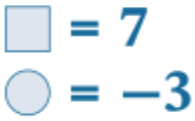
Part A: Solving Systems Using Substitution

Practice 1

1.
solution:

The value of one circle in the bottom equation, $-t + 11$, can be substituted for the two circles in the first equation. Then, the value of one triangle can be found. Then, the value of the circle can be found by substituting the value of the triangle into the bottom equation. (Your student may use variables instead of shapes if they prefer.)

2.
solution:



The value of one square in the bottom equation, $-c + 4$, can be substituted for the square in the top equation. Then, use this new equation to find the value of one circle. Then, take the value of the circle and solve for the value of the square in the bottom equation. (Your student may use variables instead of shapes if they prefer.)

3.
solution: (10, 9)

$y = 2x - 11$

$$y = \frac{1}{2}x + 4$$
$$2x - 11 = \frac{1}{2}x + 4$$
$$2(2x - 11 = \frac{1}{2}x + 4)$$
$$4x - 22 = 1x + 8$$
$$4x - 1x = 8 + 22$$
$$3x = 30$$
$$x = 10$$

$$y = 2(10) - 11$$
$$y = 20 - 11$$
$$y = 9$$

4.

solution: $(5\frac{2}{3}, \frac{1}{3})$

(a, b)

$a = 2b + 5$

$$\begin{aligned} a + b &= 6 \\ (2b + 5) + b &= 6 \\ 3b + 5 &= 6 \\ 3b &= 1 \\ b &= \frac{1}{3} \end{aligned}$$

$$a = 2(\tfrac{1}{3}) + 5$$

$$a = \frac{2}{3} + 5$$

$$a = 5\frac{2}{3} = \frac{17}{3}$$

5.

No solution, lines are parallel

$y = -3x - 2$

$$\begin{aligned} 3x + y &= -5 \\ 3x + (-3x - 2) &= -5 \\ 0 - 2 &= -5 \\ -2 &= -5 \end{aligned}$$

6.

solution: $(-2, -1)$

$y = x + 1$

$$\begin{aligned} x + 2y &= -4 \\ x + 2(x + 1) &= -4 \\ x + 2x + 2 &= -4 \\ 3x + 2 &= -4 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

$$y = -2 + 1$$

$$y = -1$$

7.

solution: $(3, \frac{3}{2})$

$y = \frac{1}{2}x$

$$\begin{aligned} x - 4y &= -3 \\ x - 4(\tfrac{1}{2}x) &= -3 \\ x - 2x &= -3 \\ -x &= -3 \\ x &= 3 \end{aligned}$$

$$y = \frac{1}{2}(3)$$

$$y = \frac{3}{2}$$

8.
solution: $(-7, -1)$

$x - 7y + 7y = 0 + 7y$
$x = 7y$

$3x + 8y = -29$
$3(7y) + 8y = -29$
$21y + 8y = -29$
$29y = -29$
$y = -1$

$x = 7(-1)$
$x = -7$

9.
solution: $(5, \frac{1}{2})$

$a - 8b + 8b = 1 + 8b$
$a = 8b + 1$

$2a - 10b = 5$
$2(8b + 1) - 10b = 5$
$16b + 2 - 10b = 5$
$6b = 3$
$b = \frac{3}{6} = \frac{1}{2}$

$a = 8(\frac{1}{2}) + 1$
$a = 4 + 1$
$a = 5$

10.
solution: $(-5, 18)$

$3x - 3x + y = 3 - 3x$
$y = -3x + 3$

$7x + 2y = 1$
$7x + 2(-3x + 3) = 1$
$7x - 6x + 6 = 1$
$x = 1 - 6$
$x = -5$

$y = -3(-5) + 3$
$y = 15 + 3$
$y = 18$

11.
solution: $(\frac{2}{5}, \frac{4}{5})$

(c, d)

$4c - 2d + 2d = 0 + 2d$
$4c = 2d$
$c = \frac{2}{4}d$
$c = \frac{1}{2}d$

$3c - 4d = -2$
$3(\frac{1}{2}d) - 4d = -2$
$\frac{3}{2}d - 4d = -2$
$2(\frac{3}{2}d - 4d = -2)$
$3d - 8d = -4$
$-5d = -4$
$d = \frac{4}{5}$

$4c - 2(\frac{4}{5}) = 0$
$4c - \frac{8}{5} = 0$
$4c = \frac{8}{5}$
$c = (\frac{8}{5})(\frac{1}{4}) = \frac{2}{5}$

12.
solution: $(0, -4)$

$y = 4x - 4$

$x = -\frac{1}{2}y - 2$

$x = -\frac{1}{2}(4x - 4) - 2$

$x = -2x + 2 - 2$

$x + 2x = 0$

$3x = 0$

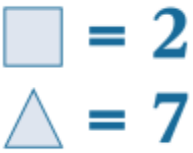
$x = 0$

$y = 4(0) - 4$

$y = -4$

Practice 2

1.
solution:



Replace each triangle in the bottom equation with two squares plus three. Solve for the value of a square. Use the value of a square to find the value of the triangle in either equation.

2.
solution: $(\frac{19}{10}, -\frac{3}{10})$

$x = \frac{1}{3}y + 2$

$2x - 4y = 5$

$2(\frac{1}{3}y + 2) - 4y = 5$

$\frac{2}{3}y + 4 - 4y = 5$

$3(\frac{2}{3}y - 4y = 5 - 4)$

$2y - 12y = 3$

$-10y = 3$

$y = -\frac{3}{10}$

$x = \frac{1}{3}(-\frac{3}{10}) + 2$

$x = -\frac{1}{10} + \frac{20}{10}$

$x = \frac{19}{10} = 1\frac{9}{10}$

3.
solution: $(74, 21)$
(a, b)
$a = 3b + 11$

$$a = 4b - 10$$
$$(3b + 11) = 4b - 10$$
$$-1b + 11 = -10$$
$$-1b = -21$$
$$b = 21$$

$$a = 3(21) + 11$$
$$a = 63 + 11$$
$$a = 74$$

4.
infinite solutions, coincident lines
$y = -\frac{5}{2}x - \frac{5}{2}$

$$5x + 2y = -5$$
$$5x + 2(-\frac{5}{2}x - \frac{5}{2}) = -5$$
$$5x - 5x - 5 = -5$$
$$-5 = -5$$

5.
solution: $(\frac{1}{4}, \frac{29}{4})$
$y = 9x + 5$

$$2x + 2y = 15$$
$$2x + 2(9x + 5) = 15$$
$$2x + 18x + 10 = 15$$
$$20x = 5$$
$$x = \frac{1}{4}$$

$$y = 9(\frac{1}{4}) + 5$$
$$y = \frac{9}{4} + \frac{20}{4}$$
$$y = \frac{29}{4} = 7\frac{1}{4}$$

6.
solution: $(-\frac{9}{2}, -\frac{3}{4})$
$m = 6n$

$$2m + 8n = -15$$
$$2(6n) + 8n = -15$$
$$12n + 8n = -15$$
$$20n = -15$$
$$n = -\frac{15}{20} = -\frac{3}{4}$$

$$m = 6(-\frac{3}{4})$$
$$m = -\frac{18}{4} = -\frac{9}{2}$$

7.
solution: $(2, -3)$
Solve for y in the top equation

$$4x - 4x + y = 5 - 4x$$
$$y = -4x + 5$$

$$2x - 3y = 13$$
$$2x - 3(-4x + 5) = 13$$
$$2x + 12x - 15 = 13$$
$$14x = 28$$
$$x = 2$$

$$4(2) + y = 5$$
$$8 + y = 5$$
$$y = -3$$

8.
solution: $(-\frac{1}{2}, -\frac{1}{2})$
(e, f)
Solve for e in the bottom equation

$$e - 3f + 3f = 1 + 3f$$
$$e = 3f + 1$$

$$3e - f = -1$$
$$3(3f + 1) - f = -1$$
$$9f + 3 - f = -1$$
$$8f = -4$$
$$f = -\frac{1}{2}$$

$$3e - (-\frac{1}{2}) = -1$$
$$3e + \frac{1}{2} = -1$$
$$3e = -1\frac{1}{2}$$
$$e = (-\frac{3}{2})(\frac{1}{3}) = -\frac{1}{2}$$

9. solution: $(-\frac{2}{3}, \frac{5}{3})$
(p, q)
Solve for q in the top equation

$$2p - q + q = -3 + q$$
$$2p = q - 3$$
$$q = 2p + 3$$

$$8p - q = -7$$
$$8p - (2p + 3) = -7$$
$$8p - 2p - 3 = -7$$
$$6p = -4$$
$$p = -\frac{4}{6} = -\frac{2}{3}$$

$$2(-\frac{2}{3}) - q = -3$$
$$-\frac{4}{3} - q = -3$$
$$-q = -\frac{9}{3} + \frac{4}{3}$$
$$-q = -\frac{5}{3}$$
$$q = \frac{5}{3}$$

10.
solution: $(7, -6)$
Solve for x

$$x + y - y = 1 - y$$
$$x = -y + 1$$

$$50x + 25y = 200$$
$$50(-y + 1) + 25y = 200$$
$$-50y + 50 + 25y = 200$$
$$-25y = 200 - 50$$
$$-25y = 150$$
$$y = -6$$

$$x + (-6) = 1$$
$$x = 7$$

11.
solution: $(-2, 5)$
(e, f)

$$e + 2f - 2f = 8 - 2f$$
$$e = -2f + 8$$

$$2e - 3f = -19$$
$$2(-2f + 8) - 3f = -19$$
$$-4f + 16 - 3f = -19$$
$$-7f = -19 - 16$$
$$-7f = -35$$
$$f = 5$$

$$e + 2(5) = 8$$
$$e + 10 = 8$$
$$e = -2$$

12.
no solution

$$5x - 4y + 4y = -1 + 4y$$
$$5x = 4y - 1$$
$$x = \frac{4}{5}y - \frac{1}{5}$$

$$4y + 6 = 5x + 5$$
$$4y + 6 = 5\left(\frac{4}{5}y - \frac{1}{5}\right) + 5$$
$$4y + 6 = 4y - 1 + 5$$
$$4y + 6 = 4y + 4$$
$$6 = 4$$

Part B: Solving Systems Using Substitution

Practice 1

1.
solution: (6, −2)
Eliminate x.

$$\begin{aligned} x + 2y &= 2 \\ +\ \underline{-x - 3y = 0} \\ -1y &= 2 \\ y &= -2 \end{aligned}$$

$$\begin{aligned} x + 2(-2) &= 2 \\ x - 4 &= 2 \\ x &= 6 \end{aligned}$$

2.
solution: (18, 48)
Write the bottom equation in standard form.
Eliminate u.

$$\begin{aligned} -u + v &= 30 \\ +\ \underline{u + v = 66} \\ 2v &= 96 \\ v &= 48 \end{aligned}$$

$$\begin{aligned} u + (48) &= 66 \\ u &= 18 \end{aligned}$$

3.
solution: (6, 3)
Eliminate a.

$$\begin{aligned} 3a - b &= 15 \\ +\ \underline{-3a - 3b = -27} \\ -4b &= -12 \\ b &= 3 \end{aligned}$$

$$\begin{aligned} 3a - (3) &= 15 \\ 3a &= 18 \\ a &= 6 \end{aligned}$$

4.
solution: (2, −1)
Eliminate x.
$(-1)(3x - 5y = 11)$

$$\begin{aligned} -3x + 5y &= -11 \\ +\ \underline{3x + 3y = 3} \\ 8y &= -8 \\ y &= -1 \end{aligned}$$

$$\begin{aligned} 3x + 3(-1) &= 3 \\ 3x - 3 &= 3 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

5.
solution: (−1, 2)
Eliminate x.
$(-1)(5x + 12y = 19)$

$$\begin{aligned} -5x - 12y &= -19 \\ +\ \underline{5x + 6y = 7} \\ -6y &= -12 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} 5x + 6(2) &= 7 \\ 5x + 12 &= 7 \\ 5x &= -5 \\ x &= -1 \end{aligned}$$

6.
solution:

△ = −3
○ = 8

7.
solution: (−15, −18)
Eliminate y, LCM(1, 3) = 3

$x - y = 3$
$-3(x - y = 3)$

$-3x + 3y = -9$
$+ 4x - 3y = -6$
$x = -15$

$(-15) - y = 3$
$-y = 3 + 15$
$y = -18$

8.
solution: (5, 2)
Eliminate x, LCM(1, 2) = 2

$x - 2y = 1$
$-2(x - 2y = 1)$

$-2x + 4y = -2$
$+ 2x + 3y = 16$
$7y = 14$
$y = 2$

$2x + 3(2) = 16$
$2x + 6 = 16$
$2x = 10$
$x = 5$

9.
solution: $(4, -\frac{14}{3})$
Eliminate y, LCM(6, 3) = 6
$(-2)(2x + 3y = -6)$

$-4x - 6y = 12$
$+ 5x + 6y = -8$
$x = 4$

$2(4) + 3y = -6$
$8 + 3y = -6$
$3y = -14$
$y = -\frac{14}{3}$

10.
solution: (−2, 2)
Eliminate x, LCM(6, 5) = 30

$(-5)(6x + 7y = 2)$
$-30x - 35y = -10$
$(6)(5x + 8y = 6)$

$30x + 48y = 36$
$+ -30x - 35y = -10$
$13y = 26$
$y = 2$

$6x + 7(2) = 2$
$6x + 14 = 2$
$6x = -12$
$x = -2$

11.
solution: $(0, -4)$
Eliminate e, LCM$(1, 4) = 4$
(e, f)
$(-4)(e + 3.5f = -14)$

$$-4e - 14f = 56$$
$$+\ 4e - 1.5f = 6$$
$$-15.5f = 62$$
$$f = -4$$

$$e + 3.5(-4) = -14$$
$$e - 14 = -14$$
$$e = 0$$

12.
solution: $(\frac{12}{7}, -\frac{5}{7})$
Write both in standard form
$(a + \frac{12}{5}c = 0)(5)$
$5a + 12c = 0$
$7a + 14c = 2$

Eliminate a, LCM$(5, 7) = 35$

$$(7)(5a + 12c = 0) = \quad 35a + 84c = 0$$
$$(-5)(7a + 14c = 2) = +\ -35a - 70c = -10$$
$$14c = -10$$
$$c = -\frac{10}{14} = -\frac{5}{7}$$

$$a = -\frac{12}{5}(-\frac{5}{7})$$
$$a = \frac{12}{7}$$

Practice 2

1.
solution: $(-4, 1)$
Eliminate x.
$(-1)(10x + 5y = -35)$

$$-10x - 5y = 35$$
$$+\ 10x + 6y = -34$$
$$y = 1$$

$$10x + 5(1) = -35$$
$$10x + 5 = -35$$
$$10x = -40$$
$$x = -4$$

2.
solution: $(22.5, -1.25)$
Eliminate n.

$$m - 2n = 25$$
$$+\ m + 2n = 20$$
$$2m = 45$$
$$m = 22.5$$

$$(22.5) + 2n = 20$$
$$2n = -2.5$$
$$n = -1.25$$

3.
solution: $(-2.5, -5)$
Eliminate y.

$$\begin{aligned} -0.6x + y &= -3.5 \\ +\ x - y &= 2.5 \\ 0.4x &= -1 \\ x &= -2.5 \end{aligned}$$

$$\begin{aligned} (-2.5) - y &= 2.5 \\ -y &= 5 \\ y &= -5 \end{aligned}$$

4.
solution: $(6, -2)$
Eliminate y.

$$\begin{aligned} x + y &= 4 \\ +\ x - y &= 8 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} (6) + y &= 4 \\ y &= -2 \end{aligned}$$

5.
solution: $(\frac{8}{3}, -\frac{20}{3})$
Write the bottom equation in standard form.

$$\begin{aligned} y &= -12 + 2x \\ -2x + y &= -12 \\ 4x + y &= 4 \end{aligned}$$

Eliminate y.

$$\begin{aligned} (-1)(-2x + y &= -12) \\ 2x - y &= 12 \\ 4x + y &= 4 \\ 6x &= 16 \\ x &= \frac{16}{6} = \frac{8}{3} \end{aligned}$$

$$\begin{aligned} y &= -12 + 2(\tfrac{8}{3}) \\ y &= -\tfrac{36}{3} + \tfrac{16}{3} \\ y &= -\tfrac{20}{3} \end{aligned}$$

6.
solution:

△ = 14
□ = 4

7.
solution: $(6, \frac{1}{3})$

Eliminate a, LCM(2, 1) = 2

$$\begin{aligned} -2(a - 6b &= 4) \\ -2a + 12b &= -8 \\ +\ 2a - 15b &= 7 \\ -3b &= -1 \\ b &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} a - 6\left(\tfrac{1}{3}\right) &= 4 \\ a - 2 &= 4 \\ a &= 6 \end{aligned}$$

8.
solution: $(3, 1)$

Eliminate x, LCM$(1, 3) = 3$
$(-3)(x + 3y = 6)$
$-3x - 9y = -18$
$+ \underline{3x + 4y = 13}$
$-5y = -5$
$y = 1$

$x + 3(1) = 6$
$x + 3 = 6$
$x = 3$

9.
solution: $(-39, -25)$
Write in standard form. (m, p)
$3m - 5p = 8$
$-2m + 3p = 3$

Eliminate m, LCM$(3, 2) = 6$
$(2)(3m - 5p = 8) = 6m - 10p = 16$
$(3)(-2m + 3p = 3) = \underline{-6m + 9p = 9}$
$-p = 25$
$p = -25$

$3m = 5(-25) + 8$
$3m = -125 + 8$
$3m = -117$
$m = -39$

10.
solution: $(3, 5)$

Eliminate d, LCM$(1, 8.5) = 8.5$
(c, d)
$(-8.5)(c + d = 8)$
$-8.5c - 8.5d = -68$
$+ \underline{10.5c + 8.5d = 74}$
$2c = 6$
$c = 3$

$(3) + d = 8$
$d = 5$

11.
solution: $(4.5, -1.5)$

Eliminate x, LCM$(2, 4) = 4$
$(2)(2x + y = 7.5)$
$4x + 2y = 15$
$\underline{-4x + 2.5y = -21.75}$
$4.5y = -6.75$
$y = -1.5$

$2x + (-1.5) = 7.5$
$2x = 9$
$x = 4.5$

12.
solution: (1.5, 2)

Eliminate g, LCM(3, 5) = 15

$(-5)(16b + 3g = 30) = -80b - 15g = -150$

$(3)(22b + 5g = 43) = \underline{+\ 66b + 15g = 129}$

$-14b = -21$

$b = \frac{-21}{-14} = \frac{3}{2} = 1.5$

$16(\frac{3}{2}) + 3g = 30$

$24 + 3g = 30$

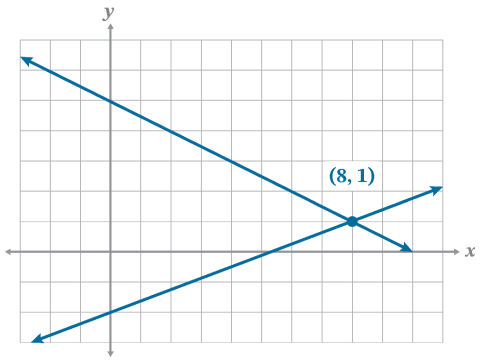
$3g = 6$

$g = 2$

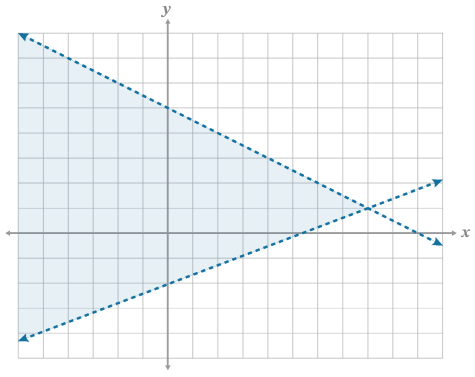
Targeted Review

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	15	15	15	15	2	7	14	10	2	11	15	15

1. solution: (8, 1)



2.



3.
Sample: The ordered pair where the lines intersect is the solution to a system of equations. To check this solution, the ordered pair needs to be substituted into both equations to check that they are both true.

4.
Sample: Systems of inequalities need to have solid or dashed lines and also need to have the region where all solutions are true shaded. Systems of equations will always have solid lines and can have zero, one, or infinitely many solutions on the line.

5.

$$\frac{2}{3}(5x + 8) = \frac{1}{15}$$
$$(15)(\frac{2}{3}(5x + 8) = \frac{1}{15})$$
$$10(5x + 8) = 1$$
$$50x + 80 = 1$$
$$50x = -79$$
$$x = -\frac{79}{50}$$

6.
□(−6, 0) No, $3(-6) - 8(0) \neq 48$
☑(8, −3) Yes, $3(8) - 8(-3) = 48$
□(16, −6) No, $3(16) - 8(-6) \neq 48$
☑(−8, −9) Yes, $3(-8) - 8(-9) = 48$

7.
Point-slope form, (day, money)
(11, 19.25), $m = 1.75$
$y - 19.25 = 1.75(x - 11)$

8.
$y = 0.32x + 85$
The slope is 0.32, which represents the cost per mile driven.
The y-intercept is 85. This represents an initial fee that the family paid (this could also be insurance).

9.
$P = 2l + 2w$; w

$$P - 2l = 2w$$
$$(\frac{1}{2})(P - 2l) = (\frac{1}{2})(2w)$$
$$\frac{P}{2} - l = w$$
$$w = \frac{P}{2} - l$$

10.

$$y - \frac{3}{5} = \frac{8}{3}(x + 9)$$
$$y - \frac{3}{5} = \frac{8}{3}x + 24$$
$$-\frac{8}{3}x + y = 24 + \frac{3}{5}$$

LCD(3, 5) = 15

$$(-15)(-\frac{8}{3}x + y = \frac{123}{5})$$
$$40x - 15y = -369$$

11. D
A. (0, 1)
B. (0, −6)
C. (0, 2)
D. (2, 0)

Distractor Rationale:
A. and B. are the y-intercepts for the lines.
C. contains the correct values but the ordered pair is not in the correct order.

12.
☑ one solution
☑ no solution
☑ infinite solutions
□ all real numbers

Distractor Rationale:
All real numbers are not a solution to the system because a solution must be a point on the line(s).