Lesson 29
Radical Expressions and Equations

Outline

Part A
Radical Expressions

- Radical Expressions with Variables
- Simplifying Radicals when the Index Is 3
- Addition and Subtraction of Radical Expressions

Part B
Radical Equations

- Rational Exponent Equations with Monomials
- Equations with Exponents
- Equations with Radicals

Targeted Review

Vocabulary

There are no new vocabulary words for this lesson.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Your student will need to write the prime factorization or make a factor tree for any numerical coefficient under the radical. This will allow them to simplify the expression completely.

Part A: Radical Expressions

Objectives

In this part of the lesson, you will learn about radical expressions.

By the end of this lesson, you will be able to do the following:

- Simplify radical expressions with variables.
- Write radical expressions using rational exponents.
- Add and subtract radical expressions.

Why?

A radical is another way to represent a rational, fractional, or exponent. Knowing how to use both forms will allow you to work with a variety of expressions and equations.

Warm Up

Write the prime factorization for the numbers below with and without exponents.

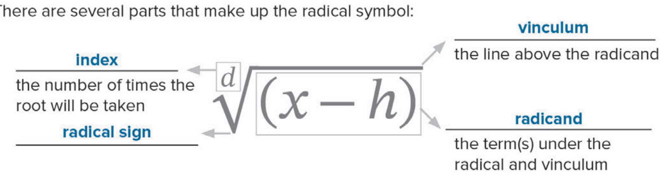
1) 72
$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
$= 23 \cdot 32$

2) 45
$45 = 3 \cdot 3 \cdot 5$
$= 3^2 \cdot 5$

3) 51
$51 = 3 \cdot 17$

Radical Expressions with Variables

- The square root symbol is also called the **radical** symbol, or sign.
- There are several parts that make up the radical symbol:



- The **index** determines the number of times the root will be taken.
  - When the index is 2, you take the **square** root of the radicand. (If no index is present, it is assumed to be 2.)
  - When the index is **3**, you take the cube root of the radicand.
- When a **radical term** is simplified, like factors are grouped by the number of the index.
- When **numerical radicands** are simplified, the factors are found and also grouped by the number of the index.

EXPLORE 29A

- For example:

Implement	Explain
$\sqrt{72}$	◀ The index is **2**.
$\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$	◀ Factors of 72 are grouped in sets of two. In this case, one set of 2s and one set of 3s.
$2 \cdot 3\sqrt{2}$	◀ The sets are simplified and then moved outside of the radical symbol. However, the leftover 2 that did not form a complete set **remains** in the **radicand**.
$6\sqrt{2}$	◀ Simplify.

- As the radicand increases, expanding expressions can become very time consuming. Another way to simplify radicals is to use the index along with **divisibility** and **exponent** rules to simplify.

- Rule 8, the final rule of exponents, establishes an important relationship between **radicals** and **exponents**.
  - Every radical can be rewritten as a **rational** exponent. The **denominator** of a fractional exponent is the index, and the **numerator** is the number of times the base is multiplied by itself.
  - This also means that the index is the **denominator** of the rational exponent.
  - For all real numbers, $\sqrt[d]{a^n} = a^{\frac{n}{d}}$.

Implement	Explain
$\sqrt{72}$	◀ The index is 2.
$\sqrt{2^3 \cdot 3^2} = 2^{\frac{3}{2}} \cdot 3^{\frac{2}{2}}$	◀ Find the prime factors of the radicand using exponents (Rule 8).
$6\sqrt{2}$	◀ Simplify.

29A EXPLORE

Example 1

Simplify. Write the term using rational exponents. The final answer should be in radical form.

$\sqrt{x^9}$

Expanding Expressions Method

Implement	Explain
$\sqrt{x^9} = \sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$	◂ Expand the expression.
$\sqrt{x^9} = \sqrt{(x \cdot x)(x \cdot x)(x \cdot x)(x \cdot x) \cdot x}$	◂ The index is 2. Simplify the radical expression by writing the base in groups of two.
$x^4\sqrt{x}$	◂ There are 4 groups of two x's, so x^4 is on the outside of the radical symbol. There is one x that remains inside the radical.

Index with Divisibility and Exponent Rules Method

Implement	Explain
$\sqrt{x^9} = x^{\frac{9}{2}}$	◂ Rule 8
$x^{\frac{9}{2}} = x^{4\frac{1}{2}}$	◂ The exponent is simplified.
$x^4\sqrt{x}$	◂ The term x^4 is placed outside of the radical and the remainder, x, is the radicand. The solution is read: "x to the fourth power times the square root of x."

> You do not need to write the exponents as a mixed number. This step can be completed using mental math. The whole number is the exponent of the term outside the radical. If there is a remainder, this is the exponent of the term that is the radicand.

Example 2

Simplify. Write the terms using rational exponents. The final answer should be in radical form.

A) $\sqrt{x^7y^5}$

Plan Rewrite each variable using a fractional exponent.

Implement	Explain
$x^{\frac{7}{2}}y^{\frac{5}{2}} = x^3y^2\sqrt{xy}$	◂ Seven divided by 2 equals 3 with a remainder of 1. Five divided by 2 equals 2 with a remainder of 1. The terms x^3 and y^2 are outside the radical, and x^1y^1, or xy, remains under the radical. (Rule 8)

> It is not necessary to write a power of 1 because it is implied.

B) $\sqrt{a^{24}b^{41}c^{18}}$

Implement	Explain
$a^{\frac{24}{2}}b^{\frac{42}{2}}c^{\frac{18}{2}} = a^{12}b^{20}c^9\sqrt{b}$	◂ The terms a and c have even exponents, so they are written outside of the radical when the expression is simplified. (Rule 8)

> If all the terms have even exponents and the index is 2, there will be no radical symbol in the simplified answer.

EXPLORE 29A

Example 3

Simplify. Write the terms using rational exponents. The final answer should be in radical form.

A) $\sqrt{24x^3y}$

Implement	Explain
$24 = 2^3 \cdot 3$	◀ Write the number as its prime factorization.
$\sqrt{2^3 \cdot 3^1 \cdot x^3 \cdot y^1}$	◀ Rewrite the radicand in factored form.
$2^{\frac{3}{2}} \cdot 2^{\frac{1}{2}} \cdot x^{\frac{3}{2}} \cdot y^{\frac{1}{2}}$	◀ Rule 8
$2x\sqrt{2 \cdot 3xy}$	
$2x\sqrt{6xy}$	

B) $\sqrt{25a^{16}}$

Implement	Explain
$\sqrt{5^2a^{16}} = 5^{\frac{2}{2}}a^{\frac{16}{2}}$	◀ Rule 8
$5a^8$	◀ Because all of the terms contain even exponents, the simplified answer does not contain a radical.

Checkpoint

Simplify. Write the term using rational exponents. The final answer should be in radical form.

$\sqrt{54a^{22}b^{13}}$

$\sqrt{2^1 \cdot 3^3a^{22}b^{13}} = 2^{\frac{1}{2}} \cdot 3^{\frac{3}{2}} \cdot a^{\frac{22}{2}} \cdot b^{\frac{13}{2}}$

$3a^{11}b^6\sqrt{(2)(3)b} = 3a^{11}b^6\sqrt{6b}$

Simplifying Radicals when the Index Is 3

- When the index is __3__ (cubed root), terms will be written with exponents that are a __multiple__ of three.
- If the index is 4, then the exponents will have a multiple of __four__, and so on.
- Remember that multiples of a given number are the numbers you get when you __skip count__ by that number.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Remember that each exponent is divided by 2 and this term is moved in front of the radical. The base(s) with remainders will remain under the radical.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How do you determine the radicand (the term under the radical)?

A: *Simplify the fractional exponents. Any base that has a remainder will be part of the radicand.*

Your student does not need to write terms with an exponent of one. However, if this helps them keep the terms organized they may include it.

29A EXPLORE

Example 4

A) $\sqrt[3]{54x^{11}y^{3}}$

Implement	Explain
$\sqrt[3]{2^{11}y^{3}}$	◂ Write the number as its prime factorization.
$2^{\frac{1}{3}} \cdot 3^{\frac{3}{3}} \cdot x^{\frac{11}{3}} \cdot y^{\frac{3}{3}}$	◂ Because the index is 3, the exponents are divided by three rather than two. (Rule 8)
$3x^{3}y\sqrt[3]{2x^{2}}$	◂ Any terms that have a remainder and any base that is raised to an exponent less than one is part of the radicand. For this expression, this is 2 and x^2.

B) $\sqrt[3]{24x^{10}}$

Implement	Explain
$\sqrt[3]{2^{3} \cdot 3^{1}x^{10}}$	◂ Write the number as its prime factorization.
$2^{\frac{3}{3}} \cdot 3^{\frac{1}{3}} \cdot x^{\frac{10}{3}}$	◂ Rule 8
$2x^{3}\sqrt[3]{3x}$	

You may have noticed that when the index increases, so does the remainder. When the index was 2, there was either no remainder or a remainder of one. When the index is 3, there can be a remainder of one or two.

Checkpoint

Simplify. Write the term using rational exponents. The final answer should be in radical form.

$\sqrt[3]{125x^{8}y^{3}}$

$\sqrt[3]{5^{3}x^{8}y^{3}} = 5^{\frac{3}{3}}x^{\frac{8}{3}}y^{\frac{3}{3}}$

$5x^{2}y\sqrt[3]{x^{2}}$

EXPLORE 29A

Addition and Subtraction of Radical Expressions

- When you **add and subtract** radical expressions, terms can only be combined when the index and the radicand are **like terms**.
  - Remember, like terms have the same **base** raised to the same **power**.
  - For example, look at the following terms: $2\sqrt{6}, 4\sqrt{6x}, 5\sqrt{6}$
    - The first $(2\sqrt{6})$ and last $(5\sqrt{6})$ terms are like terms because they have the same **index** and **radicand**.
    - If all three of these terms were written in an expression, it would simplify as follows:

      $\mathbf{2\sqrt{6} + 4\sqrt{6x} + 5\sqrt{6}}$

      $\mathbf{7\sqrt{6} + 4\sqrt{6x}}$

Example 5

Simplify.

$\sqrt{27} + 4\sqrt{3}$

Plan Simplify each radical term.
Determine if the radicals can be added together.

Implement	Explain
$\sqrt{33} = 3^{\frac{3}{2}} = 3\sqrt{3}$	◀ The square root of 27 is simplified. (Rule 8)
$\mathbf{3\sqrt{3} + 4\sqrt{3} = 7\sqrt{3}}$	◀ The coefficients are combined and written as one term.

Example 6

Simplify.

$4\sqrt{7} - 9\sqrt{7} + 2\sqrt{7x}$

> Remember the coefficient 1 is implied for any term without a coefficient.

Implement	Explain
$\mathbf{-5\sqrt{7} + 2\sqrt{7x}}$	◀ The first two terms are combined because they have the same index and radicand. The last term cannot be combined because it has a different radicand.

Checkpoint

Simplify.

$3\sqrt{2} + \sqrt{2} - \sqrt{5}$

$\mathbf{4\sqrt{2} - \sqrt{5}}$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the coefficient for the middle and last terms?

A: *One*

Q: What is the coefficient for the first term before simplifying?

A: *Three*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

2) Q: Why is there no radical symbol in the solution to this problem?

A: *There is no radical because the index is 2, which simplifies the term to a whole number without a remainder.*

5–10) For problems 5–10, your student will need to simplify the numerical values as well as the exponents. Have your student find the prime factors of the numbers to simplify them.

7, 8, 10) Make sure that your student notices the index of 3 for problems 7, 8, and 10. This will require them to divide exponents by three rather than two.

7) The number 64 can also be written as 2^6, which would simplify to $2^{\frac{6}{3}=2}$. This also simplifies to 4.

11) Q: What is the radicand of the terms?

A: $2x$

Q: What are the coefficients?

A: *8 and 7*

15) Q: What is the first step in this problem?

A: *The first step is to simplify each term.*

16) Q: What is the first step in this problem?

A: *The first step is to simplify each term.*

Q: If the first term had an index of 2, would your answer be the same? Explain.

A: *No, because like terms have the same index.*

This problem has your student extend their knowledge and simplify a cubic root expression. Recall that this means that the index is 3 and the factors of each term can be written in groups of three.

Practice 1

Complete the problems on a separate sheet of paper.

Simplify. Write the terms using rational exponents. The final answer should be in radical form.

1) $\sqrt{a^7}$ $a^3\sqrt{a}$

2) $\sqrt{a^8}$ a^4

3) $\sqrt[3]{y^{12}}$ y^4

4) $\sqrt[3]{x^8y^3}$ $x^2y\sqrt[3]{x^2}$

5) $\sqrt{75x^8y^3}$ $5x^4y\sqrt{3y}$

6) $\sqrt{8x^{16}y^9}$ $2x^8y^4\sqrt{2y}$

7) $\sqrt[3]{64x^7y^{11}}$ $4x^2y^3\sqrt[3]{xy^2}$

8) $\sqrt[3]{1000a^8b^9}$ $10a^2b^3\sqrt[3]{a\_\text{\^{}}2}$

9) $\sqrt{63x^5y^{15}}$ $3x^2y^7\sqrt{7xy}$

10) $\sqrt[3]{32a^{13}b^{75}}$ $2a^4b^{25}\sqrt[3]{4a}$

Simplify. Remember to only combine like terms.

11) $8\sqrt{2x} + 7\sqrt{2x}$ $15\sqrt{2x}$

12) $5\sqrt{3} - 8\sqrt{3}$ $-3\sqrt{3}$

13) $12\sqrt{11} + \sqrt{11} - 3\sqrt{11}$ $10\sqrt{11}$

14) $5\sqrt{3} - \sqrt{3} - 8\sqrt{7}$ $4\sqrt{3} - 8\sqrt{7}$

15) 2R3 + R9 + R12 $4\sqrt{3} + 3$

16) $\sqrt[3]{48} + 8\sqrt[3]{6} - \sqrt[3]{162}$ $7\sqrt[3]{6}$

MASTERY CHECK 29A

Mastery Check

Show What You Know

Quinn and Emmerson needed to simplify a radical expression. Both had incorrect answers.

Quinn's Work:

$$\sqrt{24x^{19}y^{10}} + \sqrt[3]{48x^{28}y^{15}}$$

$$\sqrt{2^3 \cdot 3^1 x^{19}y^{10}} + \sqrt[3]{2^4 \cdot 3^1 x^{28}y^{15}}$$

$$\left(2^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} x^{\frac{19}{2}} y^{\frac{10}{2}}\right) + \left(2^{\frac{4}{3}} \cdot 3^{\frac{1}{3}} \cdot x^{\frac{28}{3}} \cdot y^{\frac{15}{3}}\right)$$

$$2x^9y^5\sqrt{6x} + 2x^9y^5\sqrt[3]{6x}$$

$$4x^9y^5\sqrt{6x}$$

Emmerson's Work:

$$\sqrt{24x^{19}y^{10}} + \sqrt[3]{48x^{28}y^{15}}$$

$$\sqrt{2^3 \cdot 3^1 x^{19}y^{10}} + \sqrt[3]{2^4 \cdot 3^1 x^{28}y^{15}}$$

$$\left(2^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} x^{\frac{19}{2}} y^{\frac{10}{2}}\right) + \left(2^{\frac{4}{3}} \cdot 3^{\frac{1}{3}} \cdot x^{\frac{28}{3}} \cdot y^{\frac{15}{3}}\right)$$

$$2x^9y^5\sqrt{6x} + 4x^{14}y^7\sqrt{3y}$$

A) Rework the problem correctly.

$$\sqrt{24x^{19}y^{10}} + \sqrt[3]{48x^{28}y^{15}}$$

$$\sqrt{2^3 \cdot 3^1 x^{19}y^{10}} + \sqrt[3]{2^4 \cdot 3^1 x^{28}y^{15}}$$

$$\left(2^{\frac{3}{2}} 3^{\frac{1}{2}} x^{\frac{19}{2}} y^{\frac{10}{2}}\right) + \left(2^{\frac{4}{2}} 3^{\frac{1}{2}} x^{\frac{28}{2}} y^{\frac{15}{2}}\right)$$

$$2x^9y^5\sqrt{6x} + 2x^9y^5\sqrt[3]{6x}$$

B) Explain the steps you used.

Sample:
First, the prime factorization of the number needs to be simplified by the index. For the first term, the index is 2. In the second term, the index is 3. Any whole number values will become the coefficient of the radical and the remainder will be the radicand. Since the radicals have different indices, they are not like terms and cannot be combined.

C) Explain Quinn's mistake and how to fix it.

Sample:
Quinn knows how to simplify each term but incorrectly added a step. Quinn added the final two terms together even though their indexes were different.

D) Explain Emmerson's mistake and how to fix it.

Sample:
Emmerson did not use the correct index for the second term. Because the index is 3, each exponent should have been divided by three, not two.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Having your student rework the problem *before* analyzing the incorrect answers is recommended so they can think about how they would complete the problem before comparing other students' work.

B) Q: What value or values must be common in order to combine like radicals?

A: *The index and the radicand (number under the radical).*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Simplify radical expressions with variables.
- Write radical expressions using rational exponents.
- Add and subtract radical expressions.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

16) Q: Why can the radicals with the radicand of 2 not be combined?

A: *They do not have the same index. One is a square root, and the other is a cubed root, so they are not like terms.*

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Practice 2

Complete the problems on a separate sheet of paper.

Simplify. Write the terms using rational exponents. The final answer should be in radical form.

1) $\sqrt{xy^4}$ $y^2\sqrt{x}$

2) $\sqrt{a^2b^3}$ $ab\sqrt{b}$

3) $\sqrt[3]{x^9y^{15}}$ x^3y^5

4) $\sqrt[3]{x^2y^8}$ $y^2\sqrt[3]{x^2y^2}$

5) $\sqrt{18a^{20}y^4}$ $3a^{10}y^2\sqrt{2}$

6) $\sqrt{15x^{100}y^5}$ $x^{50}y^2\sqrt{15y}$

7) $\sqrt[3]{x^{19}y^{14}z^{12}}$ $x^6y^4z^4\sqrt[3]{xy^2}$

8) $\sqrt[3]{48x^{18}y^7}$ $2x^6y^2\sqrt[3]{6y}$

9) $\sqrt{20a^{20}b^3}$ $2a^{10}b\sqrt{5b}$

10) $\sqrt[3]{54x^{22}y^{20}}$ $3x^7y^2\sqrt[3]{2xy^2}$

Simplify. Remember to only combine like terms.

11) $\sqrt{5a} + 3\sqrt{5a}$ $4\sqrt{5a}$

12) $7\sqrt{2z} - \sqrt{2z}$ $6\sqrt{2z}$

13) $6\sqrt{5} - 12\sqrt{5} + \sqrt{15}$ $\sqrt{15} - 6\sqrt{5}$

14) $8\sqrt{3} + 5\sqrt{3} - 6\sqrt{2} + \sqrt{2}$ $13\sqrt{3} - 5\sqrt{2}$

15) $\sqrt[3]{125} + \sqrt{81} - \sqrt[3]{6}$ $14 - \sqrt[3]{6}$

16) $\sqrt{16} + \sqrt{8} - \sqrt[3]{1000} - \sqrt[3]{2000}$ $-6 + 2\sqrt{2} - 10\sqrt[3]{2}$

EXPLORE 29B

Part B: Radical Equations

Objectives

In this part of the lesson, you will learn about radical equations.

By the end of this lesson, you will be able to do the following:

- Solve equations with rational exponents.
- Solve second degree polynomial equations by taking the square root.
- Solve radical equations by taking the inverse of the square root.

Why?

Solving radical equations allows you to use a wide variety of formulas. This sets the foundation for learning about and using the quadratic formula and pythagorean theorem, which are integral to mathematics.

Warm Up

Simplify. Answers should contain only positive exponents.

1) $\left(2^{\frac{1}{2}}\right)\left(2^{\frac{2}{3}}\right)$

$2^{\frac{3}{6}+\frac{4}{6}}$

$2^{\frac{7}{6}}$

2) $\left(k^{-\frac{1}{2}}\right)^{-\frac{4}{3}}$

$k^{\left(-\frac{1}{2}\right)\left(-\frac{4}{3}\right)} = k^{\frac{4}{6}}$

$k^{\frac{2}{3}}$

3) $\left(3^{5}\right)^{x}$

$3^{5(x)}$

3^{5x}

Rational Exponent Equations with Monomials

- Now that you can work with all of the exponent rules on your formula sheet and can write terms using rational exponents or radicals, you will be able to solve equations with **variables** that are **exponents**.
- When solving problems with exponents, it is crucial that you make sure the **bases are equal** before using any of the exponent rules.
- The step in the equation where the base is rewritten, is up to **you**.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

2) Q: How would you write problem 2 using a radical symbol?

A: $\sqrt[3]{k^2}$

Q: What is the radicand?

A: k^2

Q: What is the index?

A: *3*

Q: Why do the exponent rules work when exponents are fractions?

A: *Because the exponent rules are true for all real numbers, and fractions are real numbers.*

3) Q: How can you simplify problem 3 so there is only one exponent?

A: *Multiply the given exponents together.*

Example 1

Some steps are interchangeable. For example, students could first write 2^3 and then divide by this term, rather than 8.

$2^5 = 2^3a$

$\frac{2^5}{2^3} = a$

$2^5 - 3 = a$

$a = 2^2 = 4$

29B EXPLORE

Example 1

Solve for a.

$2^5 = 8a$

Plan Isolate the variable, a, using the algebraic properties. Find a common base.

$\frac{2^5}{8} = a$ ◀ Multiplication Property of Equality OR $2^5 = 2^3a$ ◀ Write all bases in terms of 2.

$\frac{2^5}{2^3} = a$ ◀ Multiplication Property of Equality

Implement	Explain	Check
$\frac{2^5}{2^3} = a$	◀ Rewrite the denominator, 8, as a power of 2.	$2^5 = 8(4)$
$2^{5-3} = a$	◀ Rule $\frac{a^b}{a^c} = a^{b-c}$	$32 = 8(4)$
$2^2 = a$	◀ Simplify.	$32 = 32$ ✓
$a = 4$		

- Remember, *all* of the **bases** on both sides of the equation need to be **equal** in order to solve the equation.
- When you are solving for an unknown exponent and the bases are equal, it means that the **exponents** are equal as well.
- This allows you to solve for the variable by setting the exponents **equal** to one another.

Example 2

Solve for a.

$16 = 2^{2a}$

Implement	Explain	Check
$2^4 = 2^{2a}$	◀ Rewrite 16 as 2^4 so the bases are equal on both sides.	$16 = 2^{2(2)}$
$4 = 2a$	◀ Solve for a. If the bases are equal, their exponents are equal.	$2^2 = 2^4$ ✓
$a = 2$	◀ Multiplication Property of Equality	

EXPLORE 29B

Example 3

Solve for a.

$2^{\frac{5}{4}a} = 16^3$

This problem is very similar to the previous example, but this time sixteen also has an exponent. All bases need to be two before the value of a can be found.

Implement	Explain
$2^{\frac{5}{4}a} = (2^4)^3$	Rewrite all terms so they have the same base.
$2^{\frac{5}{4}a} = 2^{12}$	Rule 2: $(a^b)^c = a^{bc}$
$\frac{5}{4}a = 12$	Solve for a.
$a = \frac{48}{5}$	Multiplication Property of Equality

Checkpoint

Solve for a.

$2^{\frac{3}{4}} = 4^{\frac{a}{4}}$

$2^{\frac{3}{4}} = (2^2)^{\frac{a}{4}}$

$2^{\frac{3}{4}} = 2^{\frac{2a}{4}}$

$\frac{3}{4} = \frac{1a}{2}$

$a = \frac{3}{2}$

Equations with Exponents

- From Lesson 3, you know that the absolute value of x is always a positive value. Symbolically, this is represented as $|x| = \sqrt{(x^2)}$.
- When solving for a variable that has been **squared**, the result is the absolute value of x.
- When x is squared, its original sign (+ or −) is unknown. This means there are **two solutions** that could have resulted in the product.
  - The plus–minus symbol (**±**) is used rather than writing both answers separately.
- Besides factoring or graphing, another way to solve a difference of two squares is by **isolating** the squared term and then finding the **root** of both sides.

Example 3

Rule 2: Simplify the exponents in the expression on the right side of the equation.

Solve for a: Since the bases are equal, the exponents are also equal.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can you write the base 4 as a power of 2?

A: *Two to the second power,* 2^2.

Set the exponents equal to one another and simplify the fraction.

Multiply both sides by two to isolate a and simplify the fraction.

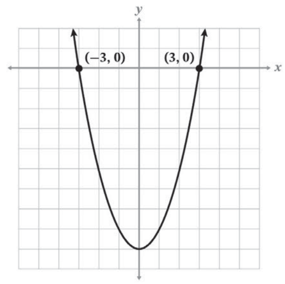
29B EXPLORE

Example 4

Solve.

$x^2 - 9 = 0$

Implement	Explain
$x^2 = 9$	Isolate the variable on one side of the equation.
$\sqrt{x^2} = \sqrt{9}$	The inverse of squaring a term is taking the square root or raising something to the $\frac{1}{2}$ power.
$\lvert x \rvert = 3$	Rule: $\lvert x \rvert = \sqrt{x^2}$
$x = 3$ OR $x = -3$	Absolute value equations have two solutions.
$x = \pm 3$	



Check

$(3^2) - 9 = 0$ $\quad$ $(-3^2) - 9 = 0$

$9 - 9 = 0$ $\quad$ $9 - 9 = 0$

$0 = 0$ $\quad$ $0 = 0$

- If the radicand is *not* a perfect square, then you will need to **simplify** the answer as you did in the beginning of this lesson.
- When solving for a variable with an even exponent, there will be **two** possible solutions because even exponents *always* result in a positive product.
  - Therefore, any equation in which the exponent is even and the product is negative has **no real solution**.
  - In other words, $x^2 = p$ will only have real solutions for a **non-negative** value of p.

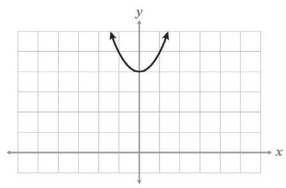
Example 5

Solve.

$x^2 = -4$ **no real solution**

There is no real number that when squared will result in a negative value.

If you look at the graph $y = x^2 + 4$, you can see that the parabola does not cross the x-axis. This is another reason why this equation has no real solution.



EXPLORE 29B

Checkpoint

Solve.

$x^2 + 1 = 9$

$x^2 = 8$

$\sqrt{x^2} = \sqrt{8}$

$|x| = 2\sqrt{2}$

$x = \pm 2\sqrt{2}$

Equations with Radicals

- Any real number squared is positive or **zero**.
- While $\sqrt{x^2}$ has two solutions, $\sqrt{x}$ only has **one solution** because the value of the radicand for an even root is non-negative.
- If a square root is set equal to a negative value, then there is **no solution**.

Example 6

Solve.

A) $\sqrt{2x - 3} = 5$

Plan $\cdot 2$, -3, $\sqrt{}$

Implement	Explain	Check
$\left(\sqrt{2x-3}\right)^2 = (5)^2$	Square both sides of the equation.	$\sqrt{2(14) - 3} = 5$
$2x - 3 = 25$	Simplify the expressions so no exponents remain.	$\sqrt{28 - 3} = 5$
$2x = 28$	Addition Property of Equality	$\sqrt{25} = 5$
$x = 14$	Multiplication Property of Equality	$5 = 5$ ✓

B) $\sqrt{x - 4} = 8$

Plan -4, $\sqrt{}$

Implement	Explain	Check
$(\sqrt{x-4})^2 = (8)^2$	Square both sides of the equation.	$\sqrt{68 - 4} = 8$
$x - 4 = 64$	Simplify the expressions so no exponents remain.	$\sqrt{64} = 8$
$x = 68$	Addition Property of Equality	$8 = 8$ ✓

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Example 6

A) Take the inverse of the square root on both sides of the equation.
Add 3 on both sides to isolate x.
Multiply both sides by $\frac{1}{2}$.

B) Take the inverse of the square root on both sides of the equation.
Add 4 to both sides to isolate x.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Take the inverse of the square root on both sides of the equation.

Subtract 2 from both sides to isolate h.

29B EXPLORE

Checkpoint

Solve using Plan, Implement, Explain.

$\sqrt{h+2} = 3$

Plan +2
$\sqrt{}$

Implement	Explain
$(\sqrt{h+2})^2 = (2)^2$	Square both sides of the equation.
$h + 2 = 9$	Simplify the expressions so no exponents remain.
$h = 7$	Addition Property of Equality

PRACTICE 1 29B

Practice 1

Complete the problems on a separate sheet of paper.

Solve for a.

1) $7^8 = (7^a)^{\frac{1}{2}}$ **$a = 16$**

2) $4^{\frac{3}{5}} = 2^a$ **$a = \frac{6}{5}$**

3) $(3^4)^{\frac{1}{2}} = 27a$ **$a = \frac{1}{3}$ OR $a = 3^{-1}$**

4) $10^{-2} = \frac{a}{100}$ **$a = 1$**

Solve for all possible solutions. Write "no real solution" if one does not exist.

5) $x^2 + 6 = 18$ **$x = \pm 2\sqrt{3}$**

6) $p^2 + 2 = 1$ **no real solution**

7) $x^2 - 75 = 0$ **$x = \pm 5\sqrt{3}$**

8) $x^2 + 5 = 30$ **$x = \pm 5$**

9) Why are there two solutions for $p^2 = 9$ but only one solution for $r = 9$ (assuming the variables are real numbers)?

Solve for all possible solutions. Write "no real solution" if one does not exist.

10) $\sqrt{b + 3} = 7$ **$b = 16$**

11) $\sqrt{r - 1} = 3$ **$r = 10$**

12) $\sqrt{\frac{1}{2}x + 1} = 5$ **$x = 48$**

13) $\sqrt{3x - 2} = 0$ **$x = \frac{2}{3}$**

Practice 1

Worked solutions for these problems are located in the Digital Pack.

2) Q: What must be equal before solving for a variable?

A: The bases must be equal.

3) This problem can be solved in more than one way. Your student may also notice that $3^2 = 9$, and $\frac{9}{27} = \frac{1}{3}$. This is also correct. If they solve this way, encourage them to try the problem using exponent rules. They may also write $3^4 = 81$.

4) Your student may want to write all terms using positive exponents.

Q: What is any base raised to the zero power?

A: One

6) Q: What needs to be isolated before you can take the square root of both sides?

A: *The term p^2 needs to be isolated.*

Q: How would problem 6 change if the problem was equal to 2?

A: *The answer would be zero because zero is neither positive nor negative.*

9) Sample:
Any value can be multiplied by itself. There are two values that when squared result in 9, −3 and 3. So p could equal 3 or −3. However, the radicand of an even root is the product of a number multiplied by itself, so it must be positive. No real number multiplied by itself is equal to −81. The only solution for r is 81.

10) Q: What is the inverse operation of a square root?

A: *Squaring, or raising something to the second power.*

13) Q: How is it possible for the square root to equal zero?

A: *Because zero-squared equals zero.*

Mastery Check

Show What You Know

Solving for an unknown variable in a formula then using the "new" formula ties together concepts from lessons 2, 19, 28, and 29.

Remind your student to use Plan, Implement, Explain. They should undo the operations happening to r as they did throughout the course. (See Lesson 2.)

A) The ± symbol is not needed here because the radius of a three dimensional figure cannot be negative.

Q: Why is it ok to omit the ± symbol for this formula?

A: Because the dimensions of a figure cannot be negative.

D) Note that this is only the height of the cylindrical portion of the silo. With the semi-sphere dome placed on top, it will be around 15 feet tall from the ground to the top of the dome.

Mastery Check

Show What You Know

Farmer Tatum knew the volume of grain that would be collected and stored in a grain silo. However, Tatum needed to determine the radius to make sure that the new grain silo would fit on the property.

A) Write the formula for the volume of a cylinder in terms of the radius, r.

$V = \pi r^2 h$

$\frac{V}{\pi h} = r^2$

$r = \sqrt{\frac{V}{\pi h}}$ **OR** $r = \left(\frac{V}{\pi h}\right)^{\frac{1}{2}}$

B) Write the formula for the volume of a semi-sphere in terms of the radius, r.

$V = \frac{2\pi r^3}{3}$

$3V = 2\pi r^3$

$\frac{3V}{2\pi} = r^3$

$r = \sqrt[3]{\frac{3V}{2\pi}}$ **OR** $r = \left(\frac{3V}{2\pi}\right)^{\frac{3}{2}}$

C) Using technology, determine the radius of the silo when the volume of grain is 486π cubic feet. Write down the formula you will use with the substituted volume before using technology.

$r = \sqrt[3]{\frac{3(486\pi)}{2\pi}}$

$r = 9$

The radius of the silo is 9 feet.

D) What is the height of the silo when the volume is 486 cubic feet? Show your work. (Hint: you need your answer from part C to find the height.)

$V = \pi r^2 h$

$V = 486\pi, r = 9$

$486\pi = \pi(9)^2 h$

$486\pi = 81\pi h$

$h = \frac{486\pi}{81\pi}$

$h = 6$

The height of the silo is 6 feet.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Solve equations with rational exponents.
- Solve second degree polynomial equations by taking the square root.
- Solve radical equations by taking the inverse of the square root.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

PRACTICE 2 29B

Practice 2

Complete the problems on a separate sheet of paper.

Solve.

1) $25^3 = 5^a$ **$a = 6$**

2) $64 = a^6$ **$a = 2$**

3) $\frac{1}{7} = 7^{\frac{a}{9}}$ **$a = -9$**

4) $\frac{1}{9^3} = (9^2)^a$ **$a = -\frac{3}{2}$**

Solve for all possible solutions. Write "no real solution" if one does not exist.

5) $x^2 - 3 = 13$ **$x = \pm 4$**

6) $c^2 + 3 = 21$ **$c = \pm 3\sqrt{2}$**

7) $x^2 - 12 = 0$ **$x = \pm 2\sqrt{3}$**

8) $x^2 + 1 = 51$ **$x = \pm 5\sqrt{2}$**

9) When is the square root taken of both expressions in an equation? Why would this result in two solutions?

Solve for all possible solutions. Write "no real solution" if one does not exist.

10) $\sqrt{k-3} = 8$ **$k = 121$**

11) $\sqrt{q+2} = 3$ **$q = 7$**

12) $\sqrt{\frac{2}{3}x + 8} = 8$ **$x = 84$**

13) $\sqrt{5x-7} = -3$ **no real solution**

Practice 2

Worked solutions for these problems are located in the Digital Pack.

3) Recall that a base raised to a negative exponent will be the reciprocal of the base.

9) Sample:
The square root is taken when solving for a variable that has been squared. Squaring the variable hides the sign of the value it represents. The value being squared could be positive or negative because both would result in the same solution. The two solutions represent both of these possibilities.

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Lesson Test

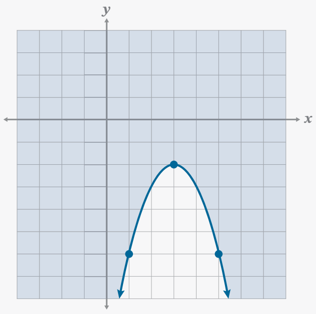
Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

5)



6) The direction of the parabola will open upwards and will be five times more narrow than the parent function. The vertex will move 11 spaces left and 26 spaces down from the origin.

7) Sample:
Quadratic equations can have zero, one, or two real solutions. Quadratic inequalities have an infinite number of solutions that are represented by the shaded region of the graph.

11) Distractor Rationale:
- A) This answer contains a negative exponent.
- C) This answer is the reciprocal of the correct answer.
- D) This answer does not follow the quotient rules for exponents and tries to apply the product rule incorrectly.

12) Distractor Rationale:
- A) This has the incorrect direction for the inequality symbol.
- C) This has the incorrect direction for the inequality symbol and represents the graph shifting right because $h = 3$.
- D) This represents the graph shifting right because $h = 3$.

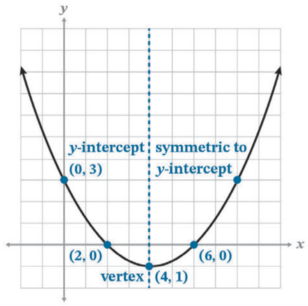
TARGETED REVIEW 29

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Copy the given graph of the parabola. Mark the vertex, y-intercept and its symmetric counterpart, and the solutions.



2) Name the domain and range for the parabola in problem 1. **domain: $(-\infty, \infty)$ range: $[-1, \infty)$**

3) Find the approximate solution(s) for the given quadratic equation. Use technology to determine the solution(s).
$y = 0.6x^2 - 1.3x - 1$ **solutions: $x = -0.602, 2.769$**

4) Find the product:
$5(x + y)^2$ **$5x^2 + 10xy + 5y^2$**

5) Graph the quadratic inequality.
$y \geq -(x - 3)^2 - 2$

6) Identify a, h, and k. Describe the transformation that will occur from the parent function.
$y = 5(x + 11)^2 - 26$ **$a = 5, h = -11, k = -26$**

7) How do the solution(s) change when graphing a quadratic inequality rather than a quadratic equation?

Simplify. Your answer should contain positive exponents only.

8) $\dfrac{2^{\frac{1}{2}} \cdot a^{-\frac{3}{4}}}{2^{-\frac{1}{2}} \cdot a^{\frac{1}{4}}}$ **$\dfrac{2}{a}$**

9) $\left(\dfrac{b^{-2}}{c^3 d^4}\right)^{-1}$ **$b^2c^3d^4$**

10) Write all terms in the numerator only.

11) $\dfrac{r^7}{q^{-6}p^3}$ **$p^{-3}q^6r^7$**

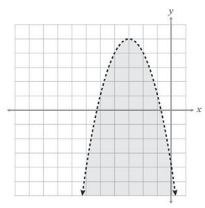
Multiple Choice

B **12)** Simplify. Your answer should contain positive exponents only.
$\dfrac{a^{18}b^{-3}}{a^9b^6}$

- A) a^9b^{-9}
- B) **$\dfrac{a^9}{b^9}$**
- C) $\dfrac{b^9}{a^9}$
- D) $\dfrac{a^{27}}{b^3}$

B **13)** Determine the inequality that matches the graph.

- A) $y > -(x + 3)^2 + 5$
- B) **$y < -(x + 3)^2 + 5$**
- C) $y > -(x - 3)^2 + 5$
- D) $y < -(x - 3)^2 + 5$



Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	26	26	26	20	27	27	27	28	28	28	28	27