Lesson 30
Exponential Functions

Outline

Part A
Modeling Exponential Functions

- Graphs of Exponential Functions
- Domain and Range of Exponential Functions
- Common Ratios for Exponential Functions

Part B
The Growth/Decay Formula

- Components of the Growth/Decay Formula
- Solving with the Growth/Decay Formula

Targeted Review

Vocabulary

- asymptote

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Reviewing this will help your students understand that a and k will affect an exponential function in a similar fashion.

30A EXPLORE

Part A: Modeling Exponential Functions

Objectives

In this part of the lesson, you will learn about modeling exponential functions.

By the end of this lesson, you will be able to do the following:

- Graph an exponential function from a table.
- Determine the domain and range of an exponential function.
- Find the common ratio for an exponential function from a table.

Why?

Representing exponents graphically shows how quickly something is increasing or decreasing. This can be helpful in many real-world scenarios.

Warm Up

Use the formula for a parabola in vertex form: $y = a(x - h)^2 + k$ to describe the transformations.

1) Describe what happens to the graph of a parabola when the value of k is changed.

Sample:
Changing k will shift the graph vertically. If k is positive, the parabola will move up, and if k is negative, the parabola will move down.

2) Describe what happens to the graph of a parabola when the value of a is negative.

When a is negative, the parabola will be reflected over the x-axis.

Graphs of Exponential Functions

- Exponential functions are **nonlinear** and increase or decrease rapidly (or exponentially).
- When exponential functions are graphed, the coordinate plane is often **scaled**.
- Exponential functions are written in the form: $y = ab^x + k$.
  - x and y represent the **coordinate plane**.
  - a is the coefficient and helps determine the **direction** and the **y-intercept** when $k = 0$.
  - b, the base, is the **common ratio** between y-coordinates, or range values. $b > 0$ because negative bases are not exponential functions.
  - k, the constant, will shift the graph **vertically**. It is also used to help determine the **range** of the function.

EXPLORE 30A

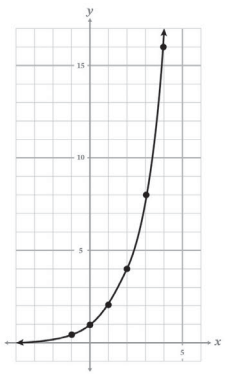
- Using a **table** will help you compare the numerical values as ordered pairs and determine the size and scale of the coordinate plane.
- The y-intercept will be **$(0, a + k)$** because $b^0 = 1$ and $ab^0 + k = a \cdot 1 + k = a + k$.

Example 1

Complete the table for the exponential function. Then graph it on the coordinate plane.

$y = 2^x$ $\quad a = 1, b = 2, k = 0$

x	$y = 2^x + k$	y
-1	$2^{-1} = \frac{1}{2^1}$	$\frac{1}{2}$
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8
4	2^4	16

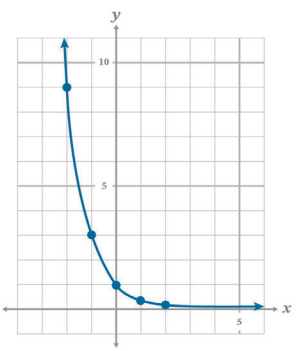


Example 2

Complete the table for the exponential function. Then graph it on the coordinate plane.

$y = \left(\frac{1}{3}\right)^x$ $\quad a = 1, b = \frac{1}{3}, k = 0$

x	$y = \left(\frac{1}{3}\right)^x$	y
-4	$\left(\frac{1}{3}\right)^{-4} = 3^4$	81
-3	$\left(\frac{1}{3}\right)^{-3} = 3^3$	27
-2	$\left(\frac{1}{3}\right)^{-2} = 3^2$	9
-1	$\left(\frac{1}{3}\right)^{-1} = 3^1$	3
0	$\left(\frac{1}{3}\right)^0$	1
1	$\left(\frac{1}{3}\right)^1$	$\frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2$	$\frac{1}{9}$



30A EXPLORE

- When the value of b is between **zero and one ($0 < b < 1$)**, the y-coordinates will be larger in the second quadrant rather than the first quadrant.
- This is because the equation in Example 2 could also be written as $y = 3^{-x}$ because a **negative** exponent represents the **reciprocal** of the base.
- Recall that when graphing quadratic equations where a is less than zero, the parabola **reflects** over the x-axis. The same is true for exponential functions when **a is less than zero**.

Example 3

Match the equations to the given graphs. Name the value for a, b, k, and the y-intercept.

A) $y = 2 \cdot 3^x$

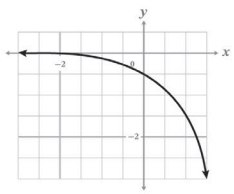
B) $y = -2 \cdot 3^x$

C) $y = \frac{1}{2} \cdot 3^x$

D) $y = -\frac{1}{2} \cdot 3^x$

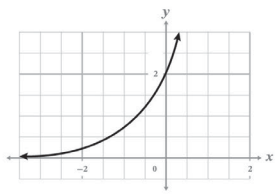
Two of the graphs are in the 3rd and 4th quadrants. These match with the equations when $a < 0$ because the graph is reflected over the x-axis. Since all of the values of b are equal to 3, the y-intercept needs to be determined to finish matching.

D 1)



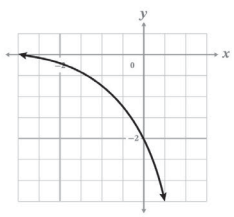
$(0, -0.5)$
$a = -\frac{1}{2}, b = 3, k = 0$

A 2)



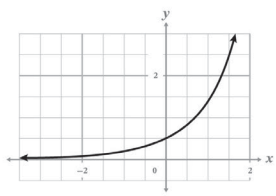
$(0, 2)$
$a = 2, b = 3, k = 0$

B 3)



$(0, -2)$
$a = -2, b = 3, k = 0$

C 4)



$(0, 0.5)$
$a = \frac{1}{2}, b = 3, k = 0$

EXPLORE 30A

Checkpoint

Name the value for a, b, k, and the y-intercept. Complete the table for the exponential function.

$y = 3 \cdot 2^x$

$a = 3, b = 2, k = 0$

$(0, 3)$

x	$y = 3 \cdot 2^x$	y
-2	$3 \cdot 2^{-2} = 3 \cdot \frac{1}{2^2} = 3 \cdot \frac{1}{4}$	$\frac{3}{4}$
-1	$3 \cdot 2^{-1} = 3 \cdot \frac{1}{2^1}$	$\frac{3}{2}$
0	$3 \cdot 2^0 = 3 \cdot 1$	3
1	$3 \cdot 2^1 = 3 \cdot 2$	6
2	$3 \cdot 2^2 = 3 \cdot 4$	12
3	$3 \cdot 2^3 = 3 \cdot 8$	24

Domain and Range of Exponential Functions

- Similar to all continuous functions in Algebra 1, exponential functions have a domain of ___$\mathbb{R}$___.
  - This means that any x-value can be selected and a corresponding ___**y-value**___ can be determined.
- For the equation $y = ab^x + k$, the ___**range**___ is determined by the value of ___k___.
  - The function gets closer and closer to k but ___**never**___ reaches it.
  - This line that the graph approaches but does not reach is called the ___**asymptote**___.
- Most often, the range for an exponential function is written as an ___**inequality**___.
- It is helpful to see the ___**graph**___ of the function to determine the domain and range because you can see what value the function approaches but never reaches.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

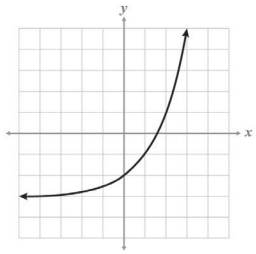
Your student can use technology to help complete the table, however completing at least this example by hand and showing the work will help connect the exponent rule to exponential functions.

30A EXPLORE

Example 4

Identify a, b, k, and the y-intercept. Determine the domain and range of the function.

Plan Use technology to create a graph to determine the domain and range.



$y = 2^x - 3$

$\boldsymbol{a = 1, b = 2, k = -3}$

Implement

$(0, a + k)$
$1 + -3 = -2$
$(0, -2)$

Domain: $\mathbb{R}$, $(-\infty, \infty)$

Range: $y > -3$ (or k)

Asymptote: $y = -3$

Example 5

Identify a, b, k, and the y-intercept. Determine the domain and range of the function.

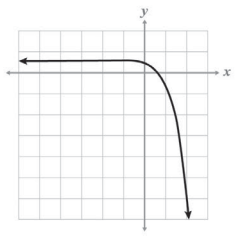
$y = -\frac{1}{3} \cdot 5^x + 1$

$\boldsymbol{a = -\frac{1}{3}, b = 5, k = 1}$

$\boldsymbol{(0, a + k)}$

$\boldsymbol{-\frac{1}{3} + 1 = \frac{2}{3}}$

$\boldsymbol{\left(0, \frac{2}{3}\right)}$



Domain: all real numbers, $\mathbb{R}$

Range: $y < 1$

Using technology to graph, we can see that this function is decreasing. As the x-values increase, the y-values rapidly decrease. This results in a range less than the k-value rather than greater than the k-value.

EXPLORE 30A

Example 6

Identify a, b, k, and the y-intercept. Determine the domain and range of the function without using technology.

$y = -\frac{1}{3} \cdot 5^x + 1$ This is the same equation as Example 5. In this example, imagine you do not have access to technology and do not already have a graph of this function. How can you solve this problem?

Implement

$a = -\frac{1}{3}, b = 5, k = 1$

a is less than 0

$-\frac{1}{3} < 0$

The graph is decreasing

Domain: all real numbers, $\mathbb{R}$

Range: $y < 1$

When $x = 0$:

$y = -\frac{1}{3} \cdot 5^0 + 1$

$y = -\frac{1}{3} \cdot 1 + 1$

$y = \frac{2}{3}$

$\left(0, \frac{2}{3}\right)$

Explain

◂ Identify a, b, and k.

◂ Decide if the graph is increasing or decreasing:

- When $a > 0$, the graph is increasing so the range will be greater than the k-value.
- When $a < 0$, the graph is decreasing, and the range will be less than the k-value.

◂ Find the y-intercept without using technology:

- Set x equal to zero.
- Solve for y.

Checkpoint

Identify a, b, k, and the y-intercept. Determine the domain and range of the function.

$y = -3^x + 5$

$a = -1, b = 3, k = 5$

$(0, a + k) = (0, 4)$

OR

$y = -3^0 + 5$

$y = -1 \cdot 1 + 5$

$y = 4$

Domain: all real numbers

Range: $y < 5$

Common Ratios for Exponential Functions

- With linear functions, the **slope formula** is used to determine the constant rate of change between any two points.

Recall that the range for linear functions is all real numbers.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

If your student writes that $b = -3$, remind them the base cannot be negative in exponential functions. This means that $a = -1$, and $b = 3$.

Q: How do you determine the direction of the inequality symbol for the range?

A: *When $a > 0$, use a greater than sign, and when $a < 0$, use a less than sign.*

Subscripts are used here to differentiate between different y-values for the same function. This is similar to the slope formula.

30A EXPLORE

- With exponential functions, there is a **common ratio** between the y-coordinates.
- The common ratio, b is found by using this formula: $b = \frac{y_{n+1}}{y_n}$

Example 7

Compare the equations $y = 10x$ and $y = 10^x$.

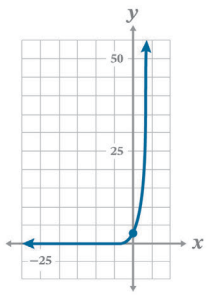
Plan Complete the tables.
Name the functions as linear or exponential.
Determine the slope or common ratio.
Use technology to sketch graphs of each.

$f(x)$: linear, $m = 10$

$g(x)$: exponential, $b = 10$

x	$f(x)$
-1	-10
0	0
1	10
2	20
3	30

x	$g(x)$
-1	$\frac{1}{10}$
0	1
1	10
2	100
3	1000



- The linear function increases by **adding** 10 to the previous number.
- The exponential function increases by **multiplying** previous number by 10.
- When you describe an exponential function, the y-coordinates **grow** (increase) or **decay** (decrease) by the common ratio, b.

Checkpoint

Determine if the table has a common ratio (b) or a constant rate of change (m). Explain whether the function is linear or exponential.

x	$f(x)$
-3	$\frac{1}{64}$
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1

This is an exponential function because the range values are increasing by a common ratio.

$b = \frac{1}{\left(\frac{1}{4}\right)}$

$1 \cdot \frac{4}{1}$

$b = 4$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Working from top to bottom, if you multiply each of the $f(x)$ values by 4, the common ratio, you will end up with the next term in the table. This is true when the x-values are consecutive integers.

PRACTICE 1 30A

Practice 1

Complete the problems on a separate sheet of paper.

Either by hand or with technology, create tables when $x = \{-2, -1, 0, 1, 2\}$ for and graph the exponential functions. Write simplified fractions rather than decimal values. An example table is shown.

1) Identify a, b, k, and the y-intercept. Create a table for the exponential function.
$f(x) = \left(\frac{1}{2}\right)^x$ **$a = 1, b = \frac{1}{2}, k = 0, (0, 1)$**

2) Graph the exponential function from problem 1. Determine the domain and range. **Domain: ($\mathbb{R}$), Range: $y > 0$**

3) Identify a, b, k, and the y-intercept. Create a table for the exponential function.
$y = 3^x - 1$ **$a = 1, b = 3, k = -1, (0, 0)$**

4) Graph the exponential function from problem 3. Determine the domain and range. **Domain: ($\mathbb{R}$), Range: $y > -1$**

5) Identify a, b, k, and the y-intercept. Create a table for the exponential function.
$h(x) = -\frac{1}{2} \cdot 10^x$ **$a = -\frac{1}{2}, b = 10, k = 0, \left(0, -\frac{1}{2}\right)$**

6) Graph the exponential function from problem 5. Determine the domain and range.

Match the equations to the correct *range* values.

B **7)** $f(x) = 7^x + 6$

C **8)** $g(x) = 7^x + \frac{1}{2}$

A **9)** $y = \frac{1}{2}(7^x)$

D **10)** $h(x) = -\frac{1}{2} \cdot 7^x$

A) $y > 0$
B) $y > 6$
C) $y > \frac{1}{2}$
D) $y < 0$

Determine whether the table shows linear or exponential behavior. If exponential, find the common ratio. If linear, find the slope.

11)

x	y
12	$\frac{1}{2}$
13	1
14	$\frac{3}{2}$
15	2

$m = \frac{2-1}{15-13} = \frac{1}{2}$
linear function, $m = \frac{1}{2}$

12)

x	y
−4	81
−3	27
−2	9
−1	3

$b = \frac{3}{9} = \frac{1}{3}$
exponential function, $b = \frac{1}{3}$

13)

x	y
0	1
1	10
2	100
3	1000

$b = \frac{1000}{100} = 10$
exponential function, $b = 10$

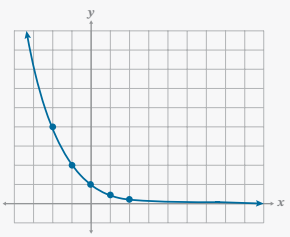
Practice 1

Worked solutions for these problems are located in the Digital Pack.

1)

x	$f(x)$
−2	4
−1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

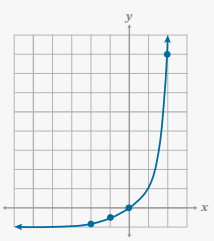
2)



3)

x	$f(x)$
−2	$-\frac{8}{9}$
−1	$-\frac{2}{3}$
0	0
1	2
2	8

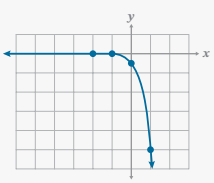
4)



5)

x	$f(x)$
−2	$-\frac{1}{200}$
−1	$-\frac{1}{20}$
0	$-\frac{1}{2}$
1	−5
2	−50

6)



Domain: all real numbers ($\mathbb{R}$)
Range: $y < 0$
Because the values are extremely small and large, your student will likely have to make a sketch rather than a precise graph.

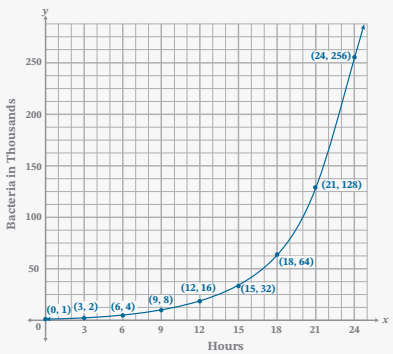
10) Having your student identify a and k will help determine the range. All of the b-values are the same for this problem.

14) (hours, bacteria in thousands)

Q: Is this a discrete or continuous function? Explain.

A: *This is a continuous function because the bacteria will continue to grow between each three hour increment.*

15)



Your student will need to estimate their ordered pairs given the scale of the graph.

If your student says 512 instead of 512,000:

Q: How many bacteria are there when the scientist begins the experiment?

A: *one thousand*

Q: What is the label of the y-axis?

A: *Bacteria in thousands*

30A PRACTICE 1

14) A scientist started with 1,000 bacteria in a dish. The number of bacteria, b, doubles every three hours, t.

- Copy and complete the table for the first 24 hours of bacterial growth.
- Write an ordered pair in words to describe the variables.

15) Use the table from problem 14 to complete the following:

- Graph the data.
- How many bacteria will there be after 27 hours? Explain.

Sample:
After 27 hours (3 hours past 24), there will be 512,000 bacteria because this is double the previous number.

t	b (in thousands)
0	**1**
3	**2**
6	**4**
9	8
12	**16**
15	**32**
18	**64**
21	**128**
24	**256**

MASTERY CHECK 30A

Mastery Check

Show What You Know

A scientist used the formula $B = P\left(2^{\frac{x}{d}}\right)$ to determine the doubling time for a certain bacteria.

B: number of bacteria, P: initial population, x: elapsed time in minutes, d: doubling time in minutes

A) If 10 bacteria cells double every five minutes, how many bacteria will there be in one hour? Define your variables and solve. Show your work.

$B = ?$ $\quad B = 10\left(2^{\frac{60}{5}}\right)$

$P = 10$ $\quad B = 10\left(2^{12}\right)$

$x = 1 \text{ hour} = 60 \text{ min}$ $\quad B = 10\,(4{,}096)$

$d = 5 \text{ min}$ $\quad B = 40{,}960$

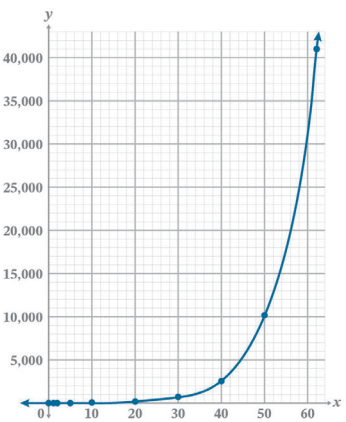
There will be 40,960 bacteria in one hour.

B) Complete the table for 10-minute increments up to one hour.

x	B
0	**10**
10	**40**
20	**160**
30	**640**
40	**2,560**
50	**10,240**
60	**40,960**

C) Graph the table from part B. Name the domain and range of the graph.

Domain: $x \geq 0$ Range: $y \geq 10$



D) When typing the formula into the calculator, a scientist accidentally typed $y = 10(2)\left(\frac{x}{5}\right)$ instead of the given formula. How does this error affect the graph of the formula?

Sample:
This formula has no exponent and is the same as the equation $y = 4x$. This would mean that the bacteria would increase by 4 bacteria every minute, rather than doubling every 5 minutes.

$y = (10 \cdot 2)\left(\frac{x}{5}\right)$

$y = \frac{20x}{5}$

$y = 4x$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

C) Q: Why does the domain begin at zero instead of being all real numbers?

A: *Because x represents time, and time starts at zero minutes.*

Your student should use a greater than or equal to symbol ($\geq$) here because the scientist started with 10 bacteria. In real life applications, functions do not always exactly match the general rule.

D) The answers would be the same for $(5, 20)$ and $(10, 40)$, but every other value representing the bacteria would be incorrect.

Your student can use technology to compare the given equation when $d = 5$, to $y = 4x$ and see what the two equations look like graphed on the same plane.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Graph an exponential function from a table.
- Determine the domain and range of an exponential function.
- Find the common ratio for an exponential function from a table.

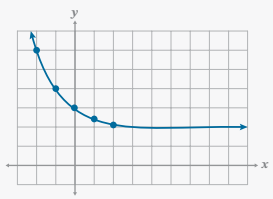
Practice 2

Worked solutions for these problems are located in the Digital Pack.

1)

x	$f(x)$
-2	6
-1	4
0	3
1	$\frac{5}{2}$
2	$\frac{9}{4}$

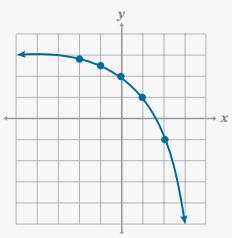
2)



3)

x	$f(x)$
-2	$\frac{11}{4}$
-1	$\frac{5}{2}$
0	2
1	1
2	-1

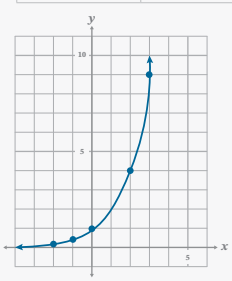
4)



5)

x	$g(x)$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

6)



Domain: all real numbers ($\mathbb{R}$)

Range: $y > 0$

30A PRACTICE 2

Practice 2

Complete the problems on a separate sheet of paper.

Either by hand or with technology, create tables when $x = \{-2, -1, 0, 1, 2\}$ for and graph the exponential functions. Write simplified fractions rather than decimal values. An example table is shown.

1) Identify a, b, k, and the y-intercept. Create a table for the exponential function.
$f(x) = \left(\frac{1}{2}\right)^x + 2$ **$a = 1, b = \frac{1}{2}, k = 2, (0, 3)$**

2) Graph the exponential function from problem 1. Determine the domain and range. **Domain: $\{\mathbb{R}\}$, Range: $y > 2$**

3) Identify a, b, k, and the y-intercept. Create a table for the exponential function.
$h(x) = -2^x + 3$ **$a = -1, b = 2, k = 3, (0, 2)$**

4) Graph the exponential function from problem 3. Determine the domain and range. **Domain: $\{\mathbb{R}\}$, Range: $y > 2$**

4) **Range: $y < 3$**

5) Identify a, b, k, and the y-intercept. Create a table for the exponential function.
$g(x) = 3^x$ **$a = 1, b = 3, k = 0, (0, 1)$**

6) Graph the exponential function from problem 5. Determine the domain and range.

Match the equations to the correct *range* values.

D **7)** $y = 5^x + 1$

B **8)** $m(x) = 5^x$

A **9)** $p(x) = -5^x$

C **10)** $w(x) = -5^x - 1$

A) $y < 0$

B) $y > 0$

C) $y < -1$

D) $y > 1$

Determine whether the table shows linear or exponential behavior. If exponential, find the common ratio. If linear, find the slope.

11)

x	y
3	8
4	16
5	32
6	64

$b = \frac{64}{32} = 2$
linear function, $b = 2$

12)

x	y
-5	1,024
-4	256
-3	64
-2	16

$b = \frac{16}{64} = \frac{1}{4}$
exponential function, $b = \frac{1}{4}$

13)

x	y
-1	-2
0	0
1	2
2	4

$m = \frac{4-2}{2-1} = 2$
linear function, $m = 2$

PRACTICE 2 30A

14) Suppose that m represents the mass in grams of a substance that halves in size each month, x. After running an experiment, a scientist wrote the equation below to determine the mass.

$m = 200(0.5)^x$

- Copy and complete the table using the equation.
- Define your variables as a verbal ordered pair.

15) Use the table from problem 14 to complete the following:

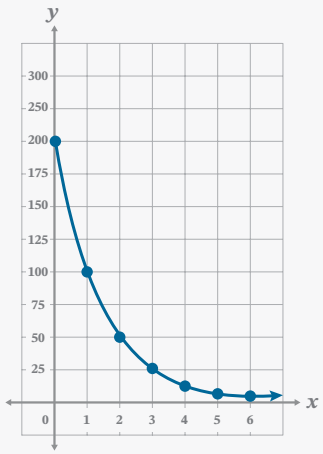
- Graph the data.
- Explain why the graph will never reach the x-axis, or $y = 0$.

Sample:
The graph will never reach $y = 0$, because the range is $y > 0$. Theoretically, the mass will continue to get smaller each month by half and never reach zero.

x	m
0	200
1	**100**
2	**50**
3	**25**
4	**12.5**
5	**6.25**
6	3.125

14) (x, m):
(number of months, mass in grams)

15)



If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

In this part of the lesson, your student will need to determine if the function is a growth or decay function. This is similar to positive and negative slope from Unit 2.

30B EXPLORE

Part B: The Growth/Decay Formula

Objectives

In this part of the lesson, you will learn about the growth/decay formula.

By the end of this lesson, you will be able to do the following:

- Determine a, r, and t and whether the function represents growth or decay.
- Calculate exponential growth and decay using the formula.

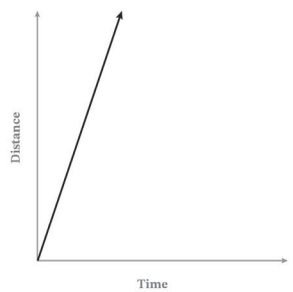
Why?

How much will you pay in interest on a loan? How much interest can you earn if you start investing? Applying the exponential growth and decay formula will help you answer questions like this related to areal-world scenarios.

Warm Up

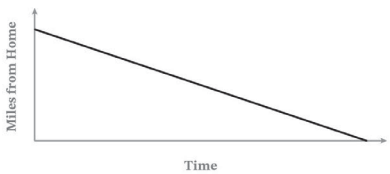
Describe the linear function. Be sure to use the words increasing and/or decreasing.

1)



Sample:
As time increases, the distance increases. This has a positive slope.

2)



Sample:
As time increases, the distance from home decreases. This has a negative slope.

Components of the Growth/Decay Formula

- The exponential **growth and decay** formula is very useful when working with quantities that increase or decrease rapidly but do not always require a graph to represent the values.
- The formula for exponential growth or decay **compounds**, or multiplies, the common ratio t times.

EXPLORE 30B

- The growth and decay formula states: $y = a(1 \pm r)^t$.
  - $y =$ final value or amount
  - $a =$ initial value or amount
  - $t =$ time in years
  - common growth ratio: $(1 + r)$
  - common decay ratio: $(1 - r)$
  - $r =$ rate
- When the rate is presented as a percentage rate, this must be converted to a decimal before solving the formula.
- When the problem represents growth rate, use the (+) symbol.
- When the problem represents decay rate, use the (−) symbol.
- Sometimes the formula will simplify the value inside the parentheses, and other times it will be written as $(1 \pm r)$.
- It is important to remember that:
  - $(1 \pm r) > 1$ represents exponential growth.
  - $(1 \pm r) < 1$ represents exponential decay.

Example 1

Given the formula, identify a, r, and t. Determine the common ratio and whether the formula represents growth or decay.

$y = 2{,}500\,(1 + 0.045)^8$

$a = 2{,}500$

$t = 8$

$r = 0.045 = 4.5\%$

$0.045 \cdot 100 = 4.5\,\%$

This equation represents exponential growth because the number in parentheses simplifies to a value greater than 1.

The common growth ratio is $(1 + 0.045)$, or 1.045.

> Remember that *per*cent means "out of 100." To convert a decimal to a percent, you multiply the value by 100.

- Growth/Decay is often used to figure out interest rate earnings. There are two general types of interest, simple and compound.
- Simple interest is calculated using $I = Prt$
  - I: interest earned, P: principal amount, r: rate of interest, t: time in years
- Compound interest is calculated using $y = a(1 + r)^t$.
  - y: total including interest, a: principal amount, r: rate of interest, t: time in years

30B EXPLORE

Example 2

Compare the growth using simple interest and compound interest.

Suppose you have \$10 and earn 10% interest yearly. Which plan will have the larger total after 20 years?

Implement

Simple Interest Formula: $I = Prt$	**Compound Interest: $y = a(1 + r)^t$**
$P = 10$	$a = 10$
$r = 0.10$	$r = 0.10$
$t = 20$	$t = 20$
$I = (10)(0.10)(20)$	$y = 10(1 + 0.10)^{20}$
$I = 20$	$y = \mathbf{67.27}$
$P + I =$ Final amount	
$10 + 20 = \mathbf{30}$	

Explain

Simple Interest	**Compound Interest**
Simple interest only multiplies the rate by the time and original amount.	Compound interest includes interest earned each year *and* then calculates the interest based on the new amount.
Simple interest earns you \$20 in interest, or \$1 per year.	Compound interest earns you \$67.27, or \$57.27 in interest.

When interest is compounded it grows much faster because you are earning interest on each year's interest. Therefore, the compound interest plan has more money after 20 years.

Example 3

Given the formula, identify a, r, and t. Determine if the growth or decay is represented and the common ratio.

$y = 375(0.82)^6$	$\mathbf{a = 375, t = 6, r = 18\%}$
$1 - r = 0.82$	Since the rate is less than 1, it is subtracted from one to find the rate.
$\mathbf{-r = -0.18}$	This also represents exponential decay, since the base is less than one.
$\mathbf{r = 0.18}$	The common ratio for this equation is **0.82**.
$\mathbf{r = 18\%}$	

EXPLORE 30B

Checkpoint

Given the formula, identify a, r, and t. Determine the common ratio and whether the formula represents growth or decay.

$y = 4630(1.02)^{25}$

$a = 4630$

$t = 25$

$r = 2\%$

$1.02 - 1 = 0.02$

The common growth ratio is 1.02.

Solving with the Growth/Decay Formula

- Using technology is important for finding the complete answer because the final values are found by compounding the **common ratio** t-number of times.
- It is common to round answers to the nearest **hundredth** because the formula is used frequently with money or to find the value of things over time.

Example 4

Ainsley purchased a car for $18,000 and was told that the car would depreciate 15% over the next ten years of owning the car. What is the value of the car after this amount of time?

Plan Define your variables for the formula.
Substitute into the exponential growth/decay formula.

Implement	**Explain**
$a = 18{,}000$	
$t = 10$	
$r = 15\% = 0.15$	
$y = 18{,}000(1 - 0.15)^{10}$	This represents decay since *de*preciate means to *de*cline or *de*cay.
$y = 3{,}543.7392$	
$y \approx 3{,}543.74$	Using technology, the number will be 3,543.7392. This rounds to 3,543.74.

After 10 years, Ainsely's car will be worth approximately $3,543.74.

What is the difference between the purchase-price of the car and the value after 10 years?

$18{,}000 - 3{,}543.74 = 14{,}456.26$

The car lost about $14,500 in value over 10 years.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can you determine if this represents growth or decay?

A: *When the common ratio is greater than one it represents growth. When the common ratio is between zero and one it represents decay.*

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the common ratio?

A: *1.089*

30B EXPLORE

Example 5

Markie received a bank statement with the current balance of a high-yield savings account. The total balance was $2,089.34 after 12 years. No extra money was added to or deducted from the account once it was opened. The bank statement included an interest rate of 2.8% but not the initial amount invested. How much money did Markie invest when the account was opened? Round to the nearest hundred.

Plan Define each variable.
Substitute using the formula.
Solve for the unknown variable.

$a = ?$

$t = 12$

$r = 2.8\% = 0.028$

$y = 2{,}089.34$

$2{,}089.34 = a(1 + 0.028)^{12}$

$2{,}089.34 = a(1.028)^{12}$

$\frac{2{,}089.34}{(1.028)^{12}} = a$

$a = 1{,}500$ **Markie initially invested $1,500.**

- The exponential growth and decay formula will change slightly when the rate is compounded more than once a year.
- Some rates will be compounded quarterly (4), semi-annually (2), monthly (12), or even daily (365).
- When this occurs, the formula is modified to include the number of times the rate is compounded.

Checkpoint

Define your variables for the formula and then substitute into the exponential growth/decay formula.

Kyle initially invested $500 into a mutual fund that earned 8.9% interest compounded yearly. What is the value of the mutual fund after 5 years? And how much interest did Kyle earn?

$a = 500$

$t = 5$

$r = 0.089$

$y = 500(1 + 0.089)^5$

$y = 765.79$

Kyle earned $265.79 in interest.
(765.79 − 500)

PRACTICE 1 30B

Practice 1

Complete the problems on a separate sheet of paper.

Given the formula, identify a, r, and t. Name the common ratio and use it to explain if growth or decay is represented.

1) $y = 35{,}000(1 + 0.027)^7$

2) $y = 125{,}500(1 - 0.084)^6$

3) $y = 632(1.26)^{14}$

4) $y = 8{,}590(0.985)^4$

State whether the problem represents exponential growth or decay. Define your variables for the problem. Use the growth/decay formula to find the missing variable. Round answers to the hundredth.

5) Jill invested in a mutual fund that had a 4.1% return each year. She invested \$3,250 and wanted to know what the final return would be after 6 years.

6) Kristin purchased a new car for \$25,450. Each year the car depreciated 12%. What is the value of the car after 8 years?

7) The population of Hometown is 12,345 people. This represents a yearly decline of 1.2% over the past 10 years. What was the population 10 years ago?

8) Generosity High School held a canned food drive for the local food cupboard. Last year they collected 500 cans. This year they saw a 250% increase in donations. What was the total number of cans collected for this year's food drive?

9) In 2004, there were 107,000 landlines in homes across Technoville. Each year saw a 16.8% decline in homes with landlines. How many homes had landlines in 2019?

How many fewer landlines were there in 2019 than 2004?

10) A car was purchased 6 years ago for a price of \$25,450. The car depreciated at a rate of 15% a year. What is the value of the car now?

What is the difference in the purchase price and the current value?

11) Beven had a savings account that he forgot about. The balance of the account now is \$983.11. The bank statement said that the interest rate was 0.5% and the account was opened 12 years ago. What was the starting balance of the account?

12) The Guthrie family purchased their home for \$215,000. They noticed that the houses in their area increased at a rate of 2.6% yearly over the last 25 years. What is the value of their house now? Round to the nearest hundred dollars.

$a = 215{,}000$
$t = 25$
$r = 2.6\% = 0.026$
$y = 215{,}000(1 + 0.026)^{25}$
$y = 408{,}434.56$

After 25 years, the value of the Guthrie home is \$408,400.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) $a = 35{,}000$ $t = 7$ $r = 2.7\%$
This represents exponential growth because the common ratio is 1.027.

2) $a = 125{,}500$ $t = 6$ $r = 8.4\%$
The represents exponential decay because the common ratio is 0.916.

3) $a = 632$ $t = 14$ $r = 26\%$
This represents exponential growth because the common ratio is 1.26.

4) $a = 8{,}590$ $t = 4$ $r = 1.5\%$
This represents exponential decay because the common ratio is 0.985.

5) Growth
$a = 3{,}250$ $t = 6$ $r = 4.1\% = 0.041$
$y = 3{,}250(1 + 0.041)^6$
Jill has \$4,136.07.

6) Decay
$a = 25{,}450$ $t = 8$ $r = 12\% = 0.12$
$y = 25{,}450(1 - 0.12)^8$
After 8 years, the value of the car will be \$9,152.70.

7) Decay
$t = 10$ $r = 1.2\% = 0.012$ $y = 12{,}345$
$12{,}345 = a(1 - 0.012)^{10}$
The population of Hometown 10 years ago was 13,929 people.

8) Growth
$a = 500$ $t = 1$ $r = 250\% = 2.5$
$y = 500(1 + 2.5)^1$
Generosity High School collected 1,750 cans this year.

9) Decay
$a = 107{,}000$ $t = 2019 - 2004 = 15$
$r = 16.8\% = 0.168$
$y = 107{,}000(1 - 0.168)^{15}$
There are 6,780 landlines in 2019.
100,220

10) Decay
$a = 25{,}450$ $t = 6$ $r = 15\% = 0.15$
$y = 25{,}450(1 - 0.15)^6$
The car is now valued at \$9,598.46.
The car lost \$15,851.54 in value.

11) Growth
$t = 12$ $r = 0.5\% = 0.005$ $y = 983.11$
$983.11 = a(1 + 0.005)^{12}$
Beven started the savings account with \$926.

Mastery Check

Show What You Know

A) Q: How many weeks are in a year?
A: *52.*

C) A credit card may have additional fees and other ways of compounding interest. This is to demonstrate the interest that occurs over time. Earning interest on a credit card is represented by a negative value because it means you owe more money than you spent.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Determine a, r, and t and whether the function represents growth or decay.
- Calculate exponential growth and decay using the formula.

30B MASTERY CHECK

Mastery Check

Show What You Know

The compound interest formula is used when the rate is compounded more than one time per year. When this happens, the variable n represents the number of times the rate will be compounded. The modified formula is:

$y = a\left(1 + \frac{r}{n}\right)^{nt}$

A) Determine what n will be equal to when interest is compounded:

weekly	$n = 52$	monthly	$n = 12$
quarterly	$n = 4$	semi-annually	$n = 2$

B) You opened a high yield savings account at the bank. The interest rate is 0.50% and compounds weekly. If you deposited $1,200 when you opened the account, what will your balance be in 2 years? Define the variables and substitute into the formula before using technology to find the value.

$a = 1{,}200$

$t = 2$

$r = 0.50\% = 0.005$

$n = 52$

$y = 1{,}200\left(1 + \frac{0.005}{52}\right)^{(52)(2)}$

$y = 1{,}212.06$

$1,212.06 will be in the account after two years.

C) The same bank offers a customer credit card with an interest rate of 18% that compounds monthly. If you have a $500 balance for the entire year, what would the payoff amount be at the end of the year? Define the variables and substitute into the formula before using technology to find the value.

$a = 500$

$t = 1$

$r = 18\% = 0.18$

$n = 12$

$y = 500\left(1 + \frac{0.18}{12}\right)^{(12 \cdot 1)}$

$y = 597.81$

The balance would be $597.81 at the end of the year.

D) The bank also offers a retirement account option that earns an average rate of 6.7% compounded quarterly. If your parent has a retirement account with $125,000 and will not retire for another 17 years, what is the expected balance at that time?

$a = 125{,}000$

$t = 17$

$r = 6.7\% = 0.067$

$n = 4$

$y = 125{,}000\left(1 + \frac{0.067}{4}\right)^{(4 \cdot 17)}$

$y = 386{,}789.19$

In 17 years, the account will have an expected value for $386,789.19.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Lesson, Unit, and Final Assessments

At the end of Unit 5, students can complete the Lesson 30 Test, Unit 5 Test, and the Final Exam.

Students should complete the Lesson 30 Test in the same manner as they have in previous lessons.

The next day, before completing the Unit 5 Test, they can review Lessons 26–30, using guided notes, Mastery Checks, and Lesson Tests.

To prepare for the Final Exam, students can:

- Review Lesson Tests and Unit Tests. Re-working test problems they missed is a great way to prepare.
- Use Unit dividers to review objectives they have mastered. If needed, they can review guided notes, examples, and Checkpoints for objectives that are less familiar.
- If not already done, complete the Targeted Reviews. The problems come from a variety of lessons, not always in the sequence they learned it, as they will on the Final Exam.

PRACTICE 2 30B

Practice 2

Complete the problems on a separate sheet of paper.

Given the formula, identify a, r, and t. Name the common ratio and use it to explain if growth or decay is represented.

1) $y = 2{,}576(1 + 0.034)^{15}$

2) $y = 405(1.015)^{8}$

3) $y = 3{,}250(0.92)^{5}$

4) $y = 25{,}000(1 - 0.13)^{9}$

State whether the problem represents exponential growth or decay. Define your variables for the problem. Use the growth/decay formula to find the missing variable. Round answers to the hundredth.

5) A certain stock price declined 3.2% yearly. If an initial investment of $5,000 was made two years ago, what is the investment worth now?

6) Twelve years ago, Jody invested $1,500 into a retirement account earning 6.5% annual interest. What is the value of the investment now?

7) The food budget increased by 15% each year as the Leighton family grew. If they currently spend $520 on groceries each month, how much did they spend six years ago? Round to the nearest dollar.

How much more does the Leighton family spend now than they did six years ago?

8) From the time a cohort group started college to graduation four years later, enrollment in the program declined at a rate of 5.3% per year. If 526 started in the program, how many will graduate in this program?

How many students did not graduate from this program?

9) In the year 2000, Rylan's retirement account had a total of $120,000. If 3.99% was the average yearly return, how much will be in the account in 30 years?

By how much did Rylan's investment increase?

10) Mrs. Neill noticed a 10% reduction in the number of photos she was printing each year. Fifteen years ago she printed 450 photos. How many did she print this year?

11) Journeysville wants to increase their walking and hiking trails 1.2% yearly over the next 10 years. If they currently have 50 miles of trails, how many will they have in 5 years? How many will they have in 10 years?

12) Suppose the people of Journeysville want to have 15 additional miles of trail in their town. How many years will it be if they continue to add trails at a rate of 1.2% a year? (Hint: you may need to try a few numbers to see how close you can get to 65 miles)

11) Growth

$a = 50 \quad t = 5 \quad r = 1.2\% = 0.012$

$y = 50(1 + 0.012)^{5}$

In 5 years, they will have 53.07 miles of trails.

In 10 years, they will have 56.33 miles of trails.

12) $50 + 15 = 65$ miles of trail

It will take 22 years to have 65 miles of trail.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note regarding Lesson and Unit Tests and the Final Exam.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

1) $a = 2{,}576 \quad t = 15 \quad r = 3.4\%$

This represents exponential growth because the common ratio is 1.034.

Recall that when the common ratio > 1, this represents exponential growth.

2) $a = 405 \quad t = 8 \quad r = 1.5\%$

This represents exponential growth because the common ratio is 1.015.

Recall that 0 < common ratio < 1 represents exponential decay.

3) $a = 3{,}250 \quad t = 5 \quad r = 8\%$

This represents exponential decay because the common ratio is 0.92.

4) $a = 25{,}000 \quad t = 9 \quad r = 13\%$

This represents exponential decay because the common ratio is 0.87.

5) Decay

$a = 5{,}000 \quad t = 2 \quad r = 3.2\% = 0.032$

$y = 4{,}685.12$

The stock is now worth $4,685.12.

6) Growth

$a = 1{,}500 \quad t = 12 \quad r = 6.5\% = 0.065$

$y = 1{,}500(1 + 0.065)^{12} = 3{,}193.64$

After 12 years, the account contained $3,193.44.

7) Growth

$t = 6 \quad r = 15\% = 0.15 \quad y = 520$

$520 = a(1 + 0.15)^{6}$

$a = 224.81$

The Leighton family spent $225 on groceries six years ago.

The Leighton family spends $295 more now.

Q: Why should you round to the nearest whole number even though the directions do not state this explicitly?

A: *Because you cannot have a fraction of a person when determining the population.*

8) Decay

$a = 526 \quad t = 4 \quad r = 5.3\% = 0.053$

$y = 526(1 - 0.053)^{4}$

423 students will graduate from the program.

103 students switch to another program.

Q: What is the time for this problem?

A: *One year.*

9) Growth

$a = 120{,}000 \quad t = 30 \quad r = 3.99\% = 0.0399$

$y = 120{,}000(1 + 0.0399)^{30}$

In 30 years, Rylan will have $388,086.55

Rylan's investment increased by $268,086.55.

10) Decay

$a = 450 \quad t = 15 \quad r = 10\% = 0.10$

$y = 450(1 - 0.10)^{15}$

This year Mrs. Neill printed 93 photos.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

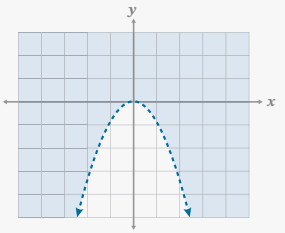
If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

4) Remember that the radicands must be like terms in order to combine radicals with the same index.

7) $a = -2, b = 7, c = -4$

$x = 0.719, 2.781$

8)



9) Distractor Rationale:

$a = -1, h = 12, k = 0$

The graph will shift right 12 spaces and be reflected over the x-axis because $a = -1$.

10) Distractor Rationale:

The third expression is not equivalent to the given expression.

$\sqrt{81x^3y^8} = \sqrt{9^2x^3y^8} = 9xy^4\sqrt{x}$

$\sqrt{81x^3y^8} = \sqrt{3^4x^3y^8}$

$\sqrt{81x^3y^8} = (81x^3y^8)^{\frac{1}{2}}$

11) Distractor Rationale:

A) The denominator of an exponent represents the index, not the numerator, and the negative sign disappears in this answer.

B) A negative sign cannot transfer from the exponent to the base.

D) The exponent cannot be separated from the base to which it is assigned.

$5^{-y} = \frac{1}{5^y} = \left(\frac{1}{5}\right)^y$

12) Distractor Rationale:

A) This answer is not in simplified radical form.

C) This is the square root of 64, ignoring the coefficient −2.

D) This would be correct if there were only one negative sign in the problem.

$x^2 = 32$

$x = \pm\sqrt{32} = \pm\sqrt{2 \cdot 4 \cdot 4}$

$x = \pm 4\sqrt{2}$

Your student should prepare for the Unit Test by reviewing their Guided Notes in Unit 5, as well as practice problems and the Targeted Review pages.

TARGETED REVIEW 30

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Simplify. Answers should contain only positive exponents.

1) $\frac{7^{-1}a^{-4}}{2^{-1}b}$ $\quad \frac{2}{7a^4b}$

2) $\frac{x^{\frac{3}{5}}y^{11}}{xy^{16}}$ $\quad \frac{x^{\frac{1}{5}}}{y^5}$

3) $\sqrt[3]{a^{17}b^{33}c^{21}}$ $\quad a^5b^{11}c^7\sqrt[3]{a^2}$

4) $\sqrt{125} + 9\sqrt{5} - \sqrt{50}$ $\quad 14\sqrt{5} - 5\sqrt{2}$

5) Solve for the value of a.

$81 = 3^{a-2}$ $\quad a = 6$

6) Solve.

$\sqrt{(3x)} = 12$ $\quad x = 48$

7) Identify a, b and c in the quadratic function. Using technology, find the solution(s) to the quadratic function.

$y = -2x^2 + 7x - 4$

8) Graph the quadratic inequality.

$y > -x^2$

Multiple Choice

9) Write all transformations that occur from the parent graph $y = x^2$ to the given equation.

$y = -(x - 12)^2$

- ☐ vertical shift
- ■ **horizontal shift**
- ■ **reflection**
- ☐ dilation

10) Write all of the following expressions that are equivalent to the given expression.

$\sqrt{81x^3y^8}$

- ■ $9xy^4\sqrt{x}$
- ■ $\sqrt{3^4x^3y^8}$
- ☐ $3xy\sqrt{xy^7}$
- ■ $(81x^3y^8)^{\frac{1}{2}}$

C **11)** Determine the equivalent expression.

5^{-y}

A) $\sqrt[y]{5}$

B) -5^y

C) $\left(\frac{1}{5}\right)^y$

D) $\frac{5}{y}$

B **12)** Solve. Simplify the answer in radical form.

$-2x^2 = -64$

A) $\pm\sqrt{32}$

B) $\pm 4\sqrt{2}$

C) ± 8

D) no solution

Your student should prepare for the Final Test by reviewing Unit 4 and Unit 5. However, understanding Units 1–3 is necessary for successful completion of the Final Test.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	28	28	29	29	29	29	26	27	27	29	29	29