Lesson 27
More Quadratic Graphing

Outline

Part A
Quadratic Inequalities

- Quadratic Inequalities
- Graphing Quadratic Inequalities

Part B
Quadratics in Vertex Form

- Parabolas in Vertex Form
- Vertical and Horizontal Translations of Parabolas
- Reflections and Dilations of Parabolas
- Combining Transformations of Parabolas

Targeted Review

Vocabulary

- vertex form
- transform
- translate
- reflect
- dilate

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

27A EXPLORE

Part A: Quadratic Inequalities

Objectives

In this part of the lesson, you will learn about quadratic inequalities.

By the end of this lesson, you will be able to do the following:

- Graph quadratic inequalities.

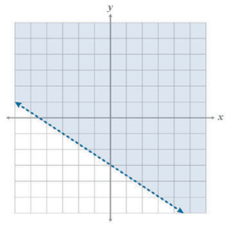
Why?

How is the shaded region of a parabola different from that of a linear inequality? This lesson sets a path toward learning about the area under a curve, which you will need for future math courses.

Warm Up

Graph the linear inequality on the coordinate plane.

$y > -\frac{2}{3}x - 3$



How did you determine the shading for the graph in problem 1?

Since the symbol is > (is greater than), the graph will be shaded above the y-intercept (above the line).
OR
Using the point $(0, 0)$, $0 > -3$, so the shading belongs on the side of the line where $(0, 0)$ is located.

Quadratic Inequalities

- Quadratic inequalities have the same features as quadratic **equations**, with two additions:
  - The parabola will be **solid or dashed** depending on the inequality symbol.
  - The graph of the quadratic inequality will be **shaded**, either inside or surrounding the parabola.
- There are four types of quadratic inequalities:

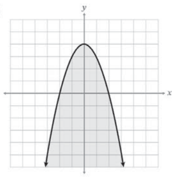
Symbol	Wording	Represented Graphically
$>$	**is greater than**	dashed graph
$<$	**is less than**	dashed graph
$\leq$	**is greater than or equal to**	solid graph
$\geq$	**is less than or equal to**	solid graph

- When determining the shading of a quadratic inequality, the **origin (0, 0)** is a reliable point to use because most parabolas do not intersect the origin.
    - When using the origin, the variables simplify out, leaving only the **constants** to compare.
    - The only time you cannot use (0, 0) as a test point is if the parabola **intersects** it.

Example 1

Match the inequalities to the correct graph. List key features about the inequality to determine the correct graphical representation.

A)



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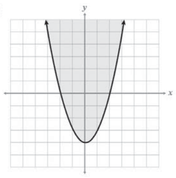

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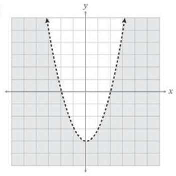
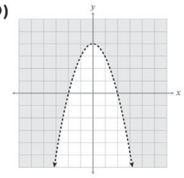


D)



Match		Inequality	Observations about the inequality	Algebraic reasoning using (0, 0)
B	1)	$y \geq x^2 - 4$	solid	$0 \geq (0)^2 - 4$
			shading above -4	$0 \geq -4$
			$a = 1$ (opens upward)	True.
			$c = -4$	Shade where $(0, 0)$ is located.
C	2)	$y < x^2 - 4$	dashed	**$0 < (0)^2 - 4$**
			shading below -4	**$0 < -4$**
			$a = 1$ (opens upward)	**False.**
			$c = -4$	**Do not shade where $(0, 0)$ is located.**
D	3)	$y > -x^2 + 4$	**dashed**	$0 > (0)^2 - 4$
			shading above $+4$	$0 > 4$
			$a = -1$ (opens downward)	False.
			$c = 4$	Do not shade where $(0, 0)$ is located.
A	4)	$y \leq -x^2 + 4$	**solid**	**$0 \leq -(0)^2 + 4$**
			shading below $+4$	**$0 \leq 4$**
			$a = -1$ (opens downward)	**True.**
			$c = 4$	**Shade where $(0, 0)$ is located.**

27A EXPLORE

- Another way to determine the shading of a parabola is to use the ___***y*-intercept**___ and the ___**direction**___ of the parabola (a).
- Sketching gives you an idea of what the graph will look like to help you determine if the graph is ___**reasonable**___ and that all the components match.

Example 2

Make a simple sketch of the given inequality.

A) $y \geq -x^2 + 2$
$a = -1, b = 0, c = 2$

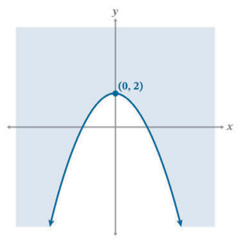
graph: ≥ , solid

direction: down

shading: above the y-intercept, outside

algebraic reasoning:
$0 \geq 2$ false

Shading is located where $(0, 0)$ is not located.



B) $y < -x^2$
$\boldsymbol{a = -1, b = 0, c = 2}$

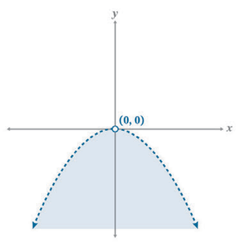
graph: **< , dashed**

direction: **down**

shading: **below the *y*-intercept, inside**

algebraic reasoning:
$0 < 0$ false (but lies on the parabola)

The point $(1, 0)$ shows $0 < -1$ which is false.
Shading is located inside the parabola.



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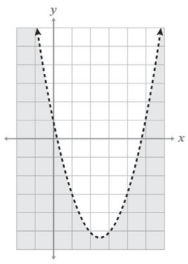
EXPLORE 27A

Checkpoint

Determine which inequality is the best representation of the given graph.

A) $y > x^2 - 5x + 1$

B) $y < x^2 - 5x + 1$



Explain your reasoning.

The correct answer is B.
The graph is shaded below the y-intercept ($c = 1$). Shading below the y-intercept when the parabola opens upward ($a = +1$) results in shading around the parabola.

Using $(0, 0)$:

A) $0 > 1$ **False**

B) $0 < 1$ **True**

Graphing Quadratic Inequalities

- When shading a graph of a parabola, the shading will either be **inside** the parabola or **surrounding** the parabola.
- Where you shade will be determined by the **inequality** symbol as well as the **inequality** of the parabola.
- The solution to a quadratic inequality is **every** point in the shaded region.
- Using **technology** is another way to determine whether the parabola will be solid or dashed as well as where the shading belongs.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Besides a, b, and c, what do you need to pay attention to in order to make the correct match?

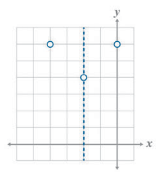
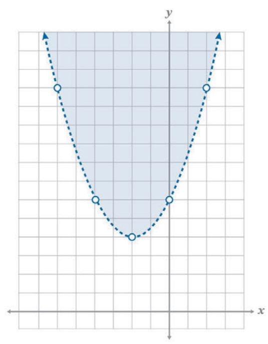
A: *The direction of the inequality symbol.*

27A EXPLORE

Example 3

Graph the quadratic inequality.

$y > \frac{1}{2}x^2 + 2x + 6$

Implement	**Explain (Steps to Graphing)**
$y > \frac{1}{2}x^2 + 2x + 6$	**1)** If needed, write the equation in standard form. $ax^2 + bx + c$
$a = \frac{1}{2}, b = 2, c = 6$	**2)** Identify:
opens upward	**A)** a, b, c
dashed	**B)** If the graph opens upward or downward (using +/− a)
shaded inside the parabola	**C)** Shading
AoS $= -\left(\frac{2}{2 \cdot \frac{1}{2}}\right) = -2$	**3)** Calculate the AoS and graph it.
vertex: $y = \frac{1}{2}(-2)^2 + 2(-2) + 6 = 4$ $(-2, 4)$	**4)** Calculate the vertex and graph it.
y-intercept: $(0, 6)$	**5)** Solve for and graph the y-intercept (c) and its counterpart.
symmetric to the y-intercept: $(-4, 6)$	
$x = (2, ?)$ $y = \frac{1}{2}(2)^2 + 2(2) + 6 = 12$ (2, 12) and the point symmetric to it is $(-6, 12)$	**6)** Solve for and graph another ordered pair. Use the x-intercepts if possible. 
	**7)** Connect the ordered pairs to make a continuous graph and shade the correct area.

EXPLORE 27A

Example 4

Graph the quadratic inequality.

$y < -2x^2 + 4x$

$a = -2, b = 4, c = 0$

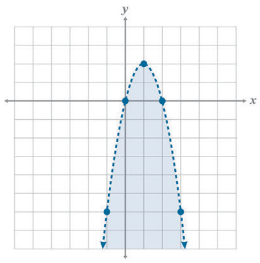
opens downward
dashed
shaded inside the parabola

$\text{AoS} = -\left(\frac{4}{2 \cdot (-2)}\right) = 1$

vertex: $y = -2(1)^2 + 4(1) = 2$
$(1, 2)$

y-intercept: $(0, 0)$
symmetric to the y-intercept: $(2, 0)$

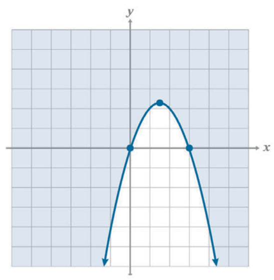
$x = (-1, ?)$ $y = -2(-1)^2 + 4(-1) = -6$
$(2, 12)$ and the point symmetric to it is $(3, -6)$



Checkpoint

Graph the inequality. Mark the vertex and x-intercepts.

$y \geq -x^2 + 3x$



Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Since this graph intersects the graph at $(0, 0)$, using the point $(1, 0)$ would be a good alternate. Your student can pick any point on the graph, but using one with zero as a coordinate will make mental math possible.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1–4) Your student should identify the key information from the inequalities before trying to match. Knowing the expected direction and shading before matching makes it a mathematical process, rather than a guess and check. Your student should not use technology for the matching.

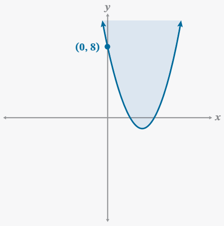
1) $a = -5$, opens downward, $c = 3$, y-int $(0, 3)$ dashed, shading above y-int or $0 > 3$, false

2) $a = 4$, opens upward, $c = -3$, y-int $(0, -3)$ dashed, shading above y-int or $0 > -3$, true

3) $a = 6$, opens upward, $c = -3$, y-int $(0, -3)$ dashed, shading below y-int or $0 < -3$, false

4) $a = -4$, opens downward, $c = 3$, y-int $(0, 3)$ dashed, shading below y-int or $0 < 3$, true

5) The parabola will open upward since $a = +1$ and will be solid because the symbol is greater than or equal to. The shading will be above the y-intercept, $c = 8$ (inside the parabola).



Using $(0, 0)$ results in a false statement, so the shading will be where $(0, 0)$ is not located.

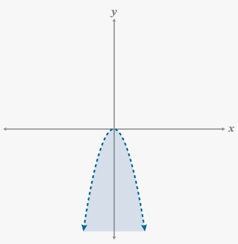
Q: What are the factors of $x^2 - 6x + 8$?

A: $(x - 4)(x - 2)$

Q: What does this factored expression tell you about the x-intercepts?

A: *The graph will cross the x-axis at 2 and 4.*

6) The parabola will open downward because $a = -1$. The parabola will be dashed because the symbol is less than. The shading will be below the y-intercept, $c = 0$ (inside the parabola). Using $(1, 0)$ results in a false statement, so the shading will be where $(1, 0)$ is not located.



The point $(0, 0)$ cannot be used since the graph intersects the origin.

7–10)

Ask the following questions if your student needs help graphing these problems:

Q: What point can you use to determine the shading for most quadratic inequalities?

A: $(0, 0)$ *the origin*

Q: What coefficient tells you the direction of the parabola?

A: *a, the number in front of* x^2

Q: Which value represents the y-intercept?

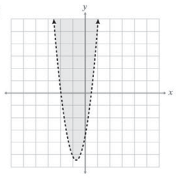
A: The value of c.

Practice 1

Complete the problems on a separate sheet of paper.

Match the quadratic inequality to the graph. List details that help determine the matching graph for the given inequality.

A)



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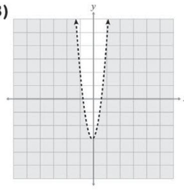
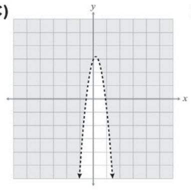
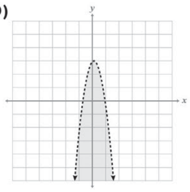


C)



D)



1) $y > -5x^2 + 2x + 3$ **C**

2) $y > 4x^2 + 6x - 3$ **A**

3) $y < 6x^2 + x - 3$ **B**

4) $y < -4x^2 + x + 3$ **D**

Sketch a graph of the given inequality. Explain your sketch.

5) $y \geq x^2 - 6x + 8$

6) $y < -x^2$

Graph the quadratic inequality. Mark the vertex, y-intercept, and x-intercepts (if possible).

7) $y > x^2 - 2x - 1$

8) $y < 2x^2 - 5x - 3$

9) $y \geq -4x^2 + 1$

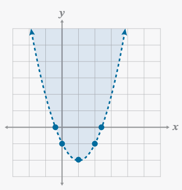
10) $y < -x^2 - 6x$

Complete the sentence using the word always, sometimes, or never. Explain your reasoning.

11) The symbol, >, will ________ result in a dashed graph. **always**

12) When the leading coefficient is positive, the graph will ______ be shaded inside the parabola. **sometimes**

7)



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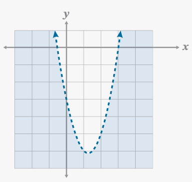

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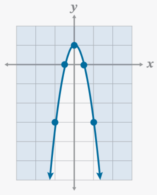
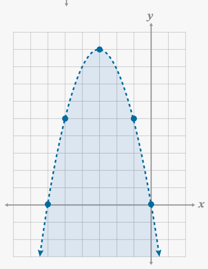


10)



11) Always; the symbol is greater than, which does not include the points on the parabola; it only includes the shaded region.

If your student needs help comparing solid and dashed parabolas, use any of the graphs from 1–10 to help them compare.

12) Sometimes; while the direction of the graph can help determine the shading, a positive leading coefficient does not always mean the graph is shaded inside. The shading is also determined by the inequality symbol.

Use the graphs from problems 7 and 8 to help guide this explanation.

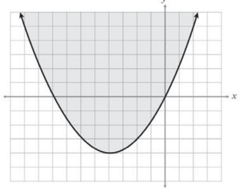
MASTERY CHECK 27A

Mastery Check

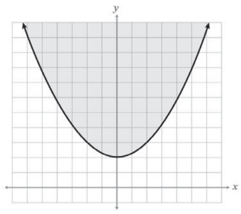
Show What You Know

Two graphs have been created to represent the quadratic inequality: $y \geq \frac{1}{4}x^2 + 2x$

Graph A:



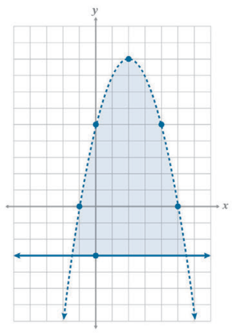
Graph B:



A) Determine which graph is correct and explain the error with the other graph.

Graph A: $c = 0$ Graph B: $c = 2$ Given Equation: $c = 0$

Graph A is correct because the y-intercept is $(0, 0)$. Graph B is incorrect because it has a y-intercept of $(0, 2)$. The x next to the two was ignored.



B) The following system contains one quadratic inequality and one linear inequality. Make a graph to find the solutions to both inequalities.

$y \geq -3$
$y < -x^2 + 4x + 5$

C) What is the slope of the line $y \geq -3$?

Since this is a horizontal line, the slope is zero.

D) Determine algebraically if the given ordered pairs are solutions to the system in part B. Explain. $\{P: (0, -3), Q: (0, 0), R: (5, 0)\}$

P: **$(0, -3)$** **$-3 \geq -3$ True** **$-3 < 5$ True**	Q: **$(0, 0)$** **$0 \geq -3$ True** **$0 < 5$ True**	R: **$(5, 0)$** **$0 \geq -3$ True** **$0 < 0$ False**
Both inequalities are true; therefore, this is a solution	**Both inequalities are true; therefore, this is a solution.**	**Only one inequality is true; this point is not a solution to the system.**

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Extend: Using technology, see if your student can recreate graph B to better describe the error. The inequality they find should be $y \geq \frac{1}{4}x^2 + 2$.

The shading will be inside the parabola.

B) Your student should be able to create a graph using what they have learned in Unit 3 and what they have learned so far in this lesson.

D) Remind your student that systems are like AND compound inequalities. A point must be true for all of the inequalities to be a solution for the system.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Graph quadratic inequalities.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

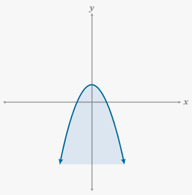
1) $a = 2$, open up, $c = -5$, y-intercept $(0, -5)$ dashed, shading inside parabola or $0 > -5$, true

2) $a = -2$, open down, $c = -5$, y-intercept $(0, -5)$ solid, shading around parabola or $0 \geq -5$, true

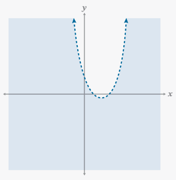
3) $a = 2$, open up, $c = 0$, y-intercept $(0, 0)$ solid, shading inside parabola, using $(1, 0)$, $0 > -3$, true

4) $a = 5$, open up, $c = 2$, y-intercept $(0, 2)$ dashed, shading inside parabola or $0 > 2$, false

5) The leading coefficient is negative so the graph will open down. The parabola will be solid because of the less than or equal to symbol. The shading will be below the y-intercept and inside the parabola. Using $(0, 0)$, results in a true statement, so the shading will be where $(0, 0)$ is located.



6) The leading coefficient is positive so the graph will open up. The parabola will be dashed because the symbol is greater than. The shading will be below the y-intercept and around the parabola. Using $(1, 0)$ results in a false statement, so the shading will be opposite of where $(1, 0)$ is located.



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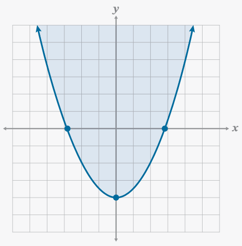

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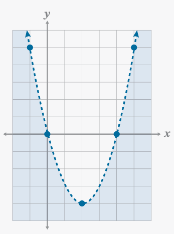

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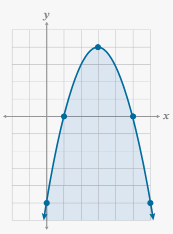
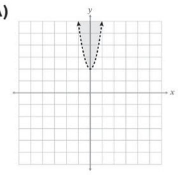
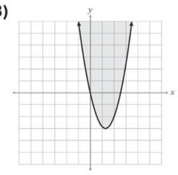
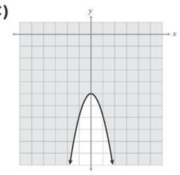
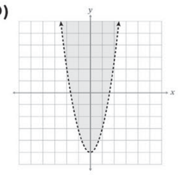


Practice 2

Complete the problems on a separate sheet of paper.

Match the quadratic inequality to the graph. List details that help determine the matching graph for the given inequality.

A)  B)  C)  D) 

1) $y > -2x^2 - 5$ **D**

2) $y \geq -2x^2 - 5$ **C**

3) $y \geq 2x^2 - 5x$ **B**

4) $y > 5x^2 + 2$ **A**

Sketch a graph of the given inequality. Explain your sketch.

5) $y \leq -3x^2 + 1$

6) $y < \frac{1}{2}x^2 - x$

Graph the quadratic inequality. Mark the vertex, y-intercept, and x-intercepts (if possible).

7) $y \geq \frac{1}{2}x^2 - 4$

8) $y < x^2 - 4x$

9) $y \leq -(x^2 - 6x + 5)$

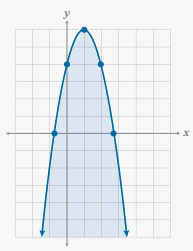
10) $y \leq -2x^2 + 4x + 4$

Complete the sentence using the word always, sometimes, or never. Explain your reasoning.

11) The inequality $y < ax^2$ will ______ be shaded on the outside of the parabola. **sometimes**

12) The vertex of a quadratic inequality is ______ the minimum or maximum point of a parabola. **always**

10)



11) Sometimes; the value of the leading coefficient will also help determine direction of the graph and whether the shading is inside or surrounding the parabola.

Remember, the direction of the parabola is determined by the value of a.

12) Always; the vertex is the minimum or maximum for quadratic inequalities just like equations.

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

EXPLORE 27B

Part B: Quadratics in Vertex Form

Objectives

In this part of the lesson, you will learn about quadratics in vertex form.

By the end of this lesson, you will be able to do the following:

- Recognize vertex form and identify and graph a, h, and k.
- Translate a parent graph vertically and horizontally and describe the transformation.
- Reflect and dilate a parent graph and describe the transformation.
- Transform a parent graph using more than one transformation (translation, reflection, dilation) and describe the transformation.

Why?

Why is the vertex not at the origin? How did the parabola get to be so narrow? Vertex form allows you to efficiently graph parabolas and describe how a graph is transformed from the parent function.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Determine the slope and the ordered pair from point-slope form.

1) $y-7=-5(x+6)$ $\quad$ $m=-5$, $(-6, 7)$

2) $y+2=\frac{1}{3}(x-12)$ $\quad$ $m=\frac{1}{3}$, $(12, -2)$

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Use point-slope form to remind your student that some forms of equations will remain somewhat unsimplified. This is because some forms provide different information about the graph that is useful in certain scenarios.

Parabolas in Vertex Form

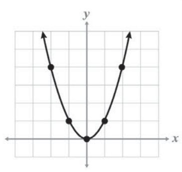
- The equation $y=x^2$ is the parent function of a quadratic function.
- The vertex of the parent function will **always** be $(0, 0)$.

It is recommended that you begin this part of the lesson by having your student explore transformations using the graphing calculator on Desmos® and the equations $y=x^2$ and $y=a(x-h)^2+k$ with sliders.

desmos.com/calculator/ivdm5muvqa

Example 1

Identify the vertex and x- and y-intercepts from the given information.

Parent Function of a Quadratic Function			
Equation	**Table**		**Graph**
$y=x^2$	x	y	
	-2	4	
	-1	1	
	0	0	
	1	1	
	2	4	

Vertex: $(0, 0)$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

First ordered pair: up 1 and over 1 on both sides of the AoS.

Second ordered pair: up 4 and over 2 on both sides of the AoS.

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Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the vertex of a parabola?

A: *The minimum $(+a)$ or maximum $(-a)$ point on the graph.*

27B EXPLORE

- Not all parabolas have a vertex at the origin, so it is important to understand how these quadratic functions are **transformed** from the parent function.
- Parabolas can be transformed by translation, reflection, or dilation.
  - To **translate** a parabola means to slide it left, right, up, or down.
  - To **reflect** a parabola means to flip it across a horizontal line.
  - To **dilate** a parabola means to compress or stretch it, making it narrower or wider.
- Transforming a parabola is more efficient when the quadratic function is written in **vertex form** instead of standard form $(y = ax^2 + bx + c)$.
  - The vertex form of a quadratic equation is: $\boldsymbol{y = a(x - h)^2 + k}$.
  - The vertex of a parabola is $\boldsymbol{(h, k)}$ and the direction and the width of the parabola are determined by the coefficient $\boldsymbol{a}$.
- The parent function written in vertex form is $\boldsymbol{y = 1(x - 0)^2 + 0}$ **OR** $\boldsymbol{y = x^2}$, where:
  - $a =$ **1**
  - $h =$ **0**
  - $k =$ **0**

Example 2

Identify a, h, and k. Name the vertex and the direction of the graph.

A) $y = -2(x - 7)^2 + 8$

$a = -2, h = 7, k = 8$

The graph will open downward, a is negative.
The vertex is $(7, 8)$.

B) $y = (x + 9)^2 - 4$

$\boldsymbol{a = 1, h = -9, k = -4}$

The graph will open upward, a is positive.
The vertex is $(-9, -4)$.

Checkpoint

Identify a, h, and k. Name the vertex and the direction of the graph.

$y = 6(x - 3)^2 - 1$

$\boldsymbol{a = 6, h = 3, k = -1}$

The graph will open upward because a is positive.

The vertex is $(3, -1)$.

EXPLORE 27B

Vertical and Horizontal Translations of Parabolas

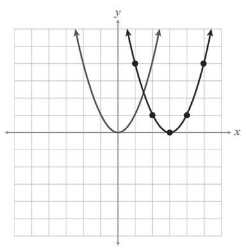
- **Horizontal** translations of parabolas will be determined by the value of h.
  - When h is **positive $(x - h)$**, the graph will translate, or slide, **right** h-spaces.
  - When h is **negative $(x + h)$**, the graph will translate, or slide, **left** h-spaces.
- **Vertical** translations of parabolas will be determined by the value of k.
  - When k is **positive**, the graph will translate **up** k-spaces.
  - When k is **negative**, the graph will translate **down** k-spaces.
- When graphing transformations, first graph the parent function as a **reference** for the transformed parabola.

Example 3

Translate the graph vertically or horizontally using vertex form.

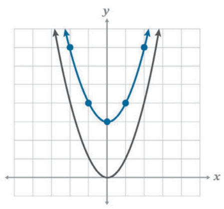
A) $y = (x - 3)^2$

Plan Determine a, h, k from the equation.
Use a, h, k to determine the direction of the graph and the translation of the vertex.



Implement	Explain
$a = 1$	◀ The graph opens upward.
$h = 3$	◀ The vertex moves right 3 spaces or $(3, 0)$.
$k = 0$	◀ The vertex does not move up or down.

B) $y = (x - 0)^2 + 3$



Implement	Explain
$a = 1$	◀ **The graph opens upward.**
$h = 0$	◀ **The vertex does not move left or right.**
$k = 3$	◀ **The vertex moves 3 spaces up or $(0, 3)$**

Example 3

Have your student compare vertex form and standard forming using technology. Enter the given vertex form and then the standard form, if it looks like only one parabola, the equations are the same. If you can see more than one parabola, the equations are not the same.

B) This is another way to represent the equation $y = x^2 + 3$.

Example 4

This is another representation of the equation $y = x^2 + 4x$.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What direction will the graph move when the value of h is changed?

A: h moves the graph left and right.

27B EXPLORE

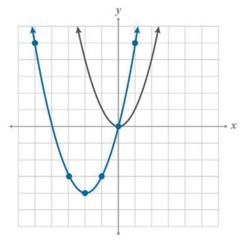
Example 4

Describe the transformation from the parent function. Then translate the graph vertically and/or horizontally using vertex form.

$y = (x + 2)^2 - 4$

Now combine two translations in one problem.

Implement	Explain
$a = 1$	The graph opens upward.
$h = -2$	The vertex moves left 2 spaces or $(-2, 0)$.
$k = -4$	The vertex moves down 4 spaces or $(-4, 0)$.



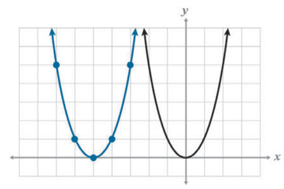
From the parent function, the vertex will shift 2 spaces left and 4 spaces down or $(-2, -4)$.

Checkpoint

Describe the transformation, then graph.

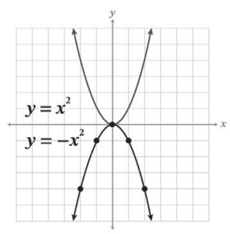
$y = (x + 5)^2$

The graph will shift left 5 spaces or $(-5, 0)$



Reflections and Dilations of Parabolas

- **Reflections** of parabolas will be determined by a.
  - When the value of a is **negative**, the parabola will be reflected across the x-axis.
- Reflected parabolas have ***y*-values** that are the **opposite** of the parent graph.
- Another transformation that occurs that is determined by the value of a is **dilation**.
  - Dilations of parabolas will be determined by $|a|$:
    - When $|a| < 1$, the graph will be **wider** than the parent graph.
    - When $|a| > 1$, the graph will be **narrower** than the parent graph.



Remember that order of operations tells you to square the term and then multiply by the coefficient, in this case, -1.

EXPLORE 27B

Example 5

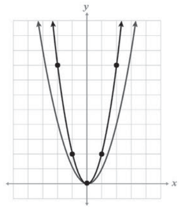
Identify a, h, and k. Determine if the graph is wider or narrower than the parent graph. Then graph.

A) $y = 2x^2$

Implement	Explain
$a = \mathbf{2}$	◂ The graph **narrows** because $2 > 1$.
$h = \mathbf{0}$	◂ The vertex does not move left or right.
$k = \mathbf{0}$	◂ The vertex does not move up or down.

The vertex remains $(0, 0)$, but the other y-values of the ordered pairs will be multiplied by a factor of 2 (the value of a).

x	y
−2	8
−1	2
0	0
1	2
2	8

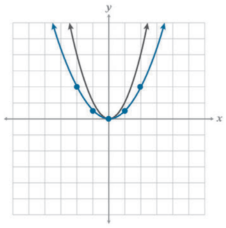


B) $y = \frac{1}{2}x^2$

Implement	Explain
$a = \frac{1}{2}$	◂ The graph **widens** because $\frac{1}{2} < 1$.
$h = \mathbf{0}$	◂ The vertex does not move left or right.
$k = \mathbf{0}$	◂ The vertex does not move up or down.

The y-values of the ordered pairs will be multiplied by a factor of $\frac{1}{2}$.

x	y
−2	2
−1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	2



Checkpoint

Complete the following sentences.
For a parabola that is dilated:

When $|a| < 1$, the parabola will be **wider than the parent graph**.

When $|a| > 1$, the parabola will be **narrower than the parent graph**.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

This is a great place to integrate technology. Have your student graph the parent graph $y = x^2$ and change the coefficient of x^2 to numbers greater than one and smaller than one. If using Desmos®, you can use the slider feature by typing in $y = ax^2$.

27B EXPLORE

Combining Transformations of Parabolas

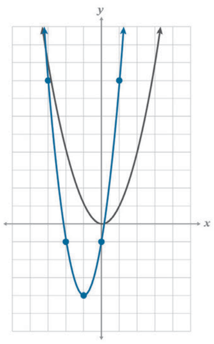
- When quadratic equations have more than one transformation, first determine how the new graph will **change** from the parent function.
- Describing the transformations before graphing gives you a chance to:
  - Think about **where** the parabola will be on the coordinate plane.
  - Decide whether or not the graph you draw **makes sense**.
- Each of the variables, will affect the graph, so be sure to do the following:
  - Name ***a*, *h*, and *k***.
  - **Describe** how they change the graph.

Example 6

Describe the transformations from the parent function. Explain your reasoning. Graph.

$y = 3(x+1)^2 - 4$

Implement	Explain
$a = 3$	The graph **narrows**.
$h = -1$	The vertex moves **left**.
$k = -4$	The vertex moves **down**.



The graph is narrower than the parent function because $a = 3$. It is shifted 1 space left because $h = -1$ and down 4 spaces because $k = -4$.

The points on the parent graph move 1 up and 1 over from the vertex. In this graph, the first point will be 3 up and 1 over since the value of $a = 3$.

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EXPLORE 27B

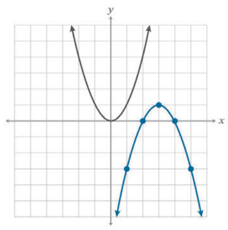
Example 7

Describe the transformations from the parent function. Explain your reasoning. Graph.

$y = -(x-3)^2 + 1$

$a = 3, h = -1, k = 1$

The graph is reflected over the x-axis because $a = -1$, then shifted 3 spaces right because $h = 3$ and up one space because $k = 1$.



Checkpoint

Describe the transformations from the parent function. Explain your reasoning. Name the vertex of the function.

$y = -\frac{1}{8}(x-12)^2 + 6$

$a = -\frac{1}{8}, h = 12, k = 6$

The parabola will be reflected and dilated by a factor of $\frac{1}{8}$. The graph will be wider than the parent function since $|a| < 1$. The graph will shift right 12 spaces and up 6 spaces because of the values of h and k. The vertex will be $(12, 6)$.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Be sure that your student identifies a, h, k before describing the graph. Your student can also use technology to help them formulate a description. They should start with $y = x^2$.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) vertex: $(-4, -8)$

2) vertex: $(1, 14)$

3–4)
Your student can also use technology to help them formulate a description. They should start with $y = x^2$.

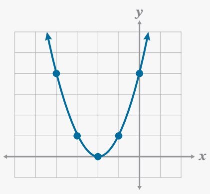
3) vertex: $(6, 0)$ The graph will shift 6 spaces right.

4) vertex: $(0, 9)$ The graph will shift up 9 spaces.

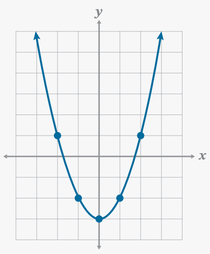
5–6)
This is a great place to integrate technology. If using Desmos®, you can use the slider feature by typing in $y = a(x - h)^2 + k$ to see how the graph changes from the parent function.

Ask your student to describe what their graph will look like *before* graphing to determine if they understand the transformations.

5) vertex: $(-2, 0)$



6) vertex: $(0, -2)$



7) The graph will be wider because $a = \frac{1}{4}$. The graph will shift 6 units left because $h = -6$. The graph will shift down 12 units because $k = -12$. The vertex is $(-6, -12)$.

8) The graph will be reflected and narrower because $a = -4$. The graph will shift 11 spaces right because $h = 11$ and will move up one space because $k = 1$. The vertex is $(11, 1)$.

9–12)
This is a great place to integrate technology. If using Desmos®, you can use the slider feature by typing in $y = a(x - h)^2 + k$ see how the graph changes from the parent function.

Ask your student to describe what their graph will look like *before* graphing to determine if they understand the transformations.

27B PRACTICE 1

Practice 1

Complete the problems on a separate sheet of paper.

Identify a, h, and k. Name the vertex.

1) $y = 6(x + 4)^2 - 8$ **$a = 6, h = -4, k = -8$**

2) $y = -\frac{2}{3}(x - 1)^2 + 14$ **$a = -\frac{2}{3}, h = 1, k = 14$**

Identify a, h, and k. Name the vertex. Describe the transformation from the parent graph.

3) $y = (x - 6)^2$ **$a = 1, h = 6, k = 0$**

4) $y = (x - 0)^2 + 9$ **$a = 1, h = 0, k = 9$**

Identify a, h, and k. Name the vertex. Graph the transformed equation.

5) $y = (x + 2)^2$ **$a = 1, h = -2, k = 0$**

6) $y = x^2 - 3$ **$a = 1, h = 0, k = -3$**

Describe the transformations from the parent function. Explain your reasoning. Name the vertex.

7) $y = \frac{1}{4}(x + 6)^2 - 12$

8) $y = -4(x - 11)^2 + 1$

Identify a, h, and k. Name the vertex. Graph the transformed equation.

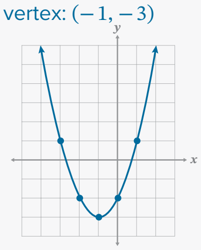
9) $y = (x + 1)^2 - 3$ **$a = 1, h = -1, k = -3$**

10) $y = -2(x - 4)^2 + 6$ **$a = -2, h = 4, k = 6$**

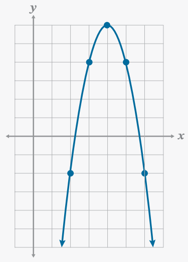
11) $y = \frac{1}{2}(x + 3)^2 + 1$ **$a = \frac{1}{2}, h = -3, k = 1$**

12) $y = -(x - 1)^2 + 4$ **$a = -1, h = 1, k = 4$**

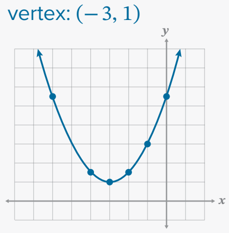
9) vertex: $(-1, -3)$



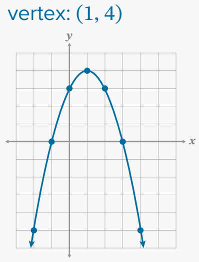
10) vertex: $(4, 6)$



11) vertex: $(-3, 1)$



12) vertex: $(1, 4)$



MASTERY CHECK 27B

Mastery Check

Show What You Know

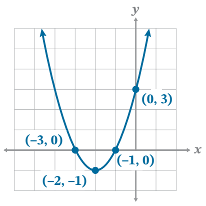
A) Using the numbers $\{-2, -1, 0, 1, 2\}$ only once, create a parabola in which the following is true:

- The vertex of the parabola is in the 3rd quadrant
- There are two solutions. Write and graph the equation.

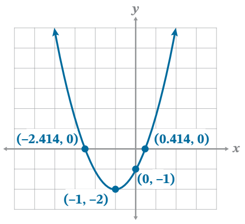
$y = (x - \square)^2 + \square$

There are two possible answers:

The vertex can be $(-2, -1)$ when $y = (x + 2)^2 - 1$



The vertex can be $(-1, -2)$ when $y = (x + 2)^2 - 2$



B) Describe how the graph in part A has been transformed from the parent graph.

For $(-2, -1)$, the graph has moved two spaces left and one space down from $(0, 0)$.

For $(-1, -2)$, the graph has moved one space left and two spaces down from $(0, 0)$.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

CONTINUE

Mastery Check

Show What You Know

This is a great place to integrate technology. Have your student graph the parent graph $y = x^2$ and on the same axis, change the coefficient to numbers greater than 1 and smaller than one. If using Desmos®, you can use the slider feature by typing in $y = a(x - h)^2 + k$.

C) The directions do not indicate that the graph needs to be created, however, using technology to assist in creating the graphs will be very helpful. Using the slider in Desmos® allows multiple values to be tried in a very brief amount of time, rather than hand graphing.

27B MASTERY CHECK

C) Using the numbers $\{-3, -2, -1, 0, 1, 2, 3\}$ only once, create a parabola that meets all of the following requirements:

- Reflected across the x-axis.
- The value of a is half that of the parent graph.
- Has one positive solution.

$$y = \frac{\square}{\square}(x - \square)^2$$

When $a = \frac{-1}{2}$, $h = 1$ or 3

When $a = \frac{1}{-2}$, $h = 1$ or 2 or 3

Sample equations:

$$y = -\frac{1}{2}(x-3)^2$$

$$y = -\frac{1}{2}(x-1)^2$$

$$y = -\frac{1}{2}(x-2)^2$$

D) Describe the transformations and equations from part C.

To widen the parent graph, $|a| < 1$. Since the width must be half that of the parent graph, the fraction $\frac{1}{2}$ will be used. The value of a must be negative, since the graph needs to be reflected over the x-axis.

Because the graph has one positive solution, the vertex must be on the x-axis. The value of h can be any of the remaining numbers greater than zero depending on the numbers that have been used for the value of a $\left(a = \frac{-1}{2} \text{ or } \frac{1}{-2}\right)$.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

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Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Recognize vertex form and identify and graph a, h, and k.
- Translate a parent graph vertically and horizontally and describe the transformation.
- Reflect and dilate a parent graph and describe the transformation.
- Transform a parent graph using more than one transformation (translation, reflection, dilation) and describe the transformation.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

PRACTICE 2 27B

Practice 2

Complete the problems on a separate sheet of paper.

Identify a, h, and k. Name the vertex.

1) $y = -(x-11)^2 - \frac{5}{2}$ **$a = -1, h = 11, k = -\frac{5}{2}$**

2) $y = 10(x+4)^2 - 6$ **$a = 10, h = -4, k = -6$**

Identify a, h, and k. Name the vertex. Describe the transformation from the parent graph.

3) $y = x^2 - 11$ **$a = 1, h = 0, k = -11$**

4) $y = (x+14)^2$ **$a = 1, h = -14, k = 0$**

Identify a, h, and k. Name the vertex. Graph the transformed equation.

5) $y = x^2 + 4$ **$a = 1, h = 0, k = 4$**

6) $y = (x-4)^2$ **$a = 1, h = 4, k = 0$**

Describe the transformations from the parent function. Explain your reasoning. Name the vertex.

7) $y = -\frac{1}{2}(x-6)^2 - 5$

8) $y = 8(x+1)^2 - 14$

Identify a, h, and k. Name the vertex. Graph the transformed equation.

9) $y = \frac{1}{3}(x-2)^2 - 5$ **$a = \frac{1}{3}, h = 2, k = -5$**

10) $y = 2(x+4)^2 - 1$ **$a = 2, h = -4, k = -1$**

11) $y = -3(x-2)^2 + 3$ **$a = -3, h = 2, k = 3$**

12) $y = -\frac{1}{2}(x+1)^2 + 2$ **$a = -\frac{1}{2}, h = -1, k = 2$**

Practice 2

Worked solutions for these problems are located in the Digital Pack.

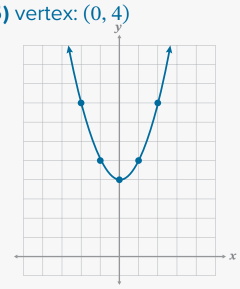
1) vertex: $\left(11, -\frac{5}{2}\right)$

2) vertex: $(-4, -6)$

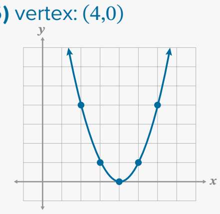
3) vertex: $(0, -11)$ The graph will move down 11 spaces.

4) vertex: $(-14, 0)$ The graph will shift left 14 spaces.

5) vertex: $(0, 4)$



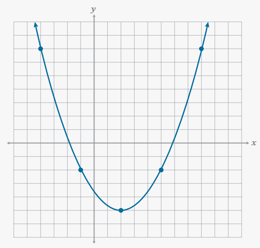
6) vertex: $(4,0)$



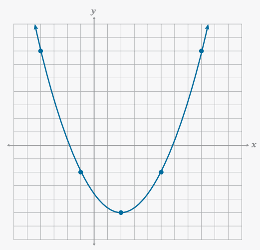
7) The graph will be reflected and be wider because $a = -\frac{1}{2}$. The graph will shift right 6 spaces because $h = 6$ and will move down 5 because $k = -5$. The vertex is $(6, -5)$.

8) The graph will be narrower $a = 8$. The graph will shift left by 1 since $h = -1$ and will shift down 14 spaces because $k = -14$. the vertex is $(-1, -14)$.

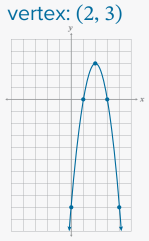
9) vertex: $(2, -5)$



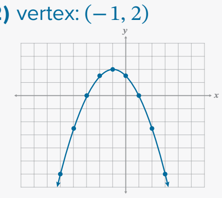
10) vertex: $(-4, -1)$



11) vertex: $(2, 3)$



12) vertex: $(-1, 2)$



If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Lesson Test

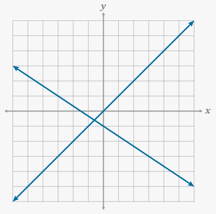
Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

3)



4) From the quadratic equation, name a, b, and c. Then, calculate the axis of symmetry using the formula $-\left(\frac{b}{2a}\right)$. Use the value of the AoS and plug this into the given equation to find the value of y. The vertex will be $\left(-\frac{b}{2a}, y\right)$.

5) Not every function can be factored but this was how quadratics were solved in Unit 4.

Quadratic functions can be solved by factoring and by graphing and finding the x-intercepts.

9) $y = 75x + 2000$

Devon earns $75 for every sale made. The minimum amount of money Devon will earn in a month, if no sales are made, is $2,000.

10) $y = 75x + 2{,}000$

$3{,}725 = 75x + 2{,}000$

$3{,}725 - 2{,}000 = 75x + 2{,}000 - 2{,}000$

$1{,}725 = 75x$

$\left(\frac{1}{75}\right)(1{,}725) = \left(\frac{1}{75}\right) 75x$

$23 = x$

Devon made 23 sales if $3,725 was earned.

11) Distractor Rationale:
The incorrect choices would be correct if the middle term was negative.

12) Distractor Rationale:

A) Has the signs of the solutions switched.

C) Would be correct if $\frac{1}{6}$ was omitted from the equation.

D) Would be correct if +14 was entered

Recall that "roots" is another word for solutions or x-intercepts. Your student can use technology to determine the solutions or substitute the values into the equation.

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

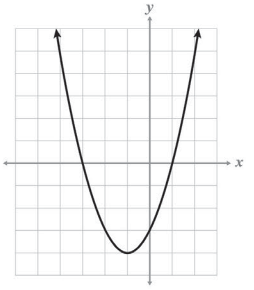
1) State the domain and range for the given graph using interval notation. **domain: $(-\infty, \infty)$ range: $[-4, \infty)$**

2) Name the solutions and vertex from the graph in problem 1. Assume the parabola crosses at grid lines. $\mathbf{x = -3, 1}$

3) Draw the parent function for a linear equation $f(x) = x$. Then on the same coordinate plane, draw a new function translated down one space with a slope of $-\frac{2}{3}$.

4) Describe how to find the vertex of a parabola given the equation $y = ax^2 + bx + c$.

5) What are two ways you have learned to solve a quadratic function? (Hint: Think about Unit 4 and Unit 5).



Use your Formula Sheet to simplify and write answers in exponential form.

6) $(p^{56}q^{64})^{\frac{1}{8}}$ $\mathbf{p^7q^8}$

7) $(a^3b^2c^8)^2 \cdot bc^{-5}$ $\mathbf{a^6b^5c^{11}}$

8) $(2^4x^8y^3)^{\frac{1}{2}}$ $\mathbf{4x^4y^{\frac{3}{2}}}$

9) Devon earns a monthly stipend of $2,000 plus $75 per sale made. Write an equation in slope-intercept form. Explain the meaning of the slope and the y-intercept in context.

10) If Devon earned $3,725 last month, how many sales were made? Use the equation you wrote in problem 9. Show your work. **Devon made 23 sales if $3,725 was earned.**

Multiple Choice

11) Determine the solution(s) to the quadratic equation. Write all that apply.

$2x^2 + 9x = 56$

- ■ $\mathbf{x = -8}$
- ☐ $x = -\frac{7}{2}$
- ■ $\mathbf{x = \frac{7}{2}}$
- ☐ $x = 8$

B **12)** Name the roots to the equation $y = \frac{1}{6}x^2 - 3x - 14$.

A) $x = -21.845, 3.845$

B) $\mathbf{x = -3.845, 21{,}845}$

C) $x = -2.531, 5.532$

D) no real solution

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	26	26	8	26	26	19	19	19	9, 10	10	25, 26	26