Lesson 28
More Exponent Rules

Outline

Part A
Negative Exponents and Zero Exponents

- Negative Exponents
- Writing Expressions with Positive Exponents

Part B
Quotient Rules for Exponents

- Distributing Exponents across a Monomial Expression
- Quotient Property for Exponents
- Using All the Exponent Rules

Targeted Review

Vocabulary

- rational expression
- reciprocal

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Q: What would the answer be if the exponent 2 was changed to $\frac{1}{2}$?

A: *The answer would be x^4y^3 because the exponents would be half of the original.*

28A EXPLORE

Part A: Negative Exponents and Zero Exponents

Objectives

In this part of the lesson, you will learn about negative exponents and zero exponents.

By the end of this lesson, you will be able to do the following:

- Write expressions with exponents so that all terms are in the numerator.
- Simplify expressions containing negative and zero exponents using only positive exponents.

Why?

How can you rearrange terms and still maintain the same value? Learning the remaining exponent rules will demonstrate how this is possible and allow you to work with even more types of problems.

Warm Up

Simplify. Use your Formula Sheet.

1) $(7x^2y)(6xy^4)$

$42x^3y^5$

2) $(x^8y^6)^2$

$x^{16}y^{12}$

Negative Exponents

- A **rational** expression is an expression in which the numerator and/or denominator contain polynomials.
- To write a rational expression as a polynomial expression, you must move all terms to the **numerator**.
- When a term with an exponent moves from the denominator to the numerator, you must use the **opposite** sign for the exponent.
- Rule 4 of exponents allows you to rewrite any term in the denominator as a term in the numerator.
  - If an exponent is **positive** in the denominator, it will be negative when it is relocated to the numerator.
  - If an exponent is **negative** in the denominator, it will be positive when it is relocated to the numerator.
  - For all real numbers, $a^b = \frac{1}{a^{-b}}$ or $a^{-b} = \frac{1}{a^b}$

EXPLORE 28A

- Notice that the negative exponent represents the **reciprocal** of the base, *not* a **negative** value.

Example 1

Write each expression so that all terms are in the numerator.

A) $\frac{1}{x^4} = x^{-4}$

Explain

The term x^4 moves from the denominator to the numerator, changing the exponent sign from $(+)$ to $(-)$.

B) $\frac{a^5b^8}{c^3}$

$a^5b^8c^{-3}$

Explain

The term c^3 moves from the denominator to the numerator, changing the exponent sign from $(+)$ to $(-)$.

The other variables do not change because they are already in the numerator.

> Remember, when there are multiple variables, list them in alphabetical order.

Example 2

Write each expression so that all terms are in the numerator.

A) $\frac{5}{n^{-8}} = 5n^8$

Explain

The term n^{-8} moves to the numerator, changing the exponent's sign.

Five is already in the numerator and has an understood exponent of 1.

B) $\frac{x^5y^{-3}}{z^{-2}}$

$x^5y^{-3}z^2$

Explain

The term z^{-2} moves to the numerator, changing the exponent's sign.

The other variables do not change because they are already in the numerator.

Checkpoint

Rewrite the expression so that all terms are in the numerator.

$\frac{a}{b^{-18}c^{12}}$

$ab^{18}c^{-12}$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the exponent assigned to the variable a?

A: *1*

Q: What happens to the exponent when a term moves from the denominator to the numerator?

A: *The sign of the exponent is the opposite of the given sign.*

28A EXPLORE

Writing Expressions with Positive Exponents

- Rule 4, $a^b = \frac{1}{a^{-b}}$, and its inverse, $a^{-b} = \frac{1}{a^b}$, allow you to move terms to either the numerator or the denominator of a fraction, depending on how the expression should be **simplified**.
- In particular, rule 4 is useful when you are asked to write an expression using only **positive** exponents.
- To write an expression with only positive exponents, move terms with negative exponents to the **other side of the fraction bar**.

Example 3

Simplify. Write the expression using only positive exponents.

A) $\frac{x^{-3}y^2}{z^{-4}} = \frac{y^2z^4}{x^3}$

Explain

The terms x and z are moved to the other side of the fraction bar to change their exponents from negative to positive.

B) $\frac{8^{-2}p^{\frac{1}{2}}}{9^{-1}q^{-\frac{1}{2}}r^3}$

$\frac{9p^{\frac{1}{2}}}{8^2q^{\frac{1}{2}}r^3} = \frac{9p^{\frac{1}{2}}}{64q^{\frac{1}{2}}r^3}$

Explain

Terms are moved to the other side of the fraction bar to change their exponents from negative to positive.

Simplify any coefficients that you can calculate using mental math.

- While simplifying expressions so that all exponents are positive, you may encounter a base raised to the **zero** power.
  - Since zero is neither positive nor negative, **any base** to the zero power must be simplified from the expression.
- Rule 5 of exponents allows you to simplify terms raised to the zero power.
  - Any base raised to the zero power is equal to **one**.
  - For all real numbers, $a^0 = 1$.

EXPLORE 28A

Example 4

Simplify. Write the expression using only positive exponents.

A) $\frac{-5x^3y^0}{z}$

$\frac{-5x^3 \cdot 1}{z} = \frac{-5x^3}{z}$

Explain

$y^0 = 1$ (rule 5)

$x^3 \cdot 1 = x^3$ (Identity Property)

The coefficient -5 has an implied exponent of positive 1, so it remains in the numerator.

B) $\frac{a^{-3}b^{\frac{1}{2}}}{c^0}$

$\frac{b^{\frac{1}{2}}}{1 \cdot a^3} = \frac{b^{\frac{1}{2}}}{a^3}$

Explain

Move a to the other side of the fraction bar to change its exponent from negative to positive (rule 4)

$c^0 = 1$ (rule 5)

If all terms must move to the denominator so that the exponents are positive, then the numerator will contain the constant 1.

Checkpoint

Simplify. Write the expression using only positive exponents.

$\frac{7^{-2}z^0}{3^{-2}xy^{-4}}$

$\frac{3^2y^4 \cdot 1}{7^2x}$

$\frac{9y^4}{49x}$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

If your student does not simplify the coefficients, this is still a correct solution because the terms will still contain positive exponents.

When a term has zero as its exponent, it is also not necessary to replace it with 1 unless this is the only remaining term in the numerator. Since any value multiplied by 1 is itself (Multiplicative Identity Property), the 1 will often simplify out of the expression.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1–6)

Recall that the exponent 1 is not necessary to write in the expression. It has been omitted in the answers. Answers with more than one variable should be written alphabetically.

10) Notice the term $15y^0$. Only the y is raised to the zero power. The coefficient is 15^1, therefore, 15 will remain in the denominator.

Q: What is the exponent of the coefficient 15?

A: *1*

12) When the coefficient 4 is multiplied by a negative exponent, the base remains positive. The exponent tells you the number of times the base repeats and whether to take the reciprocal. In this case the base moves to the numerator in the final answer because the reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$.

28A PRACTICE 1

Practice 1

Complete the problems on a separate sheet of paper.

Write each expression so that all terms are in the numerator.

1) $\frac{a^{-9}b^3}{c^5d^{-7}}$ $a^{-9}b^3c^{-5}d^7$

2) $\frac{x^{-1}y^6}{z^{-3}}$ $x^{-1}y^6z^3$

3) $\frac{f^5}{e^{\frac{3}{2}}g^{-\frac{1}{2}}}$ $e^{-\frac{3}{2}}f^5g^{\frac{1}{2}}$

4) $\frac{p^5}{q^6r^{-7}}$ $p^5q^{-6}r^7$

5) $\frac{\left(x^4y^{\frac{1}{2}}\right)^2}{z^{-6}}$ x^8yz^6

6) $\frac{y^2z}{(w^8x^6)^{-\frac{1}{2}}}$ $w^4x^3y^2z$

Write the expressions using only positive exponents.

7) $\frac{2^{-12}x^4}{3^{-8}y}$ $\frac{3^8x^4}{2^{12}y}$

8) $\frac{x^{-\frac{1}{2}}y^{-3}}{z^{-4}}$ $\frac{z^4}{x^{\frac{1}{2}}y^3}$

9) $\frac{2^{-8}c^{-5}}{-14^0a^{11}b^{-2}}$ $\frac{b^2}{2^8a^{11}c^5}$

10) $\frac{\left(5x^2y^{\frac{1}{2}}\right)^0}{15y^0}$ $\frac{1}{15}$

11) $\frac{(2x^3)^2}{y^{-3}z^{-\frac{1}{2}}}$ $4x^6y^3z^{\frac{1}{2}}$

12) $\frac{a^2b^{-3}}{(4^2c^6)^{-\frac{1}{2}}}$ $\frac{4a^2c^3}{b^3}$

MASTERY CHECK 28A

Mastery Check

Show What You Know

A civil engineer used the following formula to determine the crushing load for a wooden square pillar:

$L = \frac{25T^4}{H^2}$

H: height of the post in feet
L: crushing load in tons
T: thickness of the wood in inches

A) Suppose the engineer has a post that is 6 inches thick and 8 feet tall. What is the crushing load? Use a calculator to determine the value.

$T = 6, H = 8$

$L = \frac{25(6)^4}{8^2} = 506.25$ **tons**

B) Rewrite the formula so that all terms are in the numerator.

$L = \frac{25T^4}{H^2}$

$L = 25T^4H^{-2}$

C) Use your formula from part B to calculate the crushing load for a 2-inch post that is 2 feet tall. Find this value *without* a calculator.

$T = 2, H = 2$

$L = 25T^4H^{-2}$

$L = 25(2^4)(2^{-2})$

$L = 25(2^2)$

$L = 100$ **tons**

D) Why can part C be simplified without using a calculator, but it is more efficient to use a calculator for part A?

Sample:
The numbers with exponents in part C have the same base. Because of this, it can be rewritten as one term 2^2 or 4. In part A, the bases are not the same, and the numbers are very large so using a calculator is more efficient.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Q: What variables already have given values? What are the values?

A: $T = 6, H = 8$

B) Q: What happens to the exponent when a base is moved from the denominator to the numerator?

A: *The sign of the exponent changes, in this case from positive to negative.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Write expressions with exponents so that all terms are in the numerator.
- Simplify expressions containing negative and zero exponents using only positive exponents.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

8) Q: What will be in the numerator if all terms move to the denominator?

A: *The numerator will contain the number 1 because it represents a rational number.*

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

28A PRACTICE 2

Practice 2

Complete the problems on a separate sheet of paper.

Write each expression so that all terms are in the numerator.

1) $\frac{e^4f^8}{g^{-12}}$ $e^4f^8g^{12}$

2) $\frac{p^3r^2}{q^4s^{-1}}$ $p^3q^{-4}r^2s$

3) $\frac{e^6f^{\frac{1}{2}}}{d^{\frac{1}{2}}}$ $d^{-\frac{1}{2}}e^6f^{\frac{1}{2}}$

4) $\frac{a^3c^{11}}{b^8f^{10}}$ $a^3b^{-8}c^{11}f^{-10}$

5) $\frac{w^6x^{-2}}{(z^{-4})^{-\frac{1}{4}}}$ $w^6x^{-2}z^{-1}$

6) $\frac{(w^{12})^{\frac{2}{3}}}{x^{-3}y^5}$ $w^8x^3y^{-5}$

Write the expressions using only positive exponents.

7) $\frac{4^{-1}x^4}{y^{-2}}$ $\frac{x^4y^2}{4}$

8) $\frac{w^{-6}z^{-11}}{x^3y^4}$ $\frac{1}{w^6x^3y^4z^{11}}$

9) $\left(\frac{-7x^{11}}{(w^{-1}z)^2}\right)^0$ 1

10) $\frac{a^{\frac{1}{2}}b^{-3}}{c^0}$ $\frac{a^{\frac{1}{2}}}{b^3}$

11) $\frac{(a^4b^{-2})^0}{5^{-1}c^7}$ $\frac{5}{c^7}$

12) $\frac{3^{-2}xz^{-1}}{(y^{-4})^{\frac{1}{2}}}$ $\frac{xy^2}{3^2z}$

EXPLORE 28B

Part B: Quotient Rules for Exponents

Objectives

In this part of the lesson, you will learn about quotient rules for exponents.

By the end of this lesson, you will be able to do the following:

- Simplify monomial expressions using the quotient rules for exponents.

Why?

Not all polynomial expressions will contain terms only in the numerator. This lesson will complete the set of exponent rules you need in order to work with all rational expressions and equations.

Warm Up

Simplify.

1) $(x^4y^8)^7$

$x^{28}y^{56}$

2) $a^{\frac{1}{2}} \cdot (a^9b^{12})^{\frac{1}{3}}$

$a^{\frac{1}{2}} \cdot a^3b^4$

$a^{\frac{3}{2}}b^4$

Distributing Exponents across a Monomial Expression

- Recall that with rule 3, the power of a product rule, if more than one base within parentheses is raised to a power, the exponent is distributed to ____**each base**____.
- Similarly, rule 6 is the ____**power of a quotient**____ rule for exponents:
  - If a fraction within parentheses is raised to a power, the exponent is distributed to ____**each base**____ in the numerator and denominator.
  - For all real numbers, $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$
- All other exponent rules are still true, so when a power is raised to a power the exponents are still ____**multiplied**____.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

This is a review of the exponent rules on the formula sheet (#1–3).

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Remember that your student may also move the terms so that only positive exponents remain and then distribute them.

28B EXPLORE

Example 1

Simplify. Write the expressions using only positive exponents.

A) $\left(\frac{x}{y^2}\right)^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{y}$

Explain

Distribute $\frac{1}{2}$ to each base in the expression.

You do not need to write an exponent for y in the final answer because $2 \cdot \frac{1}{2} = 1$.

B) $\left(\frac{2xy^3}{z^4}\right)^2$

$\frac{2^2x^2y^6}{z^8} = \frac{4x^2y^6}{z^8}$

Explain

Distribute 2 to each base. (rule 6)

If an exponent is already present, the exponents are multiplied together. (rule 2)

The number is simplified, but it could be left in exponential form as long as the exponents remain positive.

Example 2

Simplify. Write the expressions using only positive exponents.

$\left(\frac{b^{-2}c^{12}}{a^{-5}}\right)^3$

Explain

$\left(\frac{b^{-2}c^{12}}{a^{-5}}\right)^3 = \frac{b^{-6}c^{36}}{a^{-15}} = \frac{a^{15}c^{36}}{b^6}$

Distribute the exponent first.

OR

$\left(\frac{b^{-2}c^{12}}{a^{-5}}\right)^3 = \left(\frac{a^5c^{12}}{b^2}\right)^3 = \frac{a^{15}c^{36}}{b^6}$

Write terms with positive exponents first.

Both methods are correct and result in the same answer.

Checkpoint

Simplify. Write the expression using only positive exponents.

$\left(\frac{3^0c^8d^{-1}}{a^{-6}b^9}\right)^5$

$\frac{3^0c^{40}d^{-5}}{a^{-30}b^{45}}$

$\frac{a^{30}c^{40}}{b^{45}d^5}$

EXPLORE 28B

Quotient Property for Exponents

- Rule 7 is the **quotient** rule for exponents.
  - When terms with the same base are **divided**, their exponents are **subtracted** from one another.
  - For all real numbers, $\frac{a^b}{a^c} = a^{b-c}$
- When only positive exponents are in the expression, the base with the **greater** exponent will determine the location of the simplified expression.
  - If the exponent is greater in the **denominator**, then the simplified expression will go in the denominator.
  - If the exponent is greater in the **numerator**, then the simplified expression will go in the numerator.
- This rule can be used along with all previous exponent rules to write expressions with **positive** exponent values.

Example 3

Simplify the expressions using only positive exponents.

A) $\frac{x^2y^3}{x^6y^4}$

$\frac{1}{x^{6-2}y^{4-3}} = \frac{1}{x^4y}$

Explain

The greater exponent for both variables is in the denominator.

Because the numerator of the fraction must have a value, 1 is used.

When the exponents for y are subtracted, the result is 1.

B) $\frac{(11^{20})(3^5)}{(3^{14})(11^7)}$

$\frac{11^{20-7}}{3^{14-5}} = \frac{11^{13}}{3^9}$

Explain

The greater exponent for 11 is in the numerator.

The greater exponent for 3 is in the denominator.

Example 3

The middle step shown is optional because subtraction can be completed using mental math.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the result when you have x minus negative 1, $x - (-1)$?

A: *The expression becomes* $x + 1$.

- Pay close attention to the signs of exponents when working with negatives.
- Remember that subtracting a negative is the same as adding.
- Addition will be used rather than the opposite of a negative number.

> Pay careful attention when both exponents are negative. In this case, the number *closest* to zero is the *greater* exponent.

Example 4

Simplify the expressions using only positive exponents.

A) $\dfrac{3^{-4}a^{9}b^{2}}{3^{-1}a^{-3}b^{8}}$

$\dfrac{a^{9+3}}{3^{-1+4}b^{8-2}} = \dfrac{a^{12}}{3^{3}b^{6}}$

Explain

The greater exponent for 3 is in the denominator. (rule 7)

Move 3^{-4} to the other side of the fraction bar, which changes its sign from negative to positive. (rule 4)

The greater exponent for a is in the numerator. (rule 7)

Move a^{-3} to the other side of the fraction bar, which changes its sign. (rule 4)

The greater exponent for b is in the denominator. (rule 7)

Move b^{2} to the other side of the fraction bar, which changes its sign. (rule 4)

B) $\dfrac{4^{-8}a^{10}b^{-6}}{4a^{2}}$

$\dfrac{a^{10-2}}{4^{1+8}b^{6}} = \dfrac{a^{8}}{4^{9}b^{6}}$

Checkpoint

Simplify the expressions using only positive exponents.

$\dfrac{10^{5}x^{-7}}{10^{-6}x^{7}}$

$\dfrac{10^{5+6}}{x^{7+7}}$

$\dfrac{10^{11}}{x^{14}}$

EXPLORE 28B

Using All of the Exponent Rules

- All of the exponent rules work together so that **rational** expressions can be **simplified**.
- Exponents can be **fractions** (rational numbers) or **integers**.
- An expression is simplified when the **base** occurs only **once** in the given expression.
- Because all of the bases are being multiplied together, there are **multiple** ways to approach simplifying expressions.

Example 5

Simplify the expressions using only positive exponents.

$$\left(\frac{x^5z^3}{x^{11}y^{-6}}\right)\left(\frac{z^4}{x^{10}y^{-2}}\right)^{\frac{1}{2}}$$

Explain

$$\left(\frac{y^6z^3}{x^6}\right)\left(\frac{z^2}{x^5y^{-1}}\right)$$

Simplify the variables in each expression.

The first rational expression now has all positive exponents and only one occurrence of x, y, and z.

For the second rational expression, the exponent $\frac{1}{2}$ was distributed to all of the bases.

$$\frac{y^6z^{3+2}}{x^{6+5}y^{-1}} = \frac{y^{6+1}z^5}{x^{11}} = \frac{y^7z^5}{x^{11}}$$

Multiply the expressions.

Checkpoint

Simplify the expressions using only positive exponents.

$$\left(\frac{(2x)^{-1}x^{-1}y}{(3y^2)^{-4}x^5}\right)^3$$

$$\left(\frac{2^{-1}x^{-1}x^{-1}y^1}{3^{-4}y^{-8}x^5}\right)^3 = \left(\frac{2^{-1}x^{-2}y^{1+8}}{3^{-4}x^5}\right)^3 = \left(\frac{3^4y^9}{2x^{5+2}}\right)^3 = \left(\frac{3^4y^9}{2x^7}\right)^3$$

$$\frac{3^{12}y^{27}}{2^3x^{21}}$$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Remind your student to start with the innermost set of parentheses.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

10–12)
Your student should have their Formula Sheet available as reference.

10) Remember that expressions can be simplified using different approaches. This is one possible way to simplify this problem.

11) Recall: $4^{\frac{1}{2}} = 2$ and $\frac{10}{2} = 5$

28B PRACTICE 1

Practice 1

Complete the problems on a separate sheet of paper.

Simplify the expressions using only positive exponents.

1) $\left(\frac{y}{x}\right)^{-2}$ $\frac{x^2}{y^2}$

2) $\left(\frac{a^3b^8}{c}\right)^4$ $\frac{a^{12}b^{32}}{c^4}$

3) $\left(\frac{b^{-2}c^{12}}{a^{-5}}\right)^3$ $\frac{a^{15}c^{36}}{b^6}$

4) $\left(\frac{w^5y^3}{x^{-2}z^{-1}}\right)^{-3}$ $\frac{1}{w^{15}x^6y^9z^3}$

5) $\frac{x^5y^2}{x^3y^8}$ $\frac{x^2}{y^6}$

6) $\frac{x^8}{x^{-8}}$ x^{16}

7) $\frac{6^{\frac{2}{3}}x}{6^{\frac{1}{3}}x^4}$ $\frac{6^{\frac{1}{3}}}{x^3}$

8) $\frac{a^7b^{-1}c^{-5}}{a^3b^{-4}c^{-2}}$ $\frac{a^4b^3}{c^3}$

9) $\frac{5^8x^{-2}y^7}{5^5x^4y^3}$ $\frac{5^3y^4}{x^6}$

Simplify. Multiple exponent rules can be applied within each problem.

10) $\left(\frac{a^{-6}c^8}{a^{-1}b^3}\right)^2\left(\frac{a^8b^{-4}}{c}\right)^{-3}$ $\frac{b^6c^{19}}{a^{34}}$

11) $\left(\frac{x^{-3}}{(4x^8)^{\frac{1}{2}}}\right)(10x^{15})$ $5x^8$

12) $\left(\frac{(2x^3)^4 \cdot xy}{3y^{-1}z}\right)^2$ $\frac{2^8x^{26}y^4}{3^2z^2}$ or $\frac{256x^{26}y^4}{9z^2}$

MASTERY CHECK 28B

Mastery Check

Show What You Know

A) Write the expression using *only one* exponent. Expression: $\frac{x^5}{y^5}$

$\left(\frac{x}{y}\right)^5$

B) Write the expression using *only one* exponent at least *three* different ways.

Expression: $\frac{5^3}{5^3}$

$\left(\frac{5}{5}\right)^3$ or 1^3 or 1^0 or 5^0 or $\left(\frac{125}{125}\right)^1$

C) Show why $5^0 = 1$ without using the exponent rule. Explain your thinking.

$\frac{5^3}{5^3} = \frac{5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5} = 1 \cdot 1 \cdot 1 = 1$

or

$\frac{5^3}{5^3} = \frac{125}{125} = 1$

When this expression is expanded or the exponents are simplified, the same value is in the numerator and denominator. When the same number is divided by itself, the result is 1. Using the exponent rule $\frac{a^b}{c^c} = a^{b-c}$ with the expression $\frac{5^3}{5^3}$ results in $5^{3-3} = 5^0$. Since $\frac{5^3}{5^3} = 1$, then 5^0 must also equal 1.

D) Complete the following sentences with one of the following words: always, sometimes, never.

Any term raised to the zero power is **always** equal to one.

When the same base is divided, the exponent is **sometimes** subtracted.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

C) In other words, prove that the exponent rule is true. (Hint: Use part B to help show this.)

D) This is the exponent rule in words. This is always true.

Problem 9 in Practice 1 shows that some exponents will be subtracted while others will be added.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Simplify monomial expressions using the quotient rules for exponents.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

10–12)

Your student should have their formula sheet available for reference.

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Practice 2

Complete the problems on a separate sheet of paper.

Simplify the expressions using only positive exponents.

1) $\left(\frac{x}{y}\right)^7$ $\quad \frac{x^7}{y^7}$

2) $\left(\frac{b^{12}d^{-3}}{a^3c^6}\right)^{\frac{1}{3}}$ $\quad \frac{b^4}{ac^2d}$

3) $\left(\frac{a^3b^{-1}}{c^6}\right)^{-2}$ $\quad \frac{b^2c^{12}}{a^6}$

4) $\left(\frac{5^{-1}x^4}{3y^5z^{-1}}\right)^{-2}$ $\quad \frac{3^2 \cdot 5^2y^{10}}{x^8z^2}$

5) $\frac{2^{-3}x^5}{2^{-5}x^2}$ $\quad 2^2x^3 = 4x^3$

6) $\frac{a}{a^{\frac{1}{2}}}$ $\quad a^{\frac{1}{2}}$

7) $\frac{3^6a^5b}{3^8a^3b^2}$ $\quad \frac{a^2}{3^2b}$

8) $\frac{x^{25}y^2z^3}{x^{-11}y^{15}z^{-3}}$ $\quad \frac{x^{36}z^6}{y^{13}}$

9) $\frac{5^{\frac{5}{3}}y^{\frac{1}{2}}}{5^{-\frac{1}{3}}y^{\frac{1}{2}}}$ $\quad 5^2 = 25$

Simplify. Multiple exponent rules can be applied within each problem.

10) $\left(\frac{5x^6}{y^8}\right)^2\left(\frac{3x^{-11}y^3}{x^{-2}}\right)$ $\quad \frac{75x^3}{y^{13}}$

11) $\frac{\left(x^3y^{\frac{1}{2}}\right)^3(2x^5)^{-1}}{(3x)^0\left(5xy^{\frac{1}{2}}\right)^2}$ $\quad \frac{x^2y^{\frac{1}{2}}}{50}$

12) $(5a^8b^0)^{\frac{1}{2}} \cdot \left(\frac{a^4b^5}{10a^{11}(b^6)^2}\right)$ $\quad \frac{5^{\frac{1}{2}}}{10a^3b^7}$

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

TARGETED REVIEW 28

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Given the quadratic equation $y = x^2 + 5x + 6$, find the axis of symmetry, vertex, and the y-intercept.

2) Factor $0 = x^2 + 5x + 6$ to find the solutions.

3) Graph $y = x^2 + 5x + 6$.

4) Name the domain and range in interval notation for the graph in problem 3.

5) Describe the transformation of the given function from the parent graph. $g(x) = 2(x + 42)^2 - 18$

6) Identify a, h, and k in the equation $y = -(x - 3)^2 - 1$. Then describe the transformation from the parent function.

7) Graph $y = -(x - 3)^2 - 1$.

8) Evaluate.

A) $\sqrt{25}$ **5**

B) $-\sqrt{121}$ **−11**

9) Factor completely.
$30x^2 - 110x - 1{,}900$ **$10(3x + 19)(x - 10)$**

10) Simplify.
$(5x^2 + 6x + 30) - (x^2 + 2x + 13)$ **$4x^2 + 4x + 17$**

Multiple Choice

11) Select all statements that are true about the function $f(x) = (x - 5)^2 + 4$.

- ■ **$f(x)$ shifts up 4 spaces from the origin.**
- ☐ $f(x)$ shifts down 4 spaces from the origin.
- ■ **$f(x)$ shifts right 5 spaces from the origin.**
- ☐ $f(x)$ shifts left 5 spaces from the origin.

12) Select all statements that are true about the parabola $y = \frac{1}{2}x^2 + 4x - 10$.

- ☐ The range of the parabola includes all real numbers.
- ■ **The vertex is $(-4, -18)$.**
- ■ **$(-10, 0)$**
- ■ **$(2, 0)$**

Targeted Review

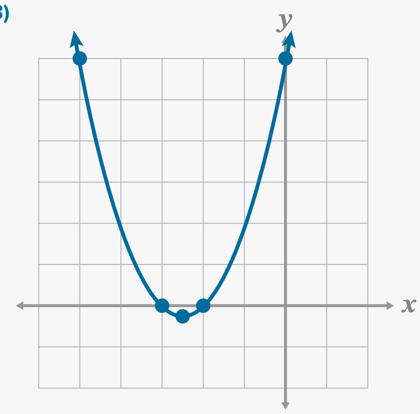
Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

1) AoS: $x = -2.5$
vertex: $(-2.5, -0.25)$
y-intercept: $(0, 6)$

2) $x = -3, -2$

3)

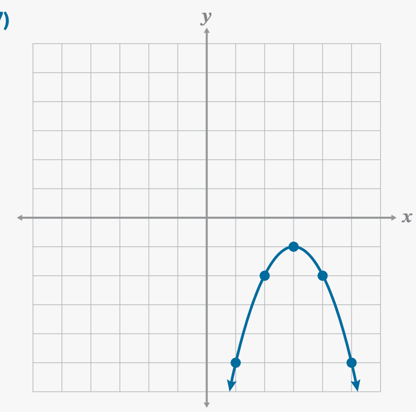


4) domain: $(-\infty, \infty)$
range: $[-0.25, \infty)$

5) The graph is dilated by a factor of 2 since $a = 2$. The graph will shift 42 spaces left and 18 spaces down from the origin because $(h, k) = (-42, -18)$

6) $a = -1$, $h = 3$, $k = -1$
The graph will reflect over the x-axis. The graph will shift right 3 spaces and down 1 space from the origin.

7)



11) Distractor Rationale:
The 2nd and 4th box represent $(-5, -4)$. This graph is $(5, 4)$ for (h, k).

12) Distractor Rationale:
The range is $[-18, \infty)$.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	26	24	26	26	27	27	27	PA	24	20	27	26