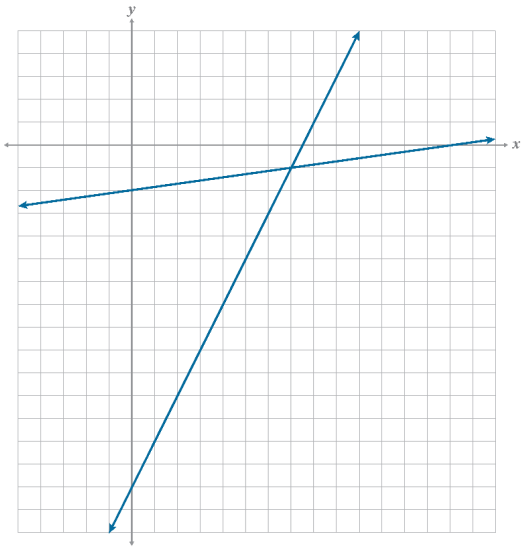
Part A: Efficiently Solving Systems

Practice 1

1.

solution: $(7, -1)$



2.

Substitution

$y = \frac{1}{7}x - 2$

$y = 2x - 15$

$(\frac{1}{7}x - 2) = 2x - 15$

Clear fractions

$7(\frac{1}{7}x - 2 = 2x - 15)$

$x - 14 = 14x - 105$

$-13x = -91$

$x = 7$

$y = \frac{1}{7}(7) - 2$

$y = 1 - 2$

$y = -1$

3.

Sample 1: Substitution was more efficient because a variable was already isolated.
Sample 2: Graphing was more efficient because the slope was a fraction, and it is quicker to graph than clearing the fractions in the equations.

4.

solution: $(3, -2)$

Eliminate y

$(-1)(2x + 3y = 0)$

$-2x - 3y = 0$

$+ \underline{-5x + 3y = -21}$

$-7x = -21$

$x = 3$

$2(3) + 3y = 0$

$6 + 3y = 0$

$3y = -6$

$y = -2$

5.

solution: $(-2, 4)$
Isolate y in the second equation. Then use substitution.

$y - 4 = x + 2$
$y = x + 6$
$y = 2x + 8$
$(x + 6) = 2x + 8$
$-2 = x$

$y = 2(-2) + 8$
$y = 4$

6.

Sample: Problem 4 is in standard form so the elimination method was used. The variable y can be eliminated first by multiplying one of the equations by -1 and then adding vertically. Problem 5 has an equation with an isolated variable. Substitution can be used to replace y with the expression $2x + 8$. The method used was determined by the setup of the given equations.

7.

solution: $(3, 5)$
$2x - y$
$2(3) - 5$
1

8. B $(4.5, 11)$

$$\begin{array}{r} x + 3y = 37.5 \\ +\ -x + 4y = 39.5 \\ \hline 7y = 77 \\ y = 11 \end{array}$$

$x + 3(11) = 37.5$
$x + 33 = 37.5$
$x = 4.5$

9. A $(-3, 4)$

$2\left(-\frac{3}{4}y\right) + 3y = 6$
$-\frac{3}{2}y + 3y = 6$
$\frac{3}{2}y = 6$
$y = 4$

$x = -\frac{3}{4}(4)$
$x = -3$

10.
$(4.5, 11)$
$x - y$
$4.5 - 11$
-6.5

11.
$(-3, 4)$
$|-3| - 3(4)$
$3 - 12$
-9

12.
Sample: Graphing is most efficient when equations are in slope-intercept form and the directions say to graph.

13.
Sample: Substitution is most efficient when the equation has a variable that is already isolated, or quickly isolated, and can replace a variable in the other equation provided.

14.
Sample: Elimination is most efficient when the equations are in standard form or the terms are aligned vertically and can be eliminated by multiplying one or both equations to find the LCM.

Practice 2

1.
Graphing this system would be very challenging because the y-intercepts are not integers. Unless the graph was rescaled to count by fifths it would be very difficult to accurately graph and find the intersection.

2.
solution: $(-\frac{1}{5}, -\frac{2}{5})$
Substitution

$y = 10x + \frac{8}{5}$
$y = -5x - \frac{7}{5}$
$10x + \frac{8}{5} = -5x - \frac{7}{5}$
$15x = -\frac{7}{5} - \frac{8}{5}$
$15x = -\frac{15}{5}$
$x = -\frac{3}{15} = -\frac{1}{5}$

$y = -5(-\frac{1}{5}) - \frac{7}{5}$
$y = \frac{5}{5} - \frac{7}{5}$
$y = -\frac{2}{5}$

3.
solution: no solution
Substitution

$$y = -2x - 11$$
$$2x + y = -11$$
$$2x + (-2x - 11) = -19$$
$$-11 = -19$$

No solution, parallel lines

4.
Sample: You can show what a solution looks like by graphing. For Problem 3, there would be a set of parallel lines on the graph, which proves there is no solution to the system.

5.

solution: (40, 40)
(c, d)

$(4)(c - d = 0) = 4c - 4d = 0$

$$4c - 4d = 0$$
$$+\ c + 4d = 200$$
$$5c = 200$$
$$c = 40$$

$(40) - d = 0$

$$d = 40$$

6.
Sample 1: Elimination was used because the equations are in standard form. Multiplying one of the equations by four will eliminate d from the equations.
Sample 2: Substitution was used because the top equation can quickly be rewritten as $c = d$. Then, c can be replaced by d, and the value of each variable can be found.

7. C

8. B

9. C

10.

solution: (13, 7)
Eliminate y

$(-1)(5x - y = 58)$

$$-5x + y = -58$$
$$+\ 8x - y = 97$$
$$3x = 39$$
$$x = 13$$

$5(13) - y = 58$

$$65 - y = 58$$
$$-y = -7$$
$$y = 7$$

11.

$$y - x$$
$$7 - 13$$
$$-6$$

12.

solution: $(-1, -1)$

$y = -2x - 3$
$4x + y = -5$
$4x + (-2x - 3) = -5$
$2x - 3 = -5$
$2x = -2$
$x = -1$

$y = -2(-1) - 3$
$y = 2 - 3$
$y = -1$

Part B: Applications of Linear Systems

Practice 1

1.

t: tomatoes, g: garlic
$8.5t + 0.5g = 26.75$
$12t = 30$

$12t = 30$
$t = \frac{30}{12} = 2.50$

$8.5(2.5) + 0.5g = 26.75$
$21.25 + 0.5g = 26.75$
$0.5g = 5.50$
$g = 11.00$

$t = 2.50$; $g = 11.00$
Tomatoes are \$2.50/pound and garlic is \$11/pound.

2.

x, y: grandmothers
$x + y = 167$
$x - y = 7$

$x + y = 167$
$+ x - y = 7$
$2x = 174$
$x = 87$

$(87) - y = 7$
$y = 80$

The ages of Balthazar's grandmothers are 80 and 87.

3.

j: jeans, h: shirt
$2j + 5h = 98$
$3j + 7h = 141$

$(-3)(2j + 5h = 98) = -6j - 15h = -294$
$(2)(3j + 7h = 141) = 6j + 14h = 282$
$-h = -12$
$h = 12$

$2j + 5(12) = 98$
$2j + 60 = 98$
$2j = 38$
$j = 19$

$h = 12$; $j = 19$
$j + h =$ \$31. It will cost \$31 to purchase a pair of jeans and a shirt.

4.

f: floor, m: mezzanine
$f + m = 525$
$m = 2f$

$m = 2f$
$f + m = 525$
$f + (2f) = 525$
$3f = 525$
$f = 175$

$(175) + m = 525$
$m = 350$

175 floor seats and 350 mezzanine seats were sold.

5.

M: Mark, S: Steven

$M + S = 3{,}974$

$S - M = 6$

$M + S = 3974$

$+ \underline{-M + S = \quad 6}$

$2S = 3800$

$S = 1990$

$(1990) - M = 6$

$M = 1984$

Mark was born in 1984 and Steven was born in 1990.

6.

x: 2-point questions, y: 10-point questions

$2x + 10y = 100$

$x + y = 30$

$(-10)(x + y = 30) = -10x - 10y = -300$

$\underline{+ 2x + 10y = 100}$

$-8x = -200$

$x = 25$

$(25) + y = 30$

$y = 5$

There were 5 questions worth 10 points each and 25 questions worth 2 points each.

7.

$3 - 2 = 1$ km/h

8.

$3 + 2 = 5$ km/h

9.

The total trip will take 6 hours. It will take 5 hours to travel upstream at a rate of 1 km/h. It will take 1 hour to travel 5 km back downstream: $5 + 1 = 6$ hours

10.

$6 - 3 = 3$ mph

11.

$6 + 3 = 9$ mph

12.

The total trip will take 8 hours. It will take 6 hours to travel upstream at a rate of 3 mph. It will take 2 hour to travel 18 miles back downstream: $6 + 2 = 8$

13.

d: dime, n: nickel

$d + n = 30$

$0.10d + 0.05n = 2.10$

$(-0.10)(d + n = 30) = -0.10d - 0.10n = -3.00$

$\underline{+ 0.10d + 0.05n = 2.10}$

$-0.05n = -0.90$

$n = 18$

$d + (18) = 30$

$d = 12$

There are 12 dimes and 18 nickels.

14.
r: letter stamp, p: postcard stamp
$0.55r + 0.35p = 38.00$
$p = r - 20$

$p = r - 20$
$0.55r + 0.35p = 38.00$
$0.55r + 0.35(r - 20) = 38.00$
$0.55r + 0.35r - 7 = 38.00$
$0.90r = 45.00$
$r = 50$

$p = (50) - 20$
$p = 30$

Calia bought 50 letter stamps and 30 postcard stamps.

15.
e: ten-dollar bill, w: twenty-dollar bill
$e + w = 20$
$10e + 20w = 330$

$(-10)(e + w = 20) = -10e - 10w = -200$
$+ 10e + 20w = 330$
$10w = 130$
$w = 13$

$e + (13) = 20$
$e = 7$

Logan has 7 ten-dollar bills and 13 twenty-dollar bills.

16.
p: plane, w: wind
$5.5 \text{ hr} = 5\frac{1}{2} = \frac{11}{2}\text{hr}$
$d = tr$

d	t	r (headwind or tailwind expression)
2,400	6	$(p - w)$ headwind
2,400	$\frac{11}{2}$	$(p + w)$ tailwind

$2{,}400 = 6(p - w)$
$2{,}400 = \frac{11}{2}(p + w)$

$(\frac{1}{6})(2{,}400 = 6(p - w)) = 400 = p - w$
$(\frac{2}{11})(2{,}400 = \frac{11}{2}(p + w)) = + 436.36 = p + w$
$836.36 = 2p$
$p = 418.18$

$436.36 = p + w$
$436.36 = (418.18) + w$
$w = 18.18$
The speed of the plane is about 418 mph, and the speed of the wind is 18 mph.

Practice 2

1.

x: multiple-choice questions, y: open-response questions

$x = 3y$

$x + y = 16$

$x = 3y$

$(3y) + y = 16$

$4y = 16$

$y = 4$

$x = 3(4)$

$x = 12$

There were 12 multiple-choice questions and 4 open-response questions.

2.

r: Tori, h: Thomas

$r - h = 3$

$\frac{1}{3}r - h = -5$

$(-1)(\frac{1}{3}r - h = -5) = + -\frac{1}{3}r + h = 5$

$r - h = 3$

$\frac{2}{3}r = 8$

$(\frac{3}{2})\frac{2}{3}r = (\frac{3}{2})(8)$

$r = 12$

$(12) - h = 3$

$h = 9$

Tori is 12 years old and Thomas is 9 years old.

3.

a: apples, p: spinach

$5a + 6p = 37.95$

$3a + 5p = 28.72$

$(3)(5a + 6p = 37.95) =$ $15a + 18p = 113.85$

$(-5)(3a + 5p = 28.72) = +$ $-15a - 25p = -143.60$

$-7p = -29.75$

$p = 4.25$

$3a + 5(4.25) = 28.72$

$3a + 21.25 = 28.72$

$3a = 7.47$

$a = 2.49$

One pound of apples costs \$2.49 and one bag of spinach costs \$4.25.

4.

b: brush, p: paint

$2b + 5p = \$52.25$

$b + 7p = \$58.75$

$(-2)(b + 7p = \$58.75) = -2b - 14p = -117.50$

$2b + 5p = \$52.25$

$-9p = -65.25$

$p = 7.25$

$b + 7(7.25) = 58.75$

$b + 50.75 = 58.75$

$b = 8.00$

(solving for b is not necessary to answer the question)

One tube of paint is \$7.25.

5.
b: brother, r: sister
$b + r = 4$
$b = 2r - 2$

$b = 2r - 2$
$b + r = 4$
$(2r - 2) + r = 4$
$3r = 6$
$r = 2$

$b = (2) = 4$
$b = 2$

The age of Magda's (twin) brother and sister are two-years old.

6.
a: adult, c: children
$25a + 16c = 219$
$c = a + 6$

$c = a + 6$
$25a + 16c = 219$
$25a + 16(a + 6) = 219$
$25a + 16a + 96 = 219$
$41a = 123$
$a = 3$

$c = (3) + 6$
$c = 9$

There were 9 meals purchased for children.

7.
d: dimes, n: nickels
$d + n = 18$
$0.10d + 0.05n = 1.25$

$(-0.10)(d + n = 18) = -0.10d - 0.10n = -1.8$
$+ 0.10d + 0.05n = 1.25$
$-0.05n = -0.55$
$n = 11$

$d + (11) = 18$
$d = 7$

Ella and Claire have 7 dimes and 11 nickels.

8.
$575 + 35 = 610$ mph

9.
$575 - 35 = 540$ mph

10.
2.5 hours; $\frac{1{,}525\ miles}{(610\ mph)} = 2.5$ hr

11.
3.25 hours; $\frac{1{,}755\ miles}{(540\ mph)} = 3.25$ hr

12.
p: boat, w: water
$d = tr$

$32 = 4(p - w)$
$32 = 4(p + w)$
$(\frac{1}{4})(32 = 4(p - w)) = \quad 8 = p - w$
$(\frac{1}{4})(32 = 4(p + w)) = \underline{+\ 8 = p + w}$
$16 = 2p$
$p = 8$

$8 = p + w$
$8 = (8) + w$
$w = 0$

The boat traveled at a speed of 8 km per hour. The water had a speed of 0 km per hour.

13.
p: plane, w: wind
$d = tr$
$3{,}500 = 8(p - w)$
$3{,}500 = 7(p + w)$
$(\frac{1}{8})(3{,}500 = 8(p - w)) = \quad 437.5 = p - w$
$(\frac{1}{7})(3{,}500 = 7(p + w)) = \underline{+\ 500 \quad = p + w}$
$937.50 = 2p$
$p = 468.75$

$500 = p + w$
$500 = (468.75) + w$
$w = 31.25$

The plane is traveling 468.75 mph.

14.
f: five-dollar bill, n: one-dollar bill
$f = 6n - 2$
$5f + 1n = 83$

$f = 6n - 2$
$5f + 1n = 83$
$5(6n-2) + 1n = 83$
$30n - 10 + 1n = 83$
$31n = 93$
$n = 3$

$f = 6(3) - 2$
$f = 16$

Fabian has 16 five-dollar bills and 3 one-dollar bills.

15.
d: dime, h: half-dollar
$0.10d + 0.50h = 4.40$
$0.50h - 0.10d = 2.60$

$0.50h + 0.10d = 4.40$
$\underline{+\ 0.50h - 0.10d = 2.60}$
$1.00h = 7.00$
$h = 7$

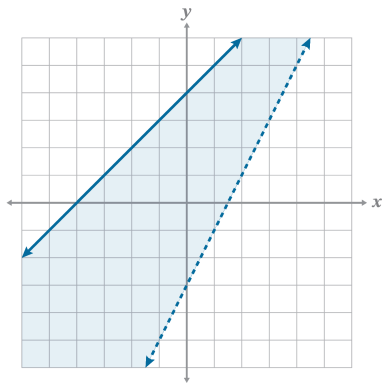
$0.10d + 0.50(7) = 4.40$
$0.10d + 3.50 = 4.40$
$0.10d = 0.90$
$d = 9$

There are 9 dimes and 7 half-dollars.

Targeted Review

Problem	1	2	3	4	5–8	9–13	14–17	18	19	20	21	22
Lesson Origin	15	15	16	16	1	12	5	1, 2	10	11	15	16

1.



2.

The solution to the system of inequalities is in all four quadrants.

3.

solution: $(-\frac{9}{2}, \frac{15}{2})$

$x = -\frac{1}{3}y - 2$

$x + y = 3$

$(-\frac{1}{3}y - 2) + y = 3$

$-\frac{1}{3}y + \frac{3}{3}y = 3 + 2$

$\frac{2}{3}y = 5$

$y = (5)(\frac{3}{2})$

$y = \frac{15}{2} = 7.5$

$x = -\frac{1}{3}(\frac{15}{2}) - 2$

$x = -\frac{5}{2} - \frac{4}{2}$

$x = -\frac{9}{2} = -4.5$

4.

solution: $(3, -4.5)$

Eliminate x

$(-1)(7x + 2y = 12)$

$-7x - 2y = -12$

$+ 7x + 4y = 3$

$2y = -9$

$y = -\frac{9}{2} = -4.5$

$7x + 2(-4.5) = 12$

$7x - 9 = 12$

$7x = 21$

$x = 3$

5.
$(-7)(-2.3)$: rational

6.
$(\sqrt{2})(½)$: irrational

7.
83 – 83: whole

8.
−26 + 3: integer

9.
$f(x)$ and $g(x)$ are _____ neither

10.
$g(x)$ and $j(x)$ are _____ perpendicular

11.
$h(x)$ and $j(x)$ are _____ neither

12.
$f(x)$ and $h(x)$ are _____ parallel

13.
$g(x)$ and $h(x)$ are _____ neither

14. 45 minutes: $\frac{45\ minutes}{60\ minutes} = \frac{3}{4}$ hour

15. 100 minutes: $\frac{100\ minutes}{60\ minutes} = \frac{5}{3}$ hour

16. 42 minutes: $\frac{42\ minutes}{60\ minutes} = \frac{7}{10}$ hour

17. 75 minutes: $\frac{75\ minutes}{60\ minutes} = \frac{5}{4}$ hour

18.

$4(9 - x) + 6x = 20$	
$36 - 4x + 6x = 20$	Distributive Property
$36 + 2x = 20$	combine like terms
$2x = -16$	Additive Property of Equality
$x = -8$	Multiplicative Property of Equality

19.
$d(t) = 55t$. A slope of 55 represents the speed of 55 mph. The y-intercept is 0 because at the start of Saul's trip he had not traveled anywhere yet.

20.

x-intercept	y-intercept
$3x - 15(0) = 45$	$3(0) - 15y = 45$
$3x = 45$	$-15y = 45$
$x = 15$	$y = -3$
$(15, 0)$	$(0, -3)$

21. B
A. $(-1, 1)$
B. $(-1, -1)$
C. $(1, -1)$
D. $(1, 1)$
Distractor Rationale:
A. is a point in Quadrant II
C. is a point in Quadrant IV
D. is a point in Quadrant I

22. D
A. −12
B. −4
C. 18
D. 77
Distractor Rationale:
A. This is the product of the constants in the given equations
B. This is the difference of x and y for (7, 11)
C. This is the sum of x and y for (7, 11)