Lesson 26
Graphing Quadratics

Outline

Part A
Introduction to Quadratic Functions

- Components of Parabolas
- Graphing Parabolas

Part B
Graphical Solutions to Quadratic Functions

- Using Technology to Solve Quadratic Functions
- Domain and Range of Quadratic Functions

Targeted Review

Vocabulary

- parabola
- solutions (to a quadratic equation)
- axis of symmetry
- vertex

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

1-2)

The concept of symmetry is a critical piece of graphing quadratic equations. Your student's figure should approximately match the figure on the other side of the dashed line (the axis of symmetry).

3) This problem reviews the concept of evaluating an equation written in function notation. Recall that the number in parentheses will replace x in the given equation.

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Part A: Introduction to Quadratic Functions

Objectives

In this part of the lesson, you will learn about quadratic functions.

By the end of this lesson, you will be able to do the following:

- Find the direction, axis of symmetry, vertex, and y-intercept of a quadratic function.
- Graph a quadratic function.

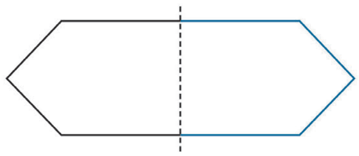
Why?

Understanding the parts that make up the graph of a quadratic function is a foundational step to graphing and interpreting the solutions to quadratic functions, which you will learn how to do in future lessons and courses.

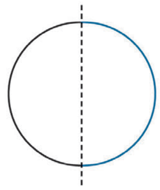
Warm Up

For problems 1–2, complete the figure so that it creates a symmetric shape.

1)



2)



3) Evaluate $f(4)$, when $f(x) = x^2 - x + 1$.

$f(4) = (4)^2 - (4) + 1$

$f(4) = 13$

Components of Parabolas

- Unlike linear functions, the graph of a quadratic function is not a **line**.
- The graph of a quadratic function is a **parabola**.
  - Parabolas are symmetric, curved graphs that either open **upward** like a "u" or a smile, or **downward** like an "n" or a mountain.
  - Parabolas can be represented by a quadratic equation in standard form:

    $y = ax^2 + bx + c$..

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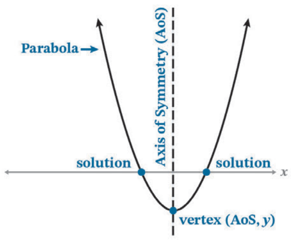
Parts of a parabola:



- The **axis of symmetry (AoS)** is an imaginary vertical line going through the center of the parabola that splits it into symmetrical halves.
- Each point on a parabola, except the vertex, has a symmetric counterpart that is the same distance from the axis of symmetry on the opposite side.
- The **vertex** is the minimum or maximum point of a parabola (depending on its direction) and **must** be on the axis of symmetry.
- The solutions to a quadratic equation are the points, or **ordered pairs**, where the parabola intersects the ***x*-axis** (x-intercepts).
  - The solutions to a quadratic function are also called roots or zeros.

Direction of a parabola:

- The opening of the parabola is determined by the **sign** of a in $y = ax^2 + bx + c$.
- When a is positive, the graph opens upward with a **minimum** vertex.
- When a is negative, the graph opens downward with a **maximum** vertex.

Plotting the parts of a parabola:

- Determine the direction of the parabola.
- Solve for the axis of symmetry.
  - The AoS is the ***x*-value** of the ordered pair for the vertex.
  - The formula for the AoS is $x = -\left(\frac{b}{2a}\right)$.
  - Draw the AoS as a dashed line because it is a **reference** used to create symmetry and not part of the quadratic equation.

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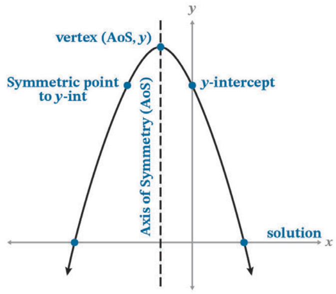
Plotting the parts of a parabola (continued):

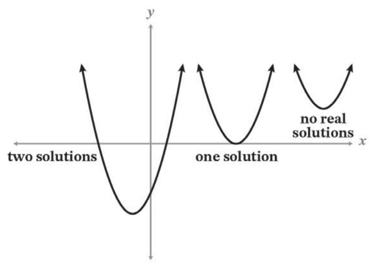
- Solve for the vertex.
  - Substitute the **AoS** into the quadratic equation to solve for ***y***.
  - The x-value of the AoS and the y-value you found form the **ordered pair** that is the vertex.
  - The formula for the vertex is: (AoS, y) **or** $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.



- Find the y-intercept and its counterpart.
  - The y-intercept of a quadratic equation in standard form is the **constant, c**.
  - The symmetric counterpart of the y-intercept (if there is one) will have the **same** y-coordinate but will be on the **opposite side** of the axis of symmetry the same distance away.

x-Intercepts on a Parabola:

- The x-intercepts, or **solutions**, are the points where the parabola crosses the x-axis.
- You can have **one**, **two**, or **zero** x-intercepts.
- The x-intercepts may need to be **estimated** if they do not go through grid lines on the coordinate plane.



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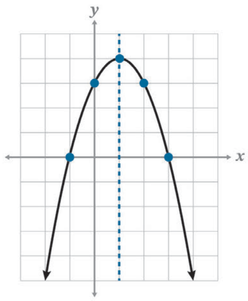
Example 1

Given the graph and equation for a parabola, mark the axis of symmetry, vertex, y-intercept, the symmetric counterpart to the y-intercept, and the solutions. Check the AoS and vertex algebraically.

$y = -x^2 + 2x + 3$



Plan Determine the values of a, b, and c.
Determine the direction of the parabola.
Draw the AoS.
Mark the vertex.
Find the y-intercept and counterpart.
Find the solutions (x-intercepts).
Solve algebraically.

Implement

$a = -1, b = 2, c = 3$

Component	Graphical Answer	Algebraic Answer
direction	**down, $a = -1$ maximum**	
AoS	**AoS: $x = 1$**	$\text{AoS} = -\left(\frac{b}{2a}\right) = -\left(\frac{2}{2(-1)}\right)$ $\text{AoS} = 1$
vertex (AoS, y)	**(1, 4)**	$y = -(1)^2 + 2(1) + 3$ $y = -1 + 2 + 3 = 4$ $(1, 4)$
y-int $(0, y)$	**(0, 3)**	$c = 3$
symmetric to y-int	**(2, 3)**	$y = -(2)^2 + 2(2) + 3$ $y = -4 + 4 + 3$ $y = 3$
solution(s)	**(−1, 0) and (3, 0)**	x-intercepts from graph or algebraically $y = -x^2 + 2x + 3$ $0 = -x^2 + 2x + 3$ $0 = -(x^2 - 2x - 3)$ $0 = -(x + 1)(x - 3)$ $x = -1, 3$

It is important to note that not all graphs will have x-intercepts. If this is the case and you are asked to provide solutions, you can write, "none" or "no real solutions."

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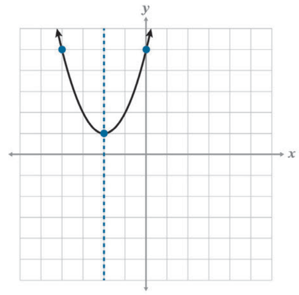
Example 2

Mark the AoS, vertex, y-intercept, point symmetric to the y-intercept, and the x-intercept(s). Check the AoS and vertex algebraically.

$f(x) = x^2 + 4x + 5$



Implement

$a = 1, b = 4, c = 5$

Component	Graphical Answer	Algebraic Answer
direction	**up, $a = 1$ minimum**	
AoS	$\text{AoS} = -2$	$\text{AoS} = -\left(\frac{b}{2a}\right) = -\left(\frac{4}{2 \cdot 1}\right)$ $\text{AoS} = -2$
vertex (AoS, y)	$(-2, 1)$	$y = (-2)^2 + 4(-2) + 5$ $y = 4 - 8 + 5 = 1$ $(-2, 1)$
y-int $(0, y)$	$(0, 5)$	$c = 5$
symmetric to y-int	$(-4, 5)$	$y = (-4)^2 + 4(-4) + 5$ $y = 16 - 16 + 5$ $y = 5$
solutions(s)	no real solutions	

- When b is equal to zero, the AoS is $x = 0$, or the y-axis $\left(\text{AoS} = -\left(\frac{0}{2a}\right) = 0\right)$.
- If the vertex is the y-intercept and the x-intercept, more ordered pairs need to be marked to understand how the parabola is formed.
  - Create a table of values to complete the parabola, using one of these options:
    - If the AoS is known, you can complete the table using symmetric points.
    - If the AoS is unknown, ordered pairs can be calculated by picking values for x and solving for y, or $g(x)$.

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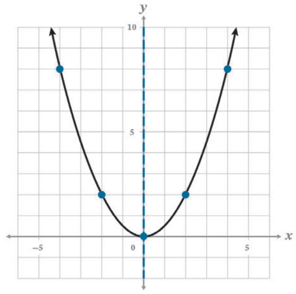
Example 3

Mark the AoS, vertex, y-intercept, point symmetric to the y-intercept, and the x-intercept(s). Check the AoS and vertex algebraically.

$g(x) = \frac{1}{2}x^2$

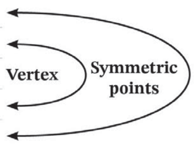


Implement

$a = \frac{1}{2}, b = 0, c = 0$

Component	Graphical Answer	Algebraic Answer
direction	**up, $a = \frac{1}{2}$** **minimum**	
AoS	$\mathbf{AoS = 0}$	$\mathbf{AoS = -\left(\frac{b}{2a}\right) = -\left(\frac{0}{2 \cdot \frac{1}{2}}\right)}$ $\mathbf{AoS = 0}$
vertex (AoS, y)	**(0, 0)**	$\mathbf{g(0) = \frac{1}{2}(0)^2 = 0}$ **(0, 0)**

x	$g(x)$
−4	8
−2	2
0	0
2	2
4	8



Complete a table using symmetric points. Then plot them on the graph.

Complete a table by picking values for x and solving for y, or $g(x)$. Then plot them on the graph.

Component	Graphical Answer	Algebraic Answer
y-int (0, y)	**(0, 0)**	
symmetric to y-int	none	
solution(s)	**(0, 0)**	

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What are the values of a, b, and c for $y = ax^2 + bx + c$?

A: *$a = 1$, $b = -6$, $c = 8$*

Q: What does the value of a tell you?

A: *The value of a tells you whether the parabola will open upward or downward. In this case, it will open upward.*

Q: Can you determine the x-intercepts for this graph? How?

A: *Yes, the x-intercepts can be determined because the parabola intersects the graph at exact grid lines. (The x-intercepts can also be found by factoring.)*

Checkpoint

Mark the graph with the axis of symmetry, vertex, y-intercept, point symmetric to the y-intercept, and the solutions (x-intercepts). Fill in the information below.

$h(x) = x^2 - 6x + 8$

direction: **up, minimum**

AoS: $x = 3$

$\text{AoS} = -\left(\frac{b}{2a}\right) = -\left(\frac{-6}{2 \cdot 1}\right) = 3$

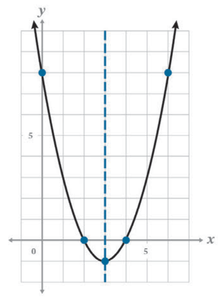
vertex: $(3, -1)$

$h(3) = (3)^2 - 6(3) + 8 = -1$

y-intercept: $(0, 8)$

symmetric to y-intercept: $(6, 8)$

solutions (x-intercepts): $(2, 0)$ **and** $(4, 0)$



Graphing Parabolas

- Steps for graphing parabolas (quadratic functions):

  1) Write the equation in **standard** form.
  2) Identify a, b, c. Determine if the graph opens up or down using **$\pm a$**.
  3) Calculate the axis of symmetry, then graph as a **dashed line**. $\text{AoS} = -\left(\frac{b}{2a}\right)$
  4) Calculate the **vertex** and graph on the AoS. (AoS, y)
  5) Solve for and graph the y-intercept, c, and its **symmetric counterpart**.
  6) Graph another ordered pair. Use the **x-intercepts** if possible.
     - Solve for the value(s) of x.
     - If you cannot use the x-intercepts, graph another point and its symmetric counterpart.
  7) Connect the **ordered pairs** to make a parabola.

- When graphing a parabola, you can either graph the components after **each step** or complete all the calculations and then graph everything **at once**.

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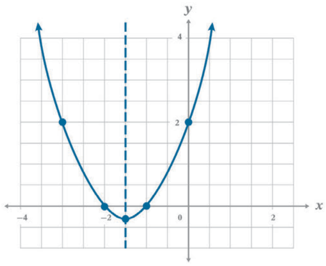
Example 4

Graph the quadratic equation. Calculate the axis of symmetry, vertex, and solution(s) algebraically.

$y = x^2 + 3x + 2$



Implement	Explain (Graphing Steps)
$y = x^2 + 3x + 2$	**1)** Write the equation in standard form.
$a = 1, b = 3, c = 2$ up, minimum vertex	**2)** Identify a, b, c. Determine if the graph opens up or down using $\pm a$.
$\text{AoS} = -\left(\frac{b}{2a}\right) = -\left(\frac{3}{2 \cdot 1}\right) = -\frac{3}{2} = -1.5$	**3)** Calculate the axis of symmetry, then graph as a dashed line.
vertex: $y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + 2$ $y = \frac{9}{4} - \frac{9}{2} + 2 = -\frac{1}{4} = -0.25$ $(-1.5, -0.25)$	**4)** Calculate the vertex and graph on the AoS.
$c = 2$, **y-intercept:** $(0, 2)$ $2 = x^2 + 3x + 2$ $0 = x^2 + 3x$ $0 = x(x + 3)$ $x = 0, x = -3$	**5)** Solve for and graph the y-intercept, c, and its counterpart.
$0 = x^2 + 3x + 2$ $0 = (x + 2)(x + 1)$ $x = -2, -1$	**6)** Graph another ordered pair using the x-intercepts if possible. **a)** Solve for the values of x by factoring. **b)** Graph the points you just found If $y = 0$, then you can find the x-intercepts by factoring.
	7) Connect the ordered pairs to make a continuous graph.

Example 5

Graph the quadratic equation. Calculate the axis of symmetry, vertex, and solutions algebraically.

$f(x) = 4x^2 - 24x + 27$

$a = 4, b = -24, c = 27$

direction: up, minimum

AoS: $\text{AoS} = -\left(\frac{b}{2a}\right) = -\left(\frac{-24}{2 \cdot 4}\right) = 3$

vertex: $f(3) = 4(3)^2 - 24(3) + 27 = -9$

$(3, -9)$

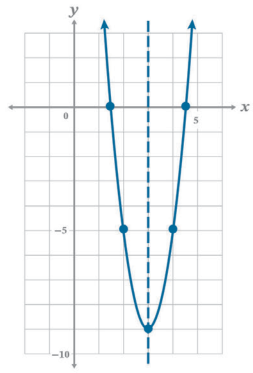
y-intercept: $(0, 27)$

$0 = 4x^2 - 24x + 27$

x-intercepts: $0 = (2x - 3)(2x - 9)$

$x = \frac{3}{2}, \frac{9}{2}$

$f(2) = 4(2)^2 - 24(2) + 27 = -5$



Because the y-intercept is so far away from the x-axis, it can be noted but will not be graphed. This means that other points must be used to create the graph.

If you are having trouble making a symmetric parabola, graph additional points by picking an x-value and solving for y.

If you average the x-values of any two symmetric points on the graph, your answer should result in the value for the axis of symmetry.

Checkpoint

Graph.

$y = -x^2 + 9$

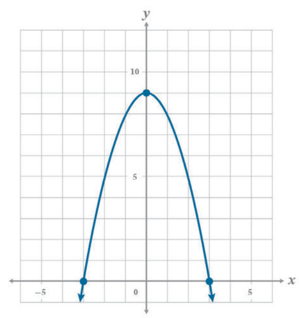
Mark the vertex, y-intercept, and solution(s). Then write them down in the space below.

Remember to check your student's graph to confirm the values they wrote match their graph.

AoS: $x = 0$

vertex: $(0, 9)$

solutions: $(-3, 0), (3, 0)$



Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Since the y-intercept has no counterpart in this graph, your student will need to factor $-x^2 + 9 = 0$ to find the x-intercepts or create a table of values to plot more points on the graph.

Q: If $(-2, 5)$ is on the graph, what point is symmetric to this?

A: *(2, 5)*

Q: Why is there not a point symmetric to the y-intercept?

A: *The y-intercept is also the vertex, which means it has no symmetric counterpart because it is on the axis of symmetry.*

PRACTICE 1 26A

Practice 1

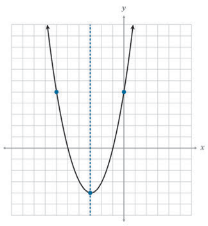
Complete the problems on a separate sheet of paper.

Determine the graph's direction and whether the vertex is a minimum or maximum.

1) $y = 3x^2 + 8x - 11$ **2)** $f(x) = -\frac{4}{3}x^2 + x$ **3)** $y = -x^2 + 4$

Copy the graph and mark the items noted. Solutions may need to be estimated.

4)



AoS: $x = -3$

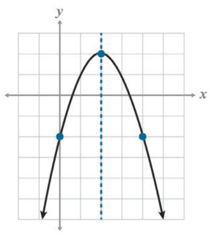
vertex: $(-3, -4)$

y-int: $(0, 5)$

symmetry to y-int: $(-6, 5)$

solution(s): $(-5, 0)$ and $(-1, 0)$

5)



AoS: $x = 2$

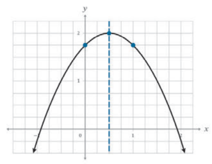
vertex: $(2, 2)$

y-int: $(0, -2)$

symmetry to y-int: $(4, -2)$

solution(s): $(0.518, 0)$ and $(3.414, 0)$

6)



Hint: Check the scale of the graph carefully.

AoS: $x = \frac{1}{2}$

vertex: $\left(\frac{1}{2}, 2\right)$

y-int: $\left(0, 1\frac{3}{4}\right)$ or $\left(0, \frac{7}{4}\right)$

symmetry to y-int: $\left(1, 1\frac{3}{4}\right)$ or $\left(1, \frac{7}{4}\right)$

Determine the graph's direction and whether the vertex is a minimum or maximum. Then, find the AoS, the vertex, and the y-intercept.

7) $y = -x^2 + 14x - 11$

Direction: **down**

Vertex: **max**

AoS: 7

Vertex (AoS, y): $(7, 38)$

$(0, y\text{-int})$: $(0, -11)$

8) $y = 4x^2 + 8x + 15$

Direction: **up**

Vertex: **min**

AoS: -1

Vertex (AoS, y): $(-1, 11)$

$(0, y\text{-int})$: $(0, 15)$

9) $y = \frac{1}{2}x^2 + x$

Direction: **up**

Vertex: **min**

AoS: -1

Vertex (AoS, y): $\left(-1, -\frac{1}{2}\right)$

$(0, y\text{-int})$: $(0, 0)$

Graph the quadratic equation. Mark the vertex, y-intercept, point symmetric to the y-intercept, and the solution(s).

10) $f(x) = x^2 - 4$ **11)** $f(x) = -2x^2 + 5x + 3$ **12)** $y = 3x^2 - 7x + 6$

Practice 1

Worked solutions for these problems are located in the Digital Pack.

For problems 1–3, you can also have your student identify a, b, and c for the given functions. This can help them when they use the formulas in later problems.

1) $a = 3$; therefore, the graph opens upward and has a minimum vertex.
$a = 3, b = 8, c = -11$

2) $a = -\frac{4}{3}$; therefore, the graph opens downward and has a maximum vertex.
$a = -\frac{4}{3}, b = 1, c = 0$

3) $a = -1$; the graph opens downward and has a maximum vertex.
$a = -1, b = 0, c = 4$

Encourage your student to mark their figures (they can use colored pencils if they prefer). This helps them interact with the graph rather than passively looking at it.

5) The x-intercepts will need to be estimated since the graph does not go through grid lines. The points $(0.5, 0)$ and $(3.5, 0)$ would be acceptable estimates.

In the next part of the lesson, your student will learn to find the intercepts using technology. It may be helpful to come back to this problem to see how well they estimated.

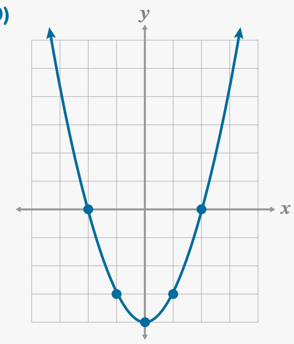
7–9)

Be sure that your student substitutes the AoS into the given equation to find the vertex, even if they use a calculator. This makes solving more efficient and results in fewer errors.

Q: What is the y-intercept when the equation is in standard form?

A: *c, the constant*

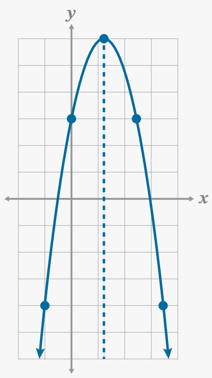
10)



Q: Does this graph have a point symmetric to the y-intercept? Explain.

A: *No, it does not because the vertex is the y-intercept.*

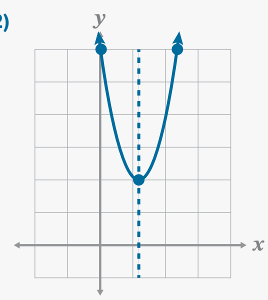
11)



Q: What is another name for the x-intercepts when working with parabolas?

A: *solutions (or roots)*

12)



Q: Why does this graph need to be estimated?

A: *The vertex and point symmetric to the y-intercept are fractions, but the graph counts by integer values.*

Mastery Check

Show What You Know

B) AoS: $x = -\left(\frac{5}{2(-1)}\right) = \frac{5}{2} = 2.5$

Vertex: $y = -(2.5)^2 + 5(2.5) + 4 = 10.25$

C) Your student can use the graph to find the symmetric point instead of solving for it algebraically.

Q: How many spaces from the axis-of-symmetry is $f(1)$?

A: *1.5 spaces*

Q: How many spaces will the symmetric point be?

A: *The symmetric point is 1.5 spaces on the opposite side.*

D) Your student can extend the graph to find the values or solve by factoring.

This means they need to find where $y = -2$. Since this is a parabola, there will be two solutions when $y = -2$.

Q: What are the approximate solutions (or x-intercepts) on this graph?

A: *Your student's answer should be as close as possible to these calculated values: $(-0.702, 0)$ and $(5.702, 0)$.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Find the direction, axis of symmetry, vertex, and y-intercept of a quadratic function.
- Graph a quadratic function.

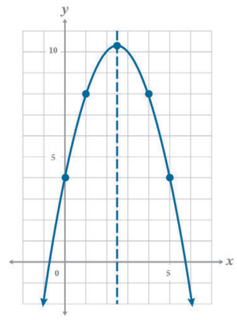
26A MASTERY CHECK

Mastery Check

Show What You Know

A) Graph.

$f(x) = -x^2 + 5x + 4$



B) Mark the axis of symmetry, vertex, and y-intercept. Then write them in the space below.

axis of symmetry: $x = 2.5$

vertex: $(2.5, 10.25)$

y-intercept: $(0, 4)$

C) Determine $f(1)$. Then find its symmetric counterpart.

$f(1) = -(1)^2 + 5(1) + 4$

$f(1) = 8$

symmetric to $f(1)$: $f(4) = 8$

D) Determine the two values for $f(x) = -2$.

$-2 = -x^2 + 5x + 4$

$0 = -x^2 + 5x + 6$

$0 = -(x^2 - 5x - 6)$

$0 = -(x - 6)(x + 1)$

$x - 6 = 0, x + 1 = 0$

$x = 6, -1$

$(6, -2)$ **and** $(-1, -2)$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

130 Lesson 26: Graphing Quadratics > Part A: Introduction to Quadratic Functions > Mastery Check Algebra 1 Student Worktext

PRACTICE 2 26A

Practice 2

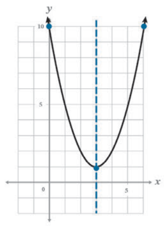
Complete the problems on a separate sheet of paper.

Determine the graph's direction and whether the vertex is a minimum or maximum.

1) $y = \frac{1}{8}x^2 + x$ **2)** $f(x) = 2x^2 - 1$ **3)** $y = -0.65x^2 + 4x - 15$

Copy the graph and mark the items noted. Solutions may need to be estimated.

4)



AoS: $x = 3$

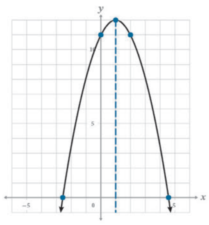
vertex: $(3, 1)$

y-int: $(0, 10)$

symmetry to y-int: $(6, 10)$

solution(s): none

5)



AoS: $x = 1$

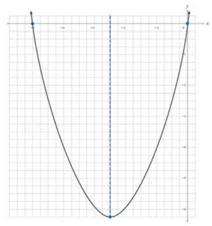
vertex: $(1, 12)$

y-int: $(0, 11)$

symmetry to y-int: $(2, 11)$

solution(s): $(-2.464, 0)$ and $(4.464, 0)$

6)



AoS: $x = -2\frac{1}{2}$

vertex: $\left(-2\frac{1}{2}, -6\frac{1}{4}\right)$ or $\left(-\frac{5}{2}, -\frac{25}{4}\right)$

y-int: $(0, 0)$

symmetry to y-int: $(-5, 0)$

solution(s): $(0, 0)$ and $(-5, 0)$

Determine the graph's direction and whether the vertex is a minimum or maximum. Then, find the AoS, the vertex, and the y-intercept.

7) $y = 6x^2 + 3x + 13$

Direction: up

Vertex: min

AoS: $-\frac{1}{4}$

Vertex (AoS, y): $\left(-\frac{1}{4}, \frac{101}{8}\right)$

$(0, y\text{-int})$: $(0, 13)$

8) $y = -x^2 - 6x - 1$

Direction: down

Vertex: max

AoS: -3

Vertex (AoS, y): $(-3, 8)$

$(0, y\text{-int})$: $(0, -1)$

9) $y = \frac{2}{3}x^2 + 5$

Direction: up

Vertex: min

AoS: 0

Vertex (AoS, y): $(0, 5)$

$(0, y\text{-int})$: $(0, 5)$

Graph the quadratic equation. Mark the vertex, y-intercept, point symmetric to the y-intercept, and the solution(s).

10) $f(x) = \frac{1}{2}x^2 + x$ **11)** $y = x^2 + 4x + 3$ **12)** $y = -x^2 + 2x$

Practice 2

Worked solutions for these problems are located in the Digital Pack.

1) $a = \frac{1}{8}$; therefore, the graph opens upward and has a minimum vertex.

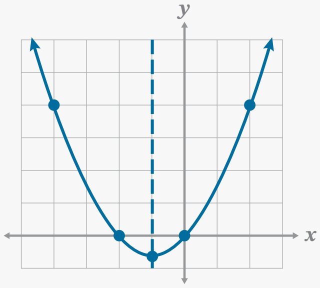
2) $a = 2$; therefore, the graph opens upward and has a minimum vertex.

3) $a = -0.65$; therefore, the graph opens downward and has a maximum vertex.

4) Another way to tell there are no solutions is if the graph faces upward and the y value of the vertex is positive, or if the graph faces downward and the y value of the vertex is negative.

5) The x-intercepts will need to be estimated since the graph does not go through grid lines. The points $(-2.5, 0)$ and $(4.5, 0)$ would be acceptable estimates.

10)



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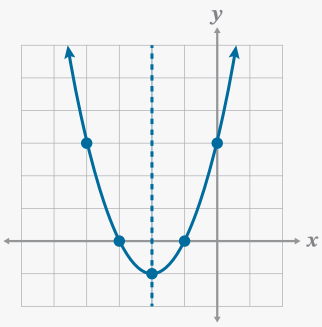

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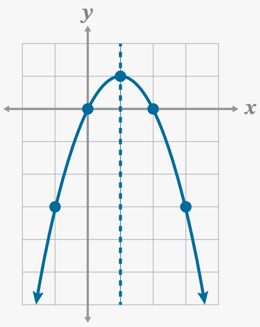


If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

The Desmos® Graphing Calculator is recommended so students can hover over the points on the parabola to see their values. Have your students use the equations from Practice 1 and 2 of Part A (problems 4–6) to practice. After students enter the information, you can compare their results with the provided graphs to determine if they entered the information correctly.

26B EXPLORE

Part B: Graphical Solutions to Quadratic Functions

Objectives

In this part of the lesson, you will learn about the graphical solutions to quadratic functions.

By the end of this lesson, you will be able to do the following:

- Find exact and approximate solutions to quadratic equations, using technology as needed.
- Determine the domain and range for quadratic equations.

Why?

Finding the exact solutions to a quadratic equation even when it cannot be determined from a graph and understanding how those solutions are formed will help you interpret quadratic graphs in future lessons.

Warm Up

Review how to graph a parabola and locate specific points using your preferred form of graphing technology (graphing calculator, graphing software, etc.).

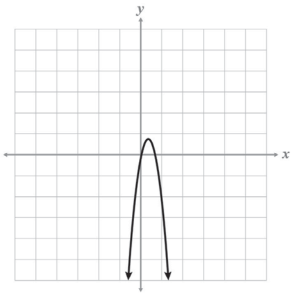
Using Technology to Solve Quadratic Functions

- The solution or solutions to a quadratic function are where the parabola crosses the **x-axis**.
- If the solutions do not cross the graph at a grid line on the coordinate plane, and the equation **cannot be factored**, you will need to use technology to help you find the exact solutions.
- Using technology to graph parabolas helps you quickly determine all of the information you need to create a graph without doing all of the **calculations** by hand.

EXPLORE 26B

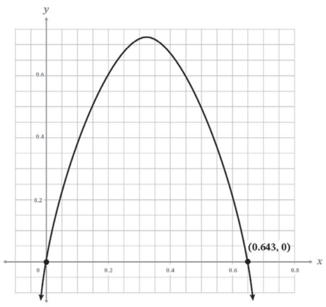
Example 1

Determine the solutions to the given quadratic function. Name the vertex.



$f(x) = -7x^2 + \frac{9}{2}x$

With the given scale of the provided graph, it is impossible to give an exact value, or even an estimate, for one of the solutions.



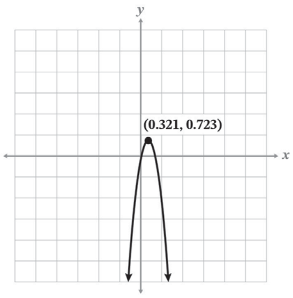
The graphing software automatically creates the parabola when you enter in the quadratic function.

However, even with graphing technology, it is still difficult to estimate the solutions accurately.

With most graphing technology, you can select any point to see its coordinates. In this case, you can identify that $(0.643, 0)$ is one of the x-intercepts.

Solutions: $(0, 0)$ and $(0.643, 0)$

Some forms of graphing technology will display this answer as $x = 0, 0.643$.



Next, determine the vertex by selecting that point on the graph.

Vertex: $(0.321, 0.723)$

What if you do not have graphing technology to solve Example 1?

- If the equation can be factored, then you can find the solution(s) as you did in Unit 4 by setting $f(x)$ equal to zero and then factoring to solve for x.
- The quadratic formula is another way to determine the solutions algebraically without technology. This formula will be covered in a later lesson.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Why do you need to know how to graph a parabola by hand even when using technology?

A: *Because you have to transfer the information onto your paper.*

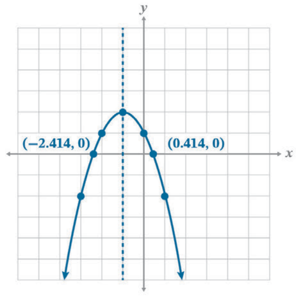
26B EXPLORE

Checkpoint

Use technology to create a graph of the equation. Then mark the axis of symmetry, vertex, y-intercept, the symmetric counterpart to the y-intercept, and the solution(s). Write the solutions below.

$y = -x^2 - 2x + 1$

Solution(s): $x = -2.414, 0.414$



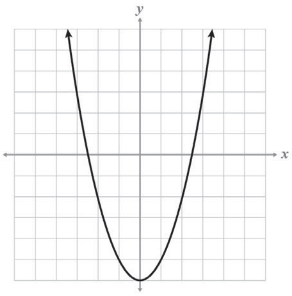
Domain and Range of Quadratic Equations

- The domain of a quadratic function is always **all real numbers**, or $(-\infty, \infty)$ in interval notation.
  - This is because **any value** of x can be substituted into the equation to find y and form an ordered pair.
- The **range** of a quadratic function is determined by the direction of the graph and by the vertex.
  - When the vertex is the **minimum** value (a is positive), the range will be $[y, \infty)$ with y being the y-value of the vertex.
  - When the vertex is a **maximum** value (a is negative), the range will be $(-\infty, y]$.

EXPLORE 26B

Example 2

Given the graph of the quadratic function, determine the domain and range using interval notation.



domain: $(-\infty, \infty)$ The graph will continue infinitely with any value of x.

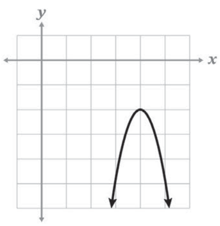
To determine the range of the graph, you must first determine the vertex.

vertex: $(0, -6)$ positive

range: $[-6, \infty)$ Since the graph is positive, the values of y will continue to increase.

Example 3

Given the graph of the quadratic function, determine the domain and range using interval notation.



domain: $(-\infty, \infty)$

range: $(-\infty, 2]$ Because the graph is negative, there is a maximum point. Therefore, all of the values of the range will be below -2.

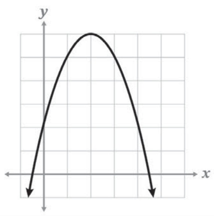
☑ Checkpoint

Use the graph to determine the domain and range.

$y = -x^2 + 4x + 2$

domain: $(-\infty, \infty)$

range: $(-\infty, 6]$



☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is another way of saying all real numbers for the domain?

A: *Negative infinity to positive infinity.*

Q: What point on the graph must you know to determine the range?

A: *The vertex.*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1–6)

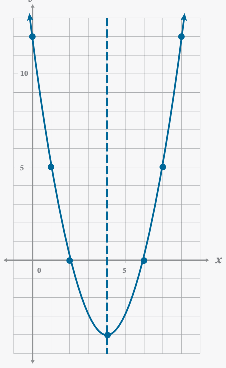
Q: How many solutions can quadratic functions have?

A: *Zero, one, or two solutions.*

Q: How is the number of solutions determined?

A: *The number of solutions is determined by the number of times the parabola crosses the x-axis.*

7)



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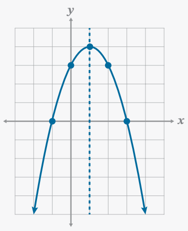

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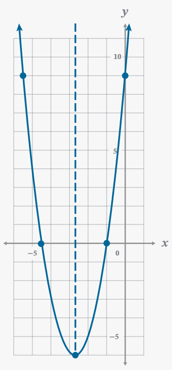


7–9)

Additional points are marked in the key to help check your student's graph.

Q: Why is seeing the graph of a parabola helpful when finding the range?

A: *Because seeing the vertex and the graph's direction helps you write the range correctly using interval notation.*

Q: Can you determine the bracket or parenthesis for the range before graphing? Explain.

A: *Yes, the a-value tells you the direction of the parabola. If the parabola opens upward, the range will be $[y, \infty)$ and if it opens downward, it will be $(-\infty, y]$.*

10–12)

Q: How are the directions, "sketch" different from "graph"?

A: *Sketch tells you that not as much detail or precision is needed compared to graphing.*

Practice 1

Complete the problems on a separate sheet of paper.

Use technology to determine the solutions for the quadratic functions. Round to the nearest thousandth.

1) $y = -x^2 + 5x + 7$ **$x = -1.14, 6.14$**

2) $f(x) = x^2 + 15x + 8$ **$x = -14.446, -0.554$**

3) $y = 0.27x^2 - 1.5x - 2.2$ **$x = -1.205, 6.761$**

4) $y = -\frac{8}{5}x^2$ **$x = 0$**

5) $h(x) = \frac{2}{3}x^2 - 11x - 6$ **$x = -0.529, 17.029$**

6) $y = 3x^2 - x + 2$ **no real solution**

Use technology to graph the given quadratic equations. Then, match the domain and range to the corresponding equation.

7) $y = x^2 - 8x + 12$ **H**

8) $y = -x^2 + 2x + 3$ **G**

9) $y = 2x^2 + 11x + 9$ **J**

G) domain: $(-\infty, \infty)$
range: $(-\infty, 4]$

H) domain: $(-\infty, \infty)$
range: $[-4, \infty)$

J) domain: $(-\infty, \infty)$
range: $\left[-\frac{49}{8}, \infty\right)$

Match each equation to the corresponding domain and range. Sketch a graph of each parabola.

10) domain: $(-\infty, \infty)$ **D**
range: $[-1, \infty)$

11) domain: $(-\infty, \infty)$ **E**
range: $[-4, \infty)$

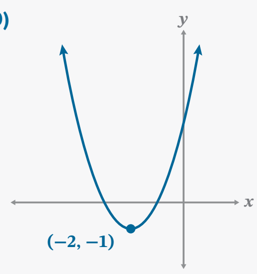
12) domain: $(-\infty, \infty)$ **F**
range: $(-\infty, 1]$

D) $y = x^2 + 4x + 3$

E) $f(x) = x^2 - 4$

F) $y = -x^2 + 2x$

10)



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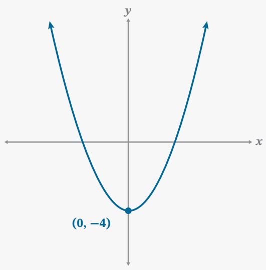

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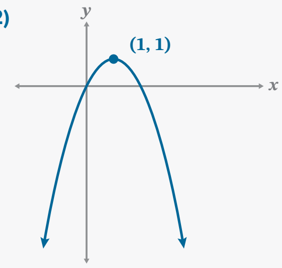


MASTERY CHECK 26B

Mastery Check

Show What You Know

A) Using the numbers $\{-2, -1, 0, 1, 2\}$ only once, complete the equation for a parabola with a domain of $(-\infty, \infty)$ and a range of $(-\infty, 1]$.

$y =$ [-1] $x^2 +$ [2] x

$y = -1x^2 + 2x$

B) Name the solution(s) to your equation.

$(0, 0)$ and $(2, 0)$ or $x = 0, 2$

C) Using the numbers $\{-2, -1, 0, 1, 2\}$ only once, complete the equation for a parabola with a domain of $(-\infty, \infty)$ and a range of $[-1, \infty)$.

$y =$ [1] $x^2 +$ [2] x

$y = 1x^2 + 2x$

D) Which of the given numbers cannot be the coefficient of x^2? Explain.

Zero cannot be the coefficient of x^2 because x^2 would simplify out of the equation, making it a linear graph.

E) The vertex of an upward-facing parabola is at the point $(6, 8)$. What is the range for this graph?

Range: $[8, \infty)$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

It is recommended that your student use technology to create their equations to quickly see if the range is correct. Your student should have multiple attempts. This will help them confirm that their answer is correct.

A) Q: Because the range represents a downward facing parabola, a must be what?

A: *Because the parabola is facing downward, a must be negative.*

Q: Where does the vertex have to be located?

A: *At the point (AoS, 1)*

B) Remember that solutions are the same as x-intercepts. Your student can find them by factoring or locating them on the graph.

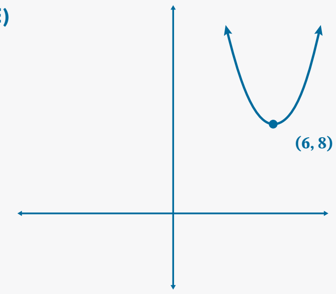
C) Q: Name the solution(s) to your equation.

A: $x = -2$ *and* 0

Q: What is the value of "c" in both equations?

A: *Zero*

E)



Remind your student that they can determine the range because they know the graph's direction and the vertex. They can make a quick sketch to help them answer this problem if needed.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Find exact and approximate solutions to quadratic equations, using technology as needed.
- Determine the domain and range for quadratic equations.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

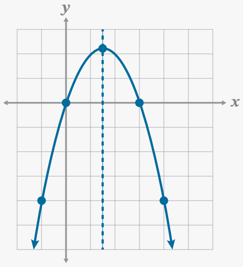
If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

7)



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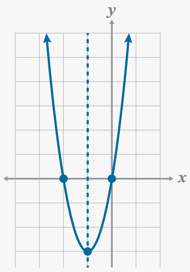

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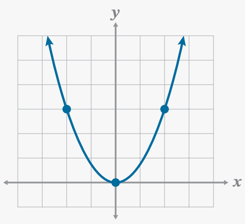

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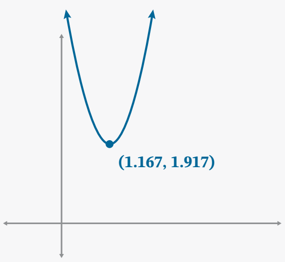

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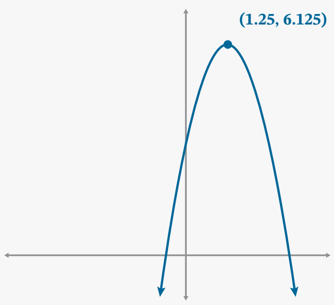

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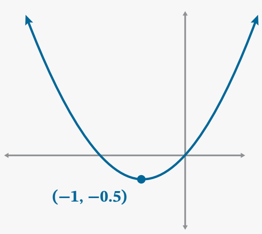


26B PRACTICE 2

Practice 2

Complete the problems on a separate sheet of paper.

Use technology to determine the solutions for the quadratic functions. Round to the nearest thousandth.

1) $y = x^2 + x - 7$ $x = -3.193, 2.193$

2) $f(x) = -x^2 + 0.5x + 3$ $x = -1.5, 2$

3) $y = -\frac{1}{5}x^2 + 8x - 1$ $x = 0.125, 39.875$

4) $g(x) = 7x^2 - 12x + 4$ $x = 0.453, 1.261$

5) $y = x^2 + x + 4$ no real solutions

6) $y = -2x^2 - 11x - 6$ $x = -4.886, -0.614$

Use technology to graph the given quadratic equations. Then, match the domain and range to the corresponding equation.

7) $f(x) = -x^2 + 3x$ M

8) $f(x) = 3x^2 + 6x$ K

9) $y = \frac{3}{4}x^2$ N

K) domain: $(-\infty, \infty)$
range: $[-3, \infty)$

M) domain: $(-\infty, \infty)$
range: $\left(-\infty, \frac{9}{4}\right]$

N) domain: $(-\infty, \infty)$
range: $[0, \infty)$

Match each equation to the corresponding domain and range. Sketch a graph of each parabola.

10) domain: $(-\infty, \infty)$ C
range: $\left[\frac{23}{12}, \infty\right)$

11) domain: $(-\infty, \infty)$ A
range: $(-\infty, 6.125]$

12) domain: $(-\infty, \infty)$ B
range: $[-0.5, \infty)$

A) $f(x) = -2x^2 + 5x + 3$

B) $g(x) = \frac{1}{2}x^2 + x$

C) $y = 3x^2 - 7x + 6$

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

TARGETED REVIEW 26

Targeted Review

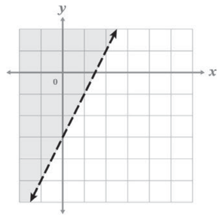
In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Factor completely.
$5x^2 + 35x - 5$
$\mathbf{5(x^2 + 7x - 1)}$

2) Factor completely.
$\frac{1}{2}x^2 + \frac{7}{2}x + \frac{10}{2}$
$\mathbf{\frac{1}{2}(x+5)(x+2)}$

3) Write the linear inequality for the provided graph. $\mathbf{y > 2x - 3}$



4) Graph the system of linear inequalities.
$y > \frac{1}{2}x - 1$
$y < 1$
$x > -2$

5) Simplify.
$(3^2y^6)(3^5y^3)$ $\mathbf{3^7y^9}$

6) Draw the parent function for a linear equation $f(x) = x$. Then, draw a new function translated up three spaces on the same coordinate plane.

7) What is the equation of the new function in problem 6? What is the relationship between the two functions? **The new function's equation is $y = x + 3$, or $g(x) = x + 3$. The two functions are parallel lines.**

8) Find the area of a rectangle with a length of $3xy^5$ and a width of $11y$ units. **$A = 33xy^6$ square units**

9) The product of two consecutive *even whole* numbers is 80. Write an equation to represent the given information. Explain your equation. $\mathbf{x(x+2) = 80}$

10) Solve your equation from problem 9. What are the two numbers with a product of 80? $\mathbf{x = -10, 8}$

Multiple Choice

11) Write all the expressions that have a term with a coefficient of 3.

- ■ $\mathbf{x^2 + 3x}$
- ☐ -3
- ☐ $6x - 3y$
- ■ $\mathbf{4x + 3y + 2z}$

12) Write all the expressions that will have two terms when simplified.

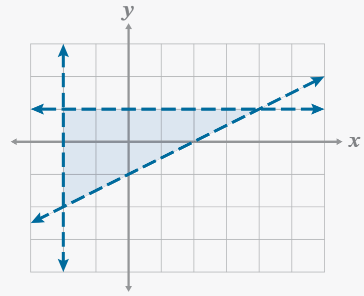
- ■ $\mathbf{2x - 5 + 3x}$
- ■ $\mathbf{(x^2 + 11x) - (x^2 + 2)}$
- ■ $\mathbf{14x + 63y}$
- ☐ x^3y^8

Targeted Review

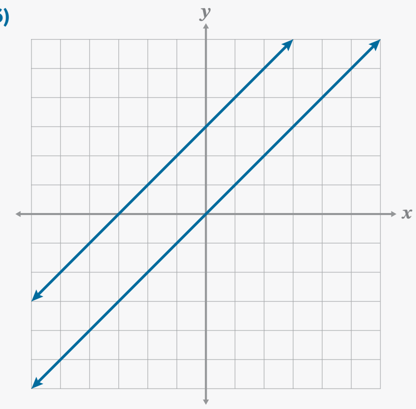
Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

4)



6)



9) The variable x represents the first even whole number, $(x + 2)$ represents the next even whole number, and 80 is the product of the factors.

10) The solution -10 is extraneous because it is not a whole number. When $x = 8$, $x + 2 = 10$. The numbers for the equation are 8 and 10 because whole numbers must be positive.

11) Distractor Rationale:

The value -3 is a constant. It has no variable, so it cannot be a coefficient.

The coefficients in $6x - 3y$ are 6 and -3.

12) Distractor Rationale:

The final expression is one term because the variables are multiplied together. Terms are separated by addition and subtraction symbols.

Problem	1	2	3	4	5	6	7	8	9–10	11	12
Lesson Origin	24	24	15	15	19	8	8, 12	19	25	PA, 1	1