Lesson 25
Applications of Quadratics

Lesson 25 Outline

Part A
Solving Quadratic Equations Not Equal to Zero

- Quadratic Equations Not Equal to Zero
- Extraneous Solutions

Part B
Quadratic Applications

- Writing Quadratic Equations Symbolically
- Solving Quadratic Equation Word Problems

Targeted Review

Vocabulary

- extraneous solution

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Q: What are the two inverse properties that allow you to solve equations?

A: *The Additive Inverse Property and the Multiplicative Inverse Property.*

Have your student review their Formula Sheet if they do not remember the algebraic properties.

Part A: Solving Quadratic Equations Not Equal to Zero

Objectives

In this part of the lesson, you will learn about solving quadratic equations not equal to zero.

By the end of this lesson, you will be able to do the following:

- Solve quadratic equations not equal to zero by factoring.

Why?

In order to be able to represent quadratic equations algebraically, you will need to be able to rewrite an equation so that it is set equal to zero.

Warm Up

Solve.

1) $5(x+8) = 10x - 2(x+6)$

$5x + 40 = 10x - 2x - 12$

$5x + 40 + 12 = 8x - 12 + 12$

$5x - 5x + 52 = 8x - 5x$

$52 = 3x$

$\frac{1}{3}(52) = \frac{1}{3}(3x)$

$\frac{52}{3} = x$

2) $(4x+1)(x-2) = (2x-3)(2x+3)$

$4x^2 - 7x - 2 = 4x^2 - 9$

$-7x - 2 = -9$

$-7x = -7$

$x = 1$

Quadratic Equations Not Equal to Zero

- All the quadratic equations you have worked with so far were equal to **zero**.
- When a quadratic equation is not equal to zero, you must move **all terms** to one side of the equals sign so that the resulting equation is equal to **zero**.
- First, determine if the terms should be on the left or right side of the equals sign:
  - Find the term with the **greatest degree**.
  - Make sure the first term is **positive** once all values are moved.
- Then, solve the equation by **factoring**.

EXPLORE 25A

Example 1

Solve.

$8x^2 + 2 = 3x^2 + 7x$

Plan Move all the terms to one side.
Factor.

Remember to write "equals zero" to remind you to solve.

Implement	**Explain**
$8x^2 \quad\quad + 2 = 3x^2 + 7x$ $-3x^2 - 7x \quad\quad -3x^2 - 7x$	Since $8x^2$ is larger than $3x^2$, move all the terms to the left side of the equation.
$5x^2 - 7x + 2 = 0$	Combine like terms.
$(\; - \;)(\; - \;) = 0$ $(5x - 2)(x - 1) = 0$	Factor to solve for the values of x. There must be two subtraction symbols in the parentheses based on the sign patterns.
$5x - 2 = 0 \quad x - 1 = 0$ $5x = 2 \quad x = 1$ $x = \frac{2}{5}$ $x = \frac{2}{5}, 1$	Set each group of terms equal to zero and solve for the values of "x."

Checkpoint

Solve.

$2x^2 + 45 = x^2 + 14x$

$\mathbf{x^2 - 14x + 45 = 0}$

$\mathbf{(x - 9)(x - 5) = 0}$

$\mathbf{x = 9, 5}$

Extraneous Solutions

- Now that you can solve quadratic equations for their values, you must determine what values make sense within the **context** of the problem.
- An **extraneous** solution is mathematically correct but is a value that is not true within the context of a problem. An extraneous solution can also be a solution that makes the original equation untrue.
  - These types of solutions often happen when working with word problems involving **measurement**.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the first step in solving this equation?

A: *Move all of the terms to one side of the equation so it can be factored.*

Q: How can you check to make sure your solutions are correct?

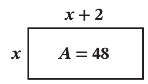
A: *Substitute the solutions in one at a time to see if the equation will be equal to zero on both sides.*

25A EXPLORE

Example 2

The length and width of a rectangle are consecutive, even natural numbers. The area of the rectangle is 48 square inches. Write an equation to find the length and width.

Plan Draw a rectangle and label the sides.
Write the area formula for a rectangle.
Substitute the known values.
Solve and evaluate the solutions.



Implement

$$A = lw$$
$$48 = x(x + 2)$$
$$48 = x^2 + 2x$$
$$0 = x^2 + 2x - 48$$
$$(x - 6)(x + 8) = 0$$
$$x - 6 = 0 \qquad x + 8 = 0$$
$$x = 6 \qquad x = -8$$

The two solutions are 6 and −8.

$x = -8$ is extraneous.

$x = 6$ is the solution.

Explain

◂ Simplify the right side by distributing x.

◂ Subtract 48 from both sides so that the equation is equal to zero.

◂ Now factor and solve for the possible values of x.

◂ Solve for x.

◂ The solution −8 cannot be a solution because the sides of a figure cannot be negative. This means that $x = 6$ is the correct solution and $x = -8$ is extraneous.

◂ This solution makes the width of the rectangle 6 feet and the length 8 feet ($6 + 2 = 8$). Since $6 \cdot 8 = 48$ and these numbers are consecutive, even, and natural, we solved correctly.

Check

$6 \cdot 8 = 48$

◂ Six and 8 are consecutive even numbers, so 6 is the correct solution.

102 Lesson 25: Applications of Quadratics > Part A: Solving Quadratic Equations Not Equal to Zero > Explore Algebra 1 Student Worktext

EXPLORE 25A

Example 3

Two rectangles have equal areas but different side lengths. The given equation shows the areas set equal to one another. Solve for the value of x.

$9x^2 - x + 1 = 25x^2 - x - 24$

Implement	Explain
$9x^2 + 1 = 25x^2 - 24$	The term $-x$ is on both sides and simplifies out of the equation.
$-9x^2 - 1 \quad -9x^2 - 1$	Move terms to the right using the Addition Property of Equality.
$0 = 16x^2 - 25$	The combined terms result in a difference of two squares equation.
$0 = (4x - 5)(4x + 5)$	Factor. The last term is negative, indicating one subtraction and one addition sign in the factored answer.
$4x - 5 = 0 \quad 4x + 5 = 0$	Solve for the values of x.
$x = \frac{5}{4} \quad x = -\frac{5}{4}$	Without knowing the side lengths, it is possible that either value of x is true. You cannot say that either answer is extraneous without knowing more about the figures.

Check

$x = \frac{5}{4}$

$9\left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right) + 1 = 25\left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right) - 24$

$13.8125 = 13.8125$

$x = -\frac{5}{4}$

$9\left(-\frac{5}{4}\right)^2 - \left(-\frac{5}{4}\right) + 1 = 25\left(-\frac{5}{4}\right)^2 - \left(-\frac{5}{4}\right) - 24$

$16.3125 = 16.3125$

Checking with a calculator shows that both answers are true. The equations are equal on both sides, and both solutions result in a positive area. If one of the solutions had resulted in a negative area, that solution would have been extraneous.

Checkpoint

A rectangle has a width of x, and a length five times the width, less three. If the area of the rectangle is 14 square feet, what are the side lengths? Solve the equation that has been started for you.

$A = lw$

$14 = x(5x - 3)$

$14 = 5x^2 - 3x$

$5x^2 - 3x - 14 = 0$

$(5x + 7)(x - 2) = 0$

$5x + 7 = 0, x - 2 = 0$

$x = -\frac{7}{5}, 2$

The width is 2 feet and the length is 7 feet ($5(2) - 3 = 7$).

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Why are both solutions not true for the rectangle?

A: *One of the solutions is negative and a side length cannot be negative.*

Remind your student to combine terms by distributing and setting the equation equal to zero before factoring and solving.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) Q: Why do you need to set the equation equal to zero?

A: *So that you can find the solutions by factoring.*

2) Q: How can you check that any solution is correct?

A: *Substitute the values back into the given equation to see if the result is true.*

4) Q: In this problem, which side of the equation should all the terms be moved to? Explain.

A: *The terms should be moved to the left side so the leading coefficient is positive.*

5) Q: When all coefficients are positive, how do you determine what side to move all of the terms to?

A: *The side with the term that has the greatest leading coefficient.*

7) The side length is 7 inches $\left(3\left(\frac{7}{3}\right) = \frac{21}{3} = 7\right)$.

The extraneous solution is $-\frac{7}{3}$ because a side length cannot be negative $\left(3\left(-\frac{7}{3}\right) = -\frac{21}{3} = -7\right)$.

Q: What is an extraneous solution?

A: *A solution that does not make sense in the context of the problem but is still correct mathematically.*

8) A side cannot be negative so $x = -6$ is not a possible solution. When $x = 4$, the sides of the rectangle are 4 feet and 6 feet $(4 + 2 = 6)$.

9) When $x = \frac{1}{3}$, each side of the square will be 3 cm. The value $-\frac{5}{3}$ is extraneous because a side cannot be -3 cm.

Remember to have your student check their solution(s) by substituting values into the original problem. Not every negative solution will be extraneous.

10) The side lengths are 1 unit $(3 - 2 = 1)$ and 6 units $(3 + 3 = 6)$. The solution $x = -4$ is extraneous.

Q: Why is it that the negative solutions tend to be extraneous for area problems?

A: *Because they usually result in a negative side length, which is not possible.*

11) When $x = 4$, the sides of the rectangle are 16 inches $(4(4) = 16)$ and 2 inches $(4 - 2 = 2)$. The extraneous solution is $x = -2$.

12) The sides of the rectangle are 9 feet $(2(5) - 1 = 9)$ and 2 feet $(5 - 3 = 2)$ long. The value $-\frac{3}{2}$ is extraneous.

Practice 1

Complete the problems on a separate sheet of paper.

Solve.

1) $x^2 + 21 = 10x$ $\quad$ $\mathbf{x = 7, 3}$

2) $x^2 = 6x$ $\quad$ $\mathbf{x = 0, 6}$

3) $3x + 10 = x^2$ $\quad$ $\mathbf{x = 5, -2}$

4) $4x^2 = 30 + 19x$ $\quad$ $\mathbf{x = -\frac{5}{4}, 6}$

5) $3x^2 + 22x = x^2 - 56$ $\quad$ $\mathbf{x = -7, -4}$

6) $20x^2 - 100 = -20$ $\quad$ $\mathbf{x = \pm 2}$

Solve. Remember to check for extraneous solutions.

7) The area of a square is 49 square inches. The sides when multiplied together are $(3x)^2$ square inches. Use the given equation to find the values of x and the side length.
$(3x)^2 = 49$ $\quad$ $\mathbf{x = \frac{7}{3}, -\frac{7}{3}}$

8) The formula for the area of a certain rectangle is $x(x + 2) = 24$. The area of the rectangle is 24 square feet. What are the values of x and the side lengths? $\quad$ $\mathbf{x = -6, 4}$

9) The sides of a square are $3x + 2$ centimeters each. The area of the square is 9 square centimeters. Find the values of x and the length of each side using the equation $(3x + 2)^2 = 9$. $\quad$ $\mathbf{x = \frac{1}{3}, -\frac{5}{3}}$

10) The equation of a rectangle is given below. What is the length and the width of the rectangle? Use the given equation to solve for x and the side lengths.
$(x - 2)(x + 3) = 6$ $\quad$ $\mathbf{x = 3, -4}$

11) The sides of a rectangle are $4x$ inches and $x - 2$ inches. The area of the rectangle is 32 inches. Use the given equation to find the values of x and the exact length and width of the sides of the rectangle.
$4x(x - 2) = 32$ $\quad$ $\mathbf{x = -2, 4}$

12) A rectangle with side lengths $2x - 1$ feet and $x - 3$ feet has an area of 18 square feet. Use the given equation to find the value of x and the side lengths of the rectangle.
$(2x - 1)(x - 3) = 18$ $\quad$ $\mathbf{x = -\frac{3}{2}, 5}$

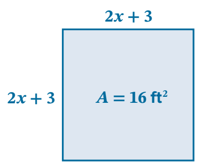
MASTERY CHECK 25A

Mastery Check

Show What You Know

The area of a square is 16 square feet. One side length of the square is $2x + 3$ feet long.

A) Draw and label the figure described.



B) Find the values of x using the equation below.
$(2x + 3)^2 = 16$

$$4x^2 + 12x + 9 = 16$$
$$4x^2 + 12x - 7 = 0$$
$$(2x - 1)(2x + 7) = 0$$
$$2x - 1 = 0,\ 2x + 7 = 0$$
$$x = \tfrac{1}{2}, -\tfrac{7}{2}$$

C) Using the values you solved for in part B, determine the side length(s) of the figure.

$$2\left(\tfrac{1}{2}\right) + 3 = 4 \text{ ft}$$
$$2\left(-\tfrac{7}{2}\right) + 3 = -4 \text{ ft}$$

The square has sides of 4 feet using $x = \frac{1}{2}$. The extraneous solution is $-\frac{7}{2}$ because the side of a figure cannot be a negative length.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

B) Q: What is the first step to solving this equation?

A: *Find the product of the binomial squared.*

C) If your student determines that both answers will be correct, discuss how a negative side length would be measured.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Solve quadratic equations not equal to zero by factoring.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

7) The side length of the square is 4 inches long $\left(5\left(\frac{6}{5}\right)-2=4\right)$.
The value $-\frac{2}{5}$ is extraneous because a side length cannot be negative.

8) The side lengths of the rectangle are $\frac{4}{3}$ feet and 9 feet $\left(3\left(\frac{4}{3}\right)+5=9\right)$.
The extraneous solution is -3.

10) The side of the square is 4 units long $\left(4\left(\frac{3}{4}\right)+1=4\right)$. The extraneous solution is $x=-\frac{3}{4}$.

11) The extraneous solution is $-\frac{11}{4}$.
The length is 9 units $(4(3)-3=9)$.
The width is 7 units $(2(3)+1=7)$.

12) The side lengths of the rectangle are $\frac{5}{6}$ cm and 6 cm $\left(6\left(\frac{5}{6}\right)+1=6\right)$. The extraneous solution is -1.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Practice 2

Complete the problems on a separate sheet of paper.

Solve.

1) $24=x^2+10x$ $\quad$ $\boldsymbol{x=-12, 2}$

2) $x^2+120=8x-60$ $\quad$ $\boldsymbol{x=-10, 18}$

3) $2x^2+x=21$ $\quad$ $\boldsymbol{x=-\frac{7}{2}, 3}$

4) $18x=-3x^2-15$ $\quad$ $\boldsymbol{x=-5, -1}$

5) $x^2-10x+300=10x+200$ $\quad$ $\boldsymbol{x=10}$

6) $5x^2+8=2x^2+31x+30$ $\quad$ $\boldsymbol{x=-\frac{2}{3}, 11}$

Solve. Remember to check for extraneous solutions.

7) The sides of a square are $5x-2$ inches each. The area of the square is 16 square inches. Find the values of x and the length of each side using the equation $(5x-2)^2=16$. $\quad$ $\boldsymbol{x=-\frac{2}{5}, \frac{6}{5}}$

8) The given area formula represents a rectangle with an area of 12 square feet. What are the side lengths of the rectangle? Find the values of x and the side lengths of the rectangle.
$x(3x+5)=12$ $\quad$ $\boldsymbol{x=\frac{4}{3}, -3}$

9) The area of a rectangle was 10 square feet. Use the given equation to find the values of x.
$x^2+1=10$ $\quad$ $\boldsymbol{x=3, -3}$

10) The area and perimeter of a certain square are equal. Use the given equation to find x and the side length of the square.
$(4x+1)^2=8x+10$ $\quad$ $\boldsymbol{x=\frac{3}{4}, -\frac{3}{4}}$

11) Find the values of x and the length and width of a rectangle using the equation below.
$(4x-3)(2x+1)=63$ $\quad$ $\boldsymbol{x=3, -\frac{11}{4}}$

12) Find the values of x and the side lengths of a rectangle using the given equation. The area of the rectangle is 5 square centimeters.
$x(6x+1)=5$ $\quad$ $\boldsymbol{x=\frac{5}{6}, -1}$

EXPLORE 25B

Part B: Quadratic Applications

Objectives

In this part of the lesson, you will learn about quadratic applications.

By the end of this lesson, you will be able to do the following:

- Translate a quadratic word problem into symbolic notation.
- Solve a quadratic word problem.

Why?

Two rooms have the same area but different dimensions. What are the dimensions of each room? This is just one type of question that you will be able to answer when you apply your knowledge of factoring.

Warm Up

1) What is an extraneous solution?

An extraneous solution is a solution that will not be true for an equation within the context of a given problem or is not true for the original equation.

2) Why is it important to check for extraneous solutions?

It is important to check for extraneous solutions because not every value will be true within the context of each problem. For example, a side length cannot be negative for a physical shape or figure.

Writing Quadratic Equations Symbolically

- To write quadratic equations for a given scenario, follow these steps:
  - **Read** the question carefully.
  - Find **key information** that will help construct the equation.
  - Define your **variable** so you know what it represents in the equation.
  - Check for **extraneous** solutions and make sure that you answer the question being asked.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

There are many problems with extraneous solutions in this lesson. Be sure that your student knows what an extraneous solution is and how to check for them.

25B EXPLORE

- Because writing and solving quadratic equations presents a new set of problem types, you will first focus on **writing** the equations and setting them equal to **zero**.
  - You will practice solving these equations in the next section.

Example 1

The product of the quantity three times a number plus two and the quantity of the same number minus seven is negative eleven. Write an equation in standard form to represent this scenario. Set your equation equal to zero but do not solve it.

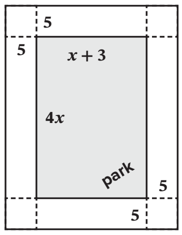
Implement	Explain
$(\quad)(\quad)$	"Product" and "quantity" mean that groups of terms will be multiplied, so start with two sets of parentheses.
$(\quad)(\quad) = -11$	The equation is equal to -11, so the product of the quantities will be equal to -11.
$(3x + 2)(x - 7) = -11$	"Three times a number" will be $3x$. "Plus two," tells you to add 2 to $3x$, so $3x + 2$. "The same number" means "x" is used again. "Minus seven" tells you to subtract 7 from x in the second set of parentheses, so $x - 7$.
$(3x + 2)(x - 7) = -11$	"Product" means the quantities will be multiplied with a result of -11.
$3x^2 - 19x - 14 = -11$	Multiply the binomials together by distributing.
$+11 \quad +11$	Move all the terms to one side to set the equation equal to zero. (Addition Property of Equality)
$3x^2 - 19x - 3 = 0$	Check that the expression is in standard form.
	The directions say "do not solve," so this problem is finished.

> The product must be found before setting the equation equal to zero because order of operations applies to polynomials as well.

EXPLORE 25B

Example 2

A tiny park is being created on a small rectangular piece of land. The park and the sidewalk surrounding it have a combined area of 300 square yards. Use the figure to write an equation representing the total area.



Implement	Explain
$A = lw$	◀ Formula for area.
$A = 300$	◀ The combined area is 300 square yards.
$l = 4x + 10$	◀ The total length of the park and the sidewalk $(4x + 5 + 5)$.
$w = x + 13$	◀ The width of the park and the surrounding sidewalk $(x + 3 + 5 + 5)$.
$300 = (4x + 10)(x + 13)$	◀ Substitute the values into the area formula.
$300 = 4x^2 + 62x + 130$	◀ Follow the order of operations and combine like terms until the equation is equal to zero.
$0 = 4x^2 + 62x - 170$	

Example 3

Suppose a small rock is tossed off a cliff into the ocean below. The person throwing the rock is 320 feet above sea level and the initial velocity of the rock is 16 feet per second (ft/s). Write an equation using the formula given below. Set the equation equal to zero.

$d = 16t^2 + vt$

$v = 16$

$d = 320$

$320 = 16t^2 + 16t$

$0 = 16t^2 + 16t - 320$

In this example, the formula $d = 16t^2 + vt$ represents the distance an object will travel given a certain velocity (v) and the time (t) it will take for the object to fall, negating air resistance.

Checkpoint

Write an equation in standard form to represent the given information. Set your equation equal to zero. Do not solve.

A rectangle has side lengths of $2x + 3$ inches and $2x - 1$ inches with an area of 3 square inches.

$A = lw,\ l = 2x + 3,\ w = 2x - 1$

$3 = (2x + 3)(2x - 1)$

$3 = 4x^2 + 4x - 3$

$0 = 4x^2 + 4x - 6$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Remind your student to define the variables for A, l, and w before setting up the equation.

25B EXPLORE

Solving Quadratic Equation Word Problems

- When solving quadratic word problems, it is important to check all your **solutions**.
  - Depending on the context of the problem, the following is possible:
    - **One** of the solutions makes sense.
    - **Both** of the solutions make sense.
- In some cases, you may need to use your **solutions** to finish answering the question.
- Answering the question in a **sentence** helps you confirm that your answer makes sense.

Example 4

In Example 3, you wrote an equation for the following scenario:

Suppose a small rock is tossed off a cliff into the ocean below. The person throwing the rock is 320 feet above sea level and initial velocity of the rock is 16 feet per second or 16 feet per second. Write an equation using the formula.

$$d = 16t^2 + vt$$
$$320 = 16t^2 + 16t$$
$$0 = 16t^2 + 16t - 320$$

Use the formula you created to solve for the time it will take for the rock to hit the water.

Implement	Explain
$0 = 16t^2 + 16t - 320$	
$16(t^2 + t - 20) = 0$	Factor out the GCF of all the terms.
$\mathbf{16(t - 4)(t + 5) = 0}$	Factor the remaining trinomial.
$\mathbf{t - 4 = 0 \quad t + 5 = 0}$	Solve and determine if the answers make sense in the context of the problem.
$\mathbf{t = 4 \quad t = -5}$	Because t represents time, the value -5 does not make sense.
It took the rock 4 seconds to hit the water.	This means that it took the rock 4 seconds to hit the water and -5 is extraneous.

EXPLORE 25B

Example 5

The product of two consecutive odd integers is sixty-three. Write an equation and solve.

Implement	Explain
n: first number $n + 2$: next odd number	
$n(n+2) = 63$	Distribute
$n^2 + 2n = 63$	Move all terms to the left side of the equation.
$n^2 + 2n - 63 = 0$	Make the equation equal to zero.
$(n-7)(n+9) = 0$	Factor
$n - 7 = 0$ $\quad n + 9 = 0$ $n = 7$ $\quad n = -9$	Now set each expression equal to zero and solve for n. To determine the value or values that will make this problem true, go back to the equation you wrote and substitute each value one at a time.
Check $n = 7$ $\quad n = -9$ $n + 2 = 9$ $\quad n + 2 = -7$ $(7)(9) = 63$ $\quad (-9)(-7) = 63$	Both 7 and −9 are solutions.

Checkpoint

The area of a rectangle is 200 square inches. The side lengths are $3x - 5$ and $x - 2$ inches. Write and solve an equation to find the dimensions of the rectangle.

$200 = (3x-5)(x-2)$

$200 = 3x^2 - 11x + 10$

$0 = 3x^2 - 11x - 190$

$(3x+19)(x-10) = 0$

$3x + 19 = 0 \quad x - 10 = 0$

$x = -\frac{19}{3}, 10$

$-\frac{19}{3} - 2 = -\frac{25}{3}$

When $x = -\frac{19}{3}$, the solution is extraneous because side lengths cannot be negative.

$10 - 2 = 8$

$3(10) - 5 = 25$

When $x = 10$, the dimensions are 8 inches by 25 inches.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Describe briefly the steps you need to take to solve the equation.

A: *Write the equation, distribute and move all terms to one side, factor, solve for x, and check for extraneous solutions.*

Your student must combine like terms before they can factor and solve.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) Q: Why do you need to set the equations equal to zero?

A: *So that you will be able to factor.*

3) $5x^2 + 4x + 273 = 0$

Q: If the product of two numbers is negative what do you know about the signs of the factors? Explain.

A: *One would be positive and one would be negative because factors with the same sign have a positive product.*

4) Q: How do you find the total height of the frame?

A: *Add the 1 inch frame on both sides to the given height.*

7) When $x = \frac{13}{2}$, $w = \frac{13}{2} = 6.5$

$l = 2\left(\frac{13}{2}\right) + 3 = 16$

A side length cannot be −20, making this solution extraneous. The dimensions of the playground are 6.5 feet by 16 feet.

8) $n = -\frac{5}{4}$: $-54\left(4 \cdot -\frac{5}{4} - 3\right) = -54(-5-3)$

$= -54(-8) = 10$

$n = 2$: $2(4 \cdot 2 - 3) = 2(8-3) = 2(5) = 10$

Both solutions make the equation true.

Q: How is it possible that there are no extraneous solutions in this problem?

A: *Since the problem is asking for factors instead of side lengths, any number that makes the equation true will be correct.*

9) The width cannot be a negative value, making $-\frac{13}{5}$ extraneous.

If the width is 3 yards, the length is 13 yards.

$l = 5(3) - 2 = 13$

10) The side of the original sheet will be 25 inches. A negative side length is not possible; therefore, −5 is extraneous.

Q: Which part of the figure represents the bottom of the box?

A: *The inner square on the figure represents the bottom of the box.*

Q: How can you find the dimensions of the inner square?

A: *By subtracting the 5 inch corners from the side of the sheet* $(x - 10)$.

11) It cannot take −3 seconds for the object to hit the ground, so −3 is extraneous.

The object will take 2 seconds to hit the ground.

Q: What is the first step once the equation is set equal to zero?

A: *Factor out the GCF, which is 16.*

12) It took the rock 2.5 seconds to hit the water. The number −4 is extraneous.

Practice 1

Complete the problems on a separate sheet of paper.

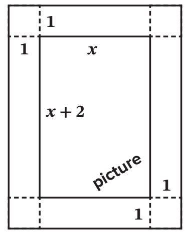
Write an equation for the given context. Define your variables. Set the equation equal to zero. *Do not* solve.

1) The area of a rectangle is 52 square feet. The length is three times the width plus two feet. $0 = 3w^2 + 2w - 52$

2) A slow-pitch softball was thrown and landed on top of home plate. The ball left the pitcher's hand 3 feet above the ground and had an initial velocity of 15 feet per second. Use the formula $d = 16t^2 + vt$ to organize the information. $0 = 16t^2 + 15t - 3$

3) The product of a number and five times that number plus 4 is −273.

4) The frame of a picture is one inch wide around the photo. The length of the opening for the picture is two more inches than the width. Write an equation to represent the given figure with a total area of 120 square inches. $0 = x^2 + 6x - 112$

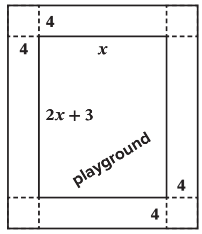


5) The height of a triangle is two times its base less three. The area of the triangle is 85 square inches. $0 = b^2 - \frac{3}{2}b - 85$

6) The product of three consecutive integers is 336. Assume x is the first term. (Hint: Your equation will be a cubic polynomial.) $x^3 + 3x^2 + 2x - 336 = 0$

Solve. Answer the questions completely and identify extraneous solutions.

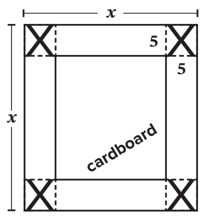
7) The playground had a 4 foot wide sidewalk surrounding it. If the total area of the sidewalk and playground was 348 square feet, what are the dimensions of the playground? Use the figure to write and solve an equation. $x = \frac{13}{2}, -20$



8) The product of a number and four times that number minus three is 10. Write and solve an equation to find the numbers that make this statement true. $n = -\frac{5}{4}, n = 2$

9) The area of a rectangle is 39 square yards. One side of the rectangle is five times the width minus 2 yards. Write an equation to find the dimensions of the rectangle. $w = -\frac{13}{5}, 3$

10) A square sheet of cardboard with side lengths of x will be folded into a box with no lid. Once folded, the area of the bottom of the box will be 225 square inches. A 5 inch square must be cut out of each corner so that the box can be folded correctly. Use the figure to write an equation and solve for the side length of the original cardboard sheet. $x = 25, -5$



11) An object is thrown from the top of a building 96 feet tall. The initial speed at which it is thrown is 16 feet per second. Negating air resistance, use the formula $d = 16t^2 + vt$ to find the time it takes the object to reach the ground. $t = -3, 2$

12) A rock was thrown off the edge of a cliff into the ocean with an initial velocity of 24 feet per second. If the rock fell 160 feet before hitting the water, how long did it take to reach the water? Negating air resistance, use the formula $d = 16t^2 + vt$ to solve. $t = \frac{5}{2}, -4$

MASTERY CHECK 25B

Mastery Check

Show What You Know

The product of a number and four more than that number is equal to the product of one less than the same number, multiplied by six less than three times the number.

A) Write and solve an equation to find all possible solutions.

x: a number

$$x(x+4) = (x-1)(3x-6)$$
$$x^2 + 4x = 3x^2 - 9x + 6$$
$$0 = 2x^2 - 13x + 6$$
$$(2x-1)(x-6) = 0$$
$$2x - 1 = 0,\ x - 6 = 0$$
$$x = \tfrac{1}{2},\ 6$$

B) Show your work to prove your solution(s) are correct. Use your solution(s) to find the product.

When $x = \frac{1}{2}$, the product is $\frac{9}{4}$.

$$x + 4 = 4\tfrac{1}{2}$$
$$x - 1 = -\tfrac{1}{2}$$
$$3x - 6 = \tfrac{3}{2} - \tfrac{12}{2} = -\tfrac{9}{2}$$
$$\left(\tfrac{1}{2}\right)\left(4\tfrac{1}{2}\right) = \left(-\tfrac{1}{2}\right)\left(-\tfrac{9}{2}\right) = \tfrac{9}{4}$$

When $x = 6$, the product is 60.

$$x + 4 = 10$$
$$x - 1 = 5$$
$$3x - 6 = 12$$
$$6 \cdot 10 = 5 \cdot 12 = 60$$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

Q: How can you prove that your solutions are correct?

A: *Substitute the values into the original equation to check if the sides of the equation will be equal.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Translate a quadratic word problem into symbolic notation.
- Solve a quadratic word problem.
- Solve quadratic equations not equal to zero by factoring.

Lesson and Unit Tests

As students reach the end of of this unit, they can complete the Lesson Test and the Unit Test.

Students should complete the Lesson Test in the same manner as they have in previous lessons.

The next day, they may need to review Lessons 19–25, using their guided notes, Mastery Checks, and Lesson Tests before beginning the Unit Test.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

7) If −13 is used, the sides of the chicken coop will be negative. This is an extraneous solution.

$l = 3\left(\frac{10}{3}\right) - 1 = 9$

$w = \left(\frac{10}{3}\right) + 2 = 3\frac{1}{3} + 2 = 5\frac{1}{3}$

The dimensions of the chicken coop are 9 feet by $5\frac{1}{3}$ feet.

8) When $x = -\frac{5}{2}, x + 1 = -\frac{3}{2}$.

The product of $-\frac{5}{2}$ and $-\frac{3}{2}$ is $\frac{15}{4}$.

Since $\left(-\frac{5}{2} - 5\right)\left(3\left(-\frac{5}{2}\right) + 7\right) = 154$, $-\frac{5}{2}$ is a solution.

When $x = 7, x + 1 = 8$. This product is 56, and $(7 - 5)(3(7) + 7) = 56$ making 7 a solution.

9) $w = \frac{5}{2}, -3$

$l = 2\left(\frac{5}{2}\right) + 1 = 5 + 1 = 6$

The width cannot be a negative measure, making −3 an extraneous solution.

When $w = \frac{5}{2}$, or $2\frac{1}{2}$ feet, the length is 6 feet.

10) $2(6) - 3 = 9$

The value of x cannot be negative because this creates a negative side length; therefore, $-\frac{3}{2}$ is extraneous.

When $x = 6$, the dimensions of the metal sheet are 6 inches by 9 inches.

11) It will take the ball 1 second to reach the ground. The solution $-\frac{5}{4}$ is extraneous.

12) The object will take 10 seconds to hit the top of the building. The solution −12 is extraneous.

Make sure your student subtracts the height of the building from the height of the helicopter to find the distance the object travelled (2,000 ft − 80 ft = 1,920 ft).

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note regarding the Lesson and Unit Tests.

25B PRACTICE 2

Practice 2

Complete the problems on a separate sheet of paper.

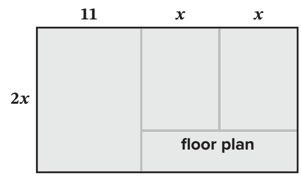
Write an equation for the given context. Define your variables. Set the equation equal to zero. *Do not* solve.

1) The area of a rectangle is 63 square inches. The length of the rectangle is four times the width plus 1 inch. Use the area formula to create your equation. $0 = 4w^2 + w - 63$

2) A rock was tossed into a deep wishing well. The initial velocity of the rock is 8 feet per second and it must travel 75 feet before reaching the water below. Use the formula $d = 16t^2 + vt$ to organize the information. $0 = 16t^2 + 8t - 75$

3) The product of two negative integers is 87. One number is nine times the other plus two. $0 = 9n^2 + 2n - 87$

4) The area of a triangle is 37 square feet. The base of the triangle is twice the height less seven. Write an equation using the formula $A = \frac{bh}{2}$. Set it equal to zero after clearing fractions. $0 = 2h^2 - 7h - 74$

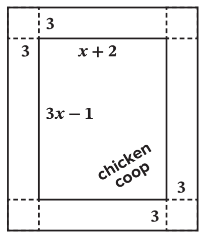


5) Write an equation to represent the floor plan of the house when the total area is 500 square feet. $0 = 4x^2 + 22x - 500$

6) The product of two consecutive *odd* integers is thirty-five. $x^2 + 2x - 35 = 0$

Solve. Answer the questions completely and identify extraneous solutions.

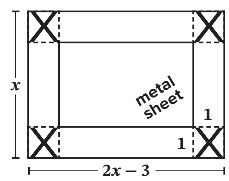
7) The total square footage of a chicken coop and sidewalk is 170 square feet. The sidewalk surrounding the coop is 3 feet wide. Find the dimensions of the chicken coop. $x = \frac{10}{3} = 3\frac{1}{3}, x = -13$



8) The product of a number and one more than the number is equal to the product of the number less five, and three times the number plus seven. Write an equation to find the solution(s). $x = -\frac{5}{2}, 7$

9) The area of a garden is 15 square feet. The length is one more than double the width of the garden. What are the dimensions of the garden? Write and solve an equation.

10) A flat, rectangular piece of metal will be formed into a box on a press. To correctly form the box, a one-inch square must be cut from each corner. Once formed, the area of the bottom of the box will be 28 square inches. Use the figure to determine the dimensions of the metal sheet before the box is formed. $x = -\frac{3}{2}, 6$



11) A child in a tree house 20 feet above the ground tossed a ball to the ground with an initial velocity of 4 feet per second. Negating air resistance, use the formula $d = 16t^2 + vt$ to find the time it takes for the ball to hit the ground. $t = -\frac{5}{4}, 1$

12) A helicopter was hovering 2,000 feet above the ground. An object was thrown from the helicopter downward and landed atop an 80 foot building. If the initial velocity was 32 feet per second, how many seconds did it take for the object to hit the building, negating air resistance? Use the formula $d = 16t^2 + vt$ to find the time in seconds. $t = 10, -12$

114 Lesson 25: Applications of Quadratics > Part B: Quadratic Applications > Practice 2 Algebra 1 Student Worktext

TARGETED REVIEW 25

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the practice problems on a separate sheet of paper.

1) Classify the polynomial expressions by degree and number of terms.

A) $6x^3 - 2$ **cubic binomial**

B) $11x^2 + 3x - 5$ **quadratic trinomial**

2) Find the product.
$3p(2p - 5)^2$

$\mathbf{12p^3 - 60p^2 + 75p}$

Solve.

3) $(2x - 3)(5x + 6) = 0$ $\mathbf{x = -\frac{6}{5}}$

4) $x^2 - 5x + 6 = 0$ $\mathbf{x = 2}$

Factor completely.

5) $25a^2 + 10a + 1$
$\mathbf{(5a + 1)(5a + 1)}$

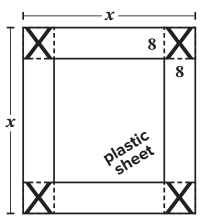
6) $8n^3 - 72n$
$\mathbf{8n(n - 3)(n + 3)}$

7) $3m^4 - 4m^2 + 1$
$\mathbf{(3m^2 - 1)(m - 1)(m + 1)}$

8) Find the value of Q.
$(5x^2 + 11x - 3) - (4x^2 - Qx + 7) = x^2 + 16x - 10$ $\mathbf{Q = 5}$

Multiple Choice

D **9)** A storage bin is being formed out of a sheet of plastic. An eight inch square needs to be cut out of each corner to allow for the box to be formed. The area of the bottom of the box is 289 square inches. Write an equation to represent the area of the box using the figure.



A) $x^2 = 289$

B) $(x + 16)^2 = 289$

C) $(x - 8)^2 = 289$

D) $\mathbf{(x - 16)^2 = 289}$

B **10)** The product of two numbers is 28. The sum of these factors is 16. What are the two numbers?

A) 1 and 28

B) **2 and 14**

C) 2 and 8

D) 4 and 7

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

9) Distractor Rationale:
- A) This does not remove the square corner.
- B) This adds 16 more inches rather than subtracting it.
- C) This only subtracts half of the square since each side is 8 inches.

10) Distractor Rationale:
- A) This results in a product of 28 and a sum of 29.
- C) This results in a product of 16 and a sum of 10.
- D) This results in a product of 28 and a sum of 11.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	20	20	24	24	24	24	24	20	25	22	24	24

11) Distractor Rationale:
- B) This answer is the opposite of both correct values.
- C) This answer is the reciprocal of the correct values.
- D) This answer is the opposite reciprocal of the correct values.

12) Distractor Rationale:
- A) This is incorrect because the GCF is not factored out.
- B) This is incorrect because the trinomial is not factored.
- C) This is incorrect because the binomials do not multiply to the given expression.

This is the last lesson before the Unit 4 Test.

TARGETED REVIEW 25

__A__ **11)** Solve:
$(6x-7)(2x+5)=0$

A) $x=-\frac{5}{2},\frac{7}{6}$

B) $x=-\frac{7}{6},\frac{5}{2}$

C) $x=-\frac{2}{5},\frac{6}{7}$

D) $x=-\frac{2}{5},\frac{6}{7}$

__D__ **12)** Choose the answer that factors the expression completely.
$4x^2+10x+6$

A) $(4x+6)(x+1)$

B) $2(2x^2+5x+3)$

C) $2(2x+1)(x+3)$

D) $2(2x+3)(x+1)$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	20	20	24	24	24	24	24	20	25	22	24	24