$$0 = 2w^2 + w - 15$$
$$0 = (2w - 5)(w + 3)$$
$$2w - 5 = 0,\ w + 3 = 0$$
$$w = \frac{5}{2},\ -3$$
$$l = 2(\frac{5}{2}) + 1 = 5 + 1 = 6$$

The width cannot be a negative measure, making −3 an extraneous solution.
When $w = \frac{5}{2}$, or $2\frac{1}{2}$ feet, the length is 6 feet.

10. $A = 28$

$$l = 2x - 3 - 1 - 1 = 2x - 5$$
$$w = x - 1 - 1 = x - 2$$
$$A = lw$$
$$28 = (2x - 5)(x - 2)$$
$$28 = 2x^2 - 9x + 10$$
$$0 = 2x^2 - 9x - 18$$
$$0 = (2x + 3)(x - 6)$$
$$2x + 3 = 0,\ x - 6 = 0$$
$$x = -\frac{3}{2},\ 6$$

The value of x cannot be negative because this creates a negative side length; therefore, $-\frac{3}{2}$ is extraneous.
When $x = 6$, the dimensions of the metal sheet are 6 inches by 9 inches.
$2(6) - 3 = 9$

11. $d = 20, v = 4$

$$d = 16t^2 + vt$$
$$20 = 16t^2 + 4t$$
$$0 = 16t^2 + 4t - 20$$
$$0 = 4(4t^2 + t - 5)$$
$$0 = 4(4t + 5)(t - 1)$$
$$4t + 5 = 0,\ t - 1 = 0$$
$$t = -\frac{5}{4},\ 1$$

It will take the ball 1 second to reach the ground. The solution $-\frac{5}{4}$ is extraneous.

12. $d = 2,000 - 80 = 1,920$

$v = 32$

$d = 16t^2 + vt$

$1,920 = 16t^2 + 32t$

$0 = 16t^2 + 32t - 1,920$

$0 = 16(t^2 + 2t - 120)$

$0 = 16(t - 10)(t + 12)$

$t - 10 = 0,\ t + 12 = 0$

$t = 10,\ -12$

The object will take 10 seconds to hit the top of the building. The solution –12 is extraneous.

Targeted Review

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	20	20	24	24	24	24	24	20	25	22	24	24

1.

a. cubic binomial

b. quadratic trinomial

2.

$3p(2p - 5)(2p - 5)$

$3p(4p^2 - 20p + 25)$

$12p^3 - 60p^2 + 75p$

3.

$2x - 3 = 0 \quad 5x + 6 = 0$

$x = \frac{3}{2} \quad x = -\frac{6}{5}$

4.

$(x - 3)(x - 2) = 0$

$x - 3 = 0 \quad x - 2 = 0$

$x = 3 \quad x = 2$

5.

$(5a + 1)(5a + 1)$

6.

$8n(n^2 - 9)$

$8n(n - 3)(n + 3)$

7.

$(3m^2 - 1)(m^2 - 1)$

$(3m^2 - 1)(m - 1)(m + 1)$

8. $(5x^2 + 11x - 3) - (4x^2 - Qx + 7) = x^2 + 16x - 10$

$11x - (-Qx) = 16x$

$11x + Qx = 16x$

$11 + Q = 16$

$Q = 5$

9. D

A) $x^2 = 289$

B) $(x + 16)^2 = 289$

C) $(x - 8)^2 = 289$

D) $(x - 16)^2 = 289$

Distractor Rationale:

A) This does not remove the square corner.

B) This adds 16 more inches rather than subtracting it.

C) This only subtracts half of the square since each side is 8 inches.

10. B

A) 1 and 28

B) 2 and 14

C) 2 and 8

D) 4 and 7

Distractor Rationale:

A) This results in a product of 28 and a sum of 29.

C) This results in a product of 16 and a sum of 10

D) This results in a product of 28 and a sum of 11.

11. A

A) $x = -\frac{5}{2}, \frac{7}{6}$

B) $x = -\frac{7}{6}, \frac{5}{2}$

C) $x = -\frac{2}{5}, \frac{6}{7}$

D) $x = -\frac{6}{7}, \frac{2}{5}$

Distractor Rationale:

B) This answer is the opposite of both correct values.

C) This answer is the reciprocal of the correct values.

D) This answer is the opposite reciprocal of the correct values.

12. D

A) $(4x + 6)(x + 1)$

B) $2(2x^2 + 5x + 3)$

C) $2(2x + 1)(x + 3)$

D) $2(2x + 3)(x + 1)$

Distractor Rationale:

A) This is incorrect because the GCF is not factored out.

B) This is incorrect because the trinomial is not factored.

C) This is incorrect because the binomials do not multiply to the given expression.