

Lesson 14: Types of Functions and Arithmetic Sequences Worked Solutions

Part A: Continuous and Discrete Functions

Practice 1

1. C

domain: $[0, 4)$

range: $[-2, 2]$

2. A

domain: $(-\infty, \infty)$

range: $(-\infty, 0]$

3. B

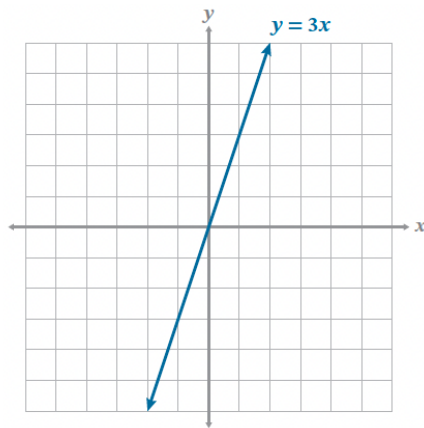
domain: $[-2, 4]$

range: $[-3, 3]$

4.

No, Graph 3 fails the vertical line test so it is not a function. Graphs 1 and 2 are functions because they pass the vertical line test. This refers back to Lesson 7.

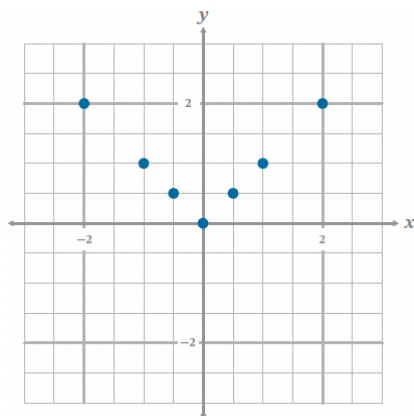
5.



domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

6.



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domain: $\{-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2\}$

range: $\{0, \frac{1}{2}, 1, 2\}$

7.

Continuous: the domain is continuous over the interval.

8.

Discrete: The domain is a set of points. The graph will also be a set of points.

9.

Continuous: The domain is time, and time is continuous.

10.

Discrete: plants are a counted value.

11.

slope intercept form (hour, inch) $m = \frac{1}{2}$, $b = 2$

$$y = \frac{1}{2}x + 2$$

This is a continuous function because snow can be measured at any time and can be any fraction of an inch in height.

12.

Point-slope, (minutes, boxes) (8, 12), (28, 42)

$$m = \frac{(42 - 12)}{(28 - 8)} = \frac{30}{20} = \frac{3}{2}$$

$$y - 42 = \frac{3}{2}(x - 28)$$

This is a discrete function because the whole box needs to be packed before it can be shipped.

13.

Slope-intercept, (hour, repair cost), $m = 49$, $b = 175$

$$y = 49x + 175$$

This is a continuous function because the shop can charge for part of an hour to complete the repairs.

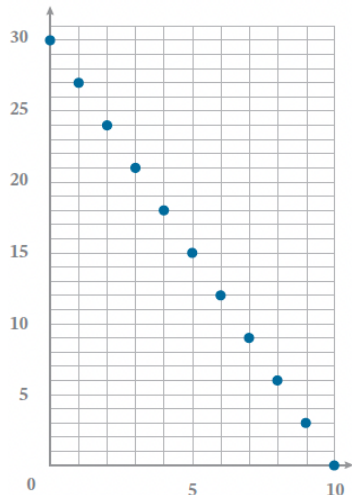
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14.

Slope-intercept form (day, money spent), $b = 30$, $m = -3$

$$y = -3x + 30$$

This is a discrete function because he is spending an exact amount each day and you cannot buy a part of a cup of coffee.



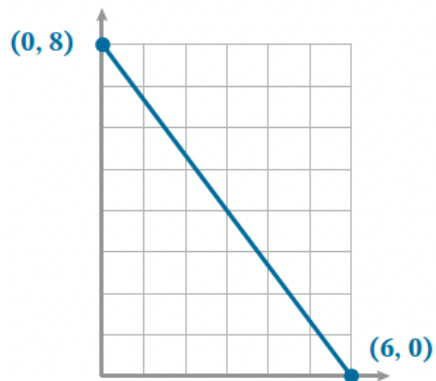
Hector will buy his last cup of coffee on day 10. The x-intercept is (10,0) which means on day 10, Hector has \$0.

15.

standard form (pound, \$) $g =$ grapes, $a =$ apples

$$4g + 3a = 24$$

This is a continuous function because you can purchase any combination of apples and grapes that fall on the graphed line.



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Practice 2

1. B

domain: $[-2, 2]$

range: $[-1, 3]$

2. C

domain: $(-\infty, 0]$

range: $(-\infty, \infty)$

3. A

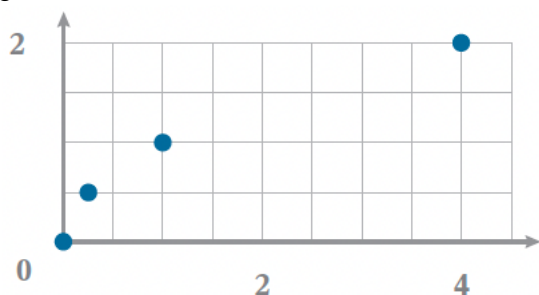
domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

4.

No, Graph B (Q1) and C (Q2) fail the vertical line test so it is not a function. Graph A (Q3) is a function because it passes the vertical line test.

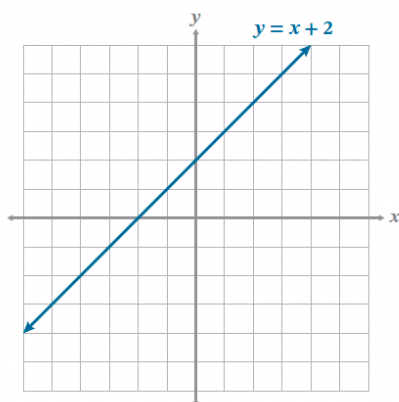
5.



domain: $\{0, \frac{1}{4}, 1, 4\}$

range: $\{0, \frac{1}{2}, 1, 2\}$

6.



domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

7.

Discrete: The domain is a set of points. The graph will also be a set of points.

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8.

Continuous: the domain is continuous over the interval.

9.

Continuous: Partial speeds (47.62 mph) are possible.

10.

Discrete: Amount of questions can be counted.

11.

Slope-intercept (people, money), $m = 25$, $b = 500$

$$y = 25x + 500$$

12.

(40, y)

$$y = 25(40) + 500$$

$$y = 1000 + 500$$

$$y = 1500$$

The cost for 40 people will be \$1500.

13.

Slope-intercept, (minutes, elevation in feet), $b = 14,360$ or $(0, 14360)$, $(75, 3110)$

$$m = \frac{(14,360 - 3,110)}{(0 - 75)} = \frac{11,250}{-75} = -150$$

$$y = -150x + 14,360$$

14.

The rate of change means that they descend 150 feet every minute.

-150 feet per minute

15.

Standard form, (songs, movies)

$$x + 5y = 24$$

(0, y)

$$0 + 5y = 24$$

$$5y = 24$$

$$y = \frac{24}{5} = 4.8$$

Brooke can purchase a maximum of 4 movies if she purchases 0 songs. This needs to be rounded down because 5 movies would be \$25, which is over her budget.

16.

(9, 3)

$$(9) + 5(3) = 24$$

$$9 + 15 = 24$$

Brooke would be able to purchase this combination of songs and movies because it is exactly \$24.

17.

Point-slope, (day, money), $m = 17$, $(9, 283)$

$$y - 283 = 17(x - 9)$$

This is a discrete function because you have to rent a car for a number of days.

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18.

The fee would be the y-intercept because this is the starting value for the equation.

$(0, y)$

$$y - 283 = 17(0 - 9)$$

$$y = 17(-9) + 283$$

$$y = 130$$

The rental fee is \$130.

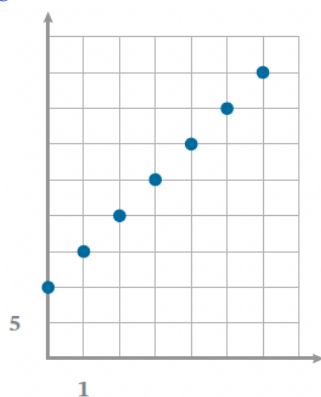
19.

This is a discrete function because fees are only charged once a month.

20.

Slope-intercept form, (month, \$ money), $m = 5$, $b = 10$

$$y = 5x + 10$$



21.

The second quadrant has negative x -coordinates. The variable x represents time in months, and it is not possible to go backwards in time to join a gym.

22.

$(x, 65)$

$$65 = 5x + 10$$

$$55 = 5x$$

$$x = 11$$

Nicole would not be able to have a full year gym membership because there are 12 months in a year and her budget only allows for 11 months.

Part B: Arithmetic Sequences

Practice 1

1. not arithmetic

2. arithmetic,

$$d = 2, a_5 = 10, a_6 = 12, a_7 = 14$$

3. arithmetic,

$$d = -0.5, a_5 = 4, a_6 = 3.5, a_7 = 3$$

4. not arithmetic

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5.

$$a_1 = 2$$

$$d = -3$$

$$a_n = 2 - 3(n - 1)$$

$$a_5 = 2 - 3(5 - 1) = 2 - 3(4) = -10$$

$$a_{10} = 2 - 3(10 - 1) = 2 - 3(9) = -25$$

$$a_{12} = 2 - 3(12 - 1) = 2 - 3(11) = -31$$

6.

$$a_1 = -5$$

$$d = \frac{1}{3}$$

$$a_n = -5 + \frac{1}{3}(n - 1)$$

$$a_5 = -5 + \frac{1}{3}(5 - 1) = -5 + \frac{1}{3}(4) = -5 + \frac{4}{3} = -\frac{11}{3} \text{ or } -3\frac{2}{3}$$

$$a_{10} = -5 + \frac{1}{3}(10 - 1) = -5 + \frac{1}{3}(9) = -2$$

$$a_{12} = -5 + \frac{1}{3}(12 - 1) = -5 + \frac{1}{3}(11) = -5 + \frac{11}{3} = -\frac{4}{3} \text{ or } -1\frac{1}{3}$$

7.

$$\{1, -1, -3, -5\}$$

8.

$$\{1, 2, 3, 4\}$$

9.

$$a_1 = 1, d = -2$$

$$a_n = 1 - 2(n - 1)$$

10.

$$a_n = 1 - 2(n - 1)$$

$$a_n = 1 - 2n + 2$$

$$a_n = -2n + 3$$

$$y = -2n + 3$$

$$f(x) = -2n + 3$$

11.

$$a_n = a_1 + d(n - 1)$$

$$5 = a_1 + \frac{1}{3}(7 - 1)$$

$$5 = a_1 + \frac{1}{3}(6)$$

$$5 = a_1 + 2$$

$$a_1 = 3$$

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12.

$$d = -4, a_{11} = -2$$

$$a_n = a_1 + d(n - 1)$$

$$-2 = a_1 - 4(11 - 1)$$

$$-2 = a_1 - 4(10)$$

$$-2 = a_1 - 40$$

$$a_1 = 38$$

13.

{2, 5, 8, ...}

14.

$$a_1 = 2, d = 3$$

$$a_n = 2 + 3(n - 1)$$

15.

$$a_{12} = 2 + 3(12 - 1)$$

$$a_{12} = 2 + 3(11)$$

$$a_{12} = 35$$

16.

$$a_n = 32, d = 32$$

$$a_n = 32 + 32(n - 1)$$

17.

$$a_{15} = 32 + 32(15 - 1)$$

$$a_{15} = 32 + 32(14)$$

$$a_{15} = 480$$

Eliza will have moved 480 pounds total.

Practice 2

1.

not arithmetic

2.

not arithmetic

3.

arithmetic,

$$d = -1, a_5 = -5, a_6 = -6, a_7 = -7$$

4.

arithmetic,

$$d = -5, a_5 = 0, a_6 = -5, a_7 = -10$$

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5.

$$a_n = 26 - \frac{1}{2}(n - 1)$$

$$a_5 = 26 - \frac{1}{2}(5 - 1) = 26 - \frac{1}{2}(4) = 24$$

$$a_{10} = 26 - \frac{1}{2}(10 - 1) = 26 - \frac{1}{2}(9) = \frac{43}{2} \text{ or } 21\frac{1}{2}$$

$$a_{12} = 26 - \frac{1}{2}(12 - 1) = 26 - \frac{1}{2}(11) = \frac{41}{2} \text{ or } 20\frac{1}{2}$$

6.

$$a_n = -5 + 4(n - 1)$$

$$a_5 = -5 + 4(5 - 1) = -5 + 4(4) = 11$$

$$a_{10} = -5 + 4(10 - 1) = -5 + 4(9) = 31$$

$$a_{12} = -5 + 4(12 - 1) = -5 + 4(11) = -39$$

7.

$$a_1 = 201, d = 7, a_n = 264$$

$$a_n = a_1 + d(n - 1)$$

$$a_n = 201 + 7(n - 1)$$

$$264 = 201 + 7n - 7$$

$$264 = 7n + 194$$

$$70 = 7n$$

$$n = 10$$

$$a_{10} = 264$$

8.

$$d = 22, a_1 = 13,426$$

$$a_n = 13,426 + 22(45 - 1)$$

$$a_n = 13,426 + 22(44)$$

$$a_n = 14,394$$

9.

$$d = -0.25, a_{21} = 6$$

$$6 = a_1 - 0.25(21 - 1)$$

$$6 = a_1 - 0.25(20)$$

$$6 = a_1 - 5$$

$$a_1 = 11$$

10.

$$d = 4, a_1 = 10$$

$$a_n = 10 + 4(n - 1)$$

11.

$$a_n = 10 + 4(n - 1)$$

$$a_n = 10 + 4n - 4$$

$$a_n = 4n + 6$$

$$y = 4n + 6$$

12.

Both d and m show the change in y over the change in x , or the slope.

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13.

{see student work, terms should increase by $\frac{1}{5}$ each time}

14.

$a_n = a_1 + \frac{1}{5}(n - 1)$, a_1 will be determined by the sequence your student wrote in part a.

15.

Your student should substitute 9 for n and solve for a_n .

Targeted Review

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---------------|----|----|----|----|---|----|----|----|----|----|----|----|
| Lesson Origin | 10 | 11 | 10 | 11 | 7 | 12 | 10 | 10 | 13 | 7 | 13 | 10 |

1.

$$m = \frac{1 - (-2)}{-3 - (-4)} = \frac{3}{1} = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - (-3))$$

$$y = 3x + 9 + 1$$

$$y = 3x + 10$$

2.

$$-3y = -2x - 6$$

$$y = -\frac{1}{3}(-2x - 6)$$

$$y = \frac{2}{3}x + 2$$

3.

Tripp is the faster painter. He paints pictures three times faster than Rook.

4.

$x = 4$ (vertical), $y = 5$ (horizontal)

5.

Yes, for every input (x) there is only one output (y).

6.

$$y = -\frac{1}{4}x + 11, \text{ through } (-3, 8);$$

$$m = -\frac{1}{4} \perp m = 4, (-3, 8)$$

$$y - 8 = 4(x - (-3))$$

$$y - 8 = 4x + 12$$

$$y = 4x + 20$$

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7.

(week, money remaining), $m = -25$, $b = 500$

Jimena was given \$500 to start the semester (0, 500). She spends \$25 per week on lunch or

$$m = \frac{-25}{1}.$$

8.

Point-slope form, (weeks, money saved)

$$m = 20, (6, 520)$$

$$y - 520 = 20(x - 6)$$

9. D

A. $r = -0.91$

B. $r = -0.42$

C. $r = 0.43$

D. $r = 0.91$

Distractor Rationale:

A and B represent a negative correlation when the graph shows a positive correlation.

B and C represent a weak correlation when this is a strong correlation.

10. A

A. (2, -3)

B. (2, 1)

C. (2, 0)

D. (2, 3)

Distractor rationale:

B (common error: taking the constants to form an ordered pair)

C (common error: thinking the coefficient of x will be the correct solution)

D (common error: not checking the signs of the values)

(2, -3): $2(2) + (-3) = 1$ $4 - 3 = 1$ $1 = 1$

(2, 1): $2(2) + 1 = 1$ $4 + 1 = 1$ 5 not equal 1

(2, 0): $2(2) + 0 = 1$ 4 not equal 1

(2, 3): $2(2) + 3 = 1$ $4 + 3 = 1$ 7 not equal 1

11. A

A. $y = -0.1x + 1.15$

B. $y = -10x + 1.15$

C. $y = 0.1x + 1.15$

D. $y = 10x + 15$

Distractor Rationale:

B and D have slopes too large for the scale of the graph.

C and D represent positive slopes when the scatter plot has negative correlation.

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12. B

A. $y = \frac{4}{3}x$

B. $y = \frac{3}{4}x$

C. $y = -\frac{4}{3}x$

D. $y = -\frac{3}{4}x$

Distractor Rationale:

A and C incorrectly calculate slope as run over rise.

C and D represent a negatively sloped function.

Using the points (0, 0) and (4, 3)

$$m = \frac{(3-0)}{(4-0)} = \frac{3}{4}$$

$$b = 0$$

$$y = \frac{3}{4}x$$