Lesson 24
More Factoring

Outline

Part A
Factoring Completely

- Factoring Completely

Part B
Solving Polynomial Equations Equal to Zero

- The Zero-Product Property and Factoring
- Solving Polynomial Equations Equal to Zero

Targeted Review

Vocabulary

There are no new vocabulary words for this lesson.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

This is a good time to have your student look back at Lessons 21–23 to review the different factoring methods. They can do this by reviewing each lesson's objectives and practice problems.

Q: What is a solution to an equation?

A: *The number, or numbers, that make the equation equal on both sides of the equation.*

Part A: Factoring Completely

Objectives

In this part of the lesson, you will learn about factoring completely.

By the end of this lesson, you will be able to the following:

- Factor polynomials completely, or state when a polynomial is not factorable.

Why?

When can you stop factoring? Knowing when an expression is factored completely is an important step in mastering how to factor quadratic expressions.

Warm Up

Name the ways you have learned to factor so far in this Unit.

The ways of factoring are finding the GCF, factoring by grouping, identifying a difference of two squares, identifying perfect square trinomials, ac-grouping, and using sign patterns.

Factoring Completely

- To factor completely means that you **cannot factor** the expression any **further**.
- To factor completely, you will **combine** multiple methods of factoring in the following order:
  1) Find the **greatest common monomial** factor (other than 1).
  2) Factor by **grouping** (when given 4 terms).
  3) Analyze the **sign** patterns.
  4) Factor **special** products (difference of two squares, perfect square trinomials).
  5) Factor using your **preferred** factoring method (ac-grouping, modeling, or mental math).
- Some expressions cannot be **factored**.

EXPLORE 24A

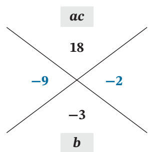
Example 1

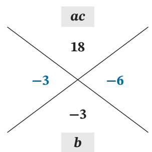
Factor Completely.	Implement	Explain
$4x^3 - 6x^2 - 36x + 54$	$4x^3 - 6x^2 - 36x + 54$	
	$2(2x^3 - 3x^2 - 18x + 27)$	◀ Find the GCF
	$2\left((2x^3 - 3x^2) + (-18x + 27)\right)$	◀ Factor by grouping
	$2\left(x^2(2x - 3) - 9(2x - 3)\right)$	
	$\mathbf{2(x^2 - 9)(2x - 3)}$	◀ Sign patterns
	$\mathbf{2(x - 3)(x + 3)(2x - 3)}$	◀ Factor special products

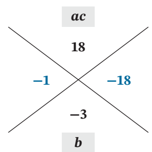
Example 2

Factor. If the expression cannot be factored, write "cannot be factored."

A) $2x^2 - 3x + 9$
Solution: cannot be factored







Why is the expression not factorable?

- There is no GCF.
- It does not have four terms.
- The sign patterns do not work.
  - Two subtraction symbols must be in the binomials.
  - None of the factor pairs add to −3.

No combination of terms multiply to +18 and add to −3.

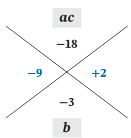
B) $2x^2 - 3x - 9$

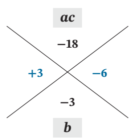
$\mathbf{(2x + 3)(x - 3)}$

> Remember, unless the directions say to use a specific method, use the factoring method that makes sense to you.

Sign patterns

- −9 and +2 will not add to −3
- 3 and −6 will add to −3





	$2x$	$+3$
x	$2x^2$	$+3x$
-3	$-6x$	-9

Check

$-6x + 3x = -3x$

☑ Checkpoint

Factor completely.

$-8x^2 + 200$

$\mathbf{-8(x^2 - 25)}$

$\mathbf{-8(x - 5)(x + 5)}$

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What step did you complete first?
A: *Factor out the GCF*

Q: What do you notice about the remaining terms after your first step?
A: *The terms are a difference of two squares.*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) This is a difference of two squares expression.

3) Q: Is the GCF always the leading coefficient? Explain.

A: *No, the GCF is the greatest common factor among all the terms, not necessarily the leading coefficient.*

6) A sum of squares cannot be factored. The sign patterns do not allow this.

Q: Why are there only two terms in a difference of two squares expression?

A: *There are only two terms because the outer and inner terms simplify out of the expression because they are additive inverses of one another (opposites).*

10) Q: What is the first step in factoring when the leading coefficient is negative?

A: *Factor out a negative one.*

12) Your student may need this question as a hint for how to factor the GCF, $\frac{1}{2}$, out of the expression. You can also suggest writing the first term as $\frac{2}{2}x^2$.

Q: What is half of 2?

A: *1*

Practice 1

Complete the problems on a separate sheet of paper.

Factor the expressions. Write "cannot be factored" for any expression that is not factorable.

1) $9x^2 - 16$ **$(3x-4)(3x+4)$**

2) $9x^2 + 16$ **cannot be factored**

3) $4x^2 + 38x + 70$ **$2(2x+5)(x+7)$**

4) $10x^2 - 13x - 77$ **$(2x-7)(5x+11)$**

5) $5x^3 + 5x^2 - 30x$ **$5x(x+3)(x-2)$**

6) $x^4 - 81$ **$(x-3)(x+3)(x^2+9)$**

7) $4v^2 + 26v + 12$ **$2(v+6)(2v+1)$**

8) $2x^2 + 8x + 1$ **cannot be factored**

9) $6x^2 + 17x - 3$ **$(6x-1)(x+3)$**

10) $-4x^2 + 28x - 40$ **$-4(x-5)(x-2)$**

11) $2x^2y^2 - 2x^2 - 18xy^3 + 18xy$ **$2x(x-9y)(y-1)(y+1)$**

12) The area of a triangle is $x^2 + \frac{13}{2}x - \frac{7}{2}$ square feet. Factor the expression to find the base and height of the triangle. Recall $A = \frac{1}{2}bh$. **The base and height are $2x-1$ and $x+7$ feet.**

13) The volume of a rectangular prism is $2x^3 + 11x^2 + 5x$ cubic units. What are the side lengths of the figure? Factor to find the side lengths. **The side lengths are x, $2x+1$, and $x+5$ units long.**

MASTERY CHECK 24A

Mastery Check

Show What You Know

A) Is $n^2 + 4$ another way to write $(n-2)(n-2)$? Explain.

$(n-2)(n-2) = n^2 - 2n - 2n + 4 = n^2 - 4n + 4$

No, The first and last terms are correct, but the middle term is not. It is not possible to factor a sum of squares.

B) The volume of a square prism is $12x^3 - 60x^2 + 75x$ cubic inches. Factor completely to find the side lengths.

$12x^3 - 60x^2 + 75x$

$3x(4x^2 - 20x + 25)$

$3x(2x-5)(2x-5)$

The side lengths are $3x$ inches on one side and $(2x-5)$ inches on two sides.

C) Given the set $\{0, 1, 2, 3, 4\}$, which of these numbers cannot be values of x for the square prism from part B? Show or explain your work.

Zero cannot be an answer because $3(0) = 0$. A side length of an object cannot be 0 inches.

1 and 2 cannot be answers because $2(1) - 5 = -3$, and $2(2) - 5 = -1$. This results in negative side lengths.

The numbers 0, 1, and 2 cannot be values of x because they result in a side length of zero or a side length that is negative. Side lengths that are not positive are not possible for a physical shape or object.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Q: What is the first thing you should look for when factoring an expression?

A: *The GCF*

C) Part C is here to get your student to think about restrictions that can occur when you are using real-life objects.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Factor polynomials completely, or state when a polynomial is not factorable.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

12) Two of the sides are equal. This results in a perfect square trinomial factor pattern.

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Practice 2

Complete the problems on a separate sheet of paper.

Factor the expressions. Write "cannot be factored" for any expression that is not factorable.

1) $3x^2 + 8x + 8$ **cannot be factored**

2) $4x^2 - 80x + 400$ $\mathbf{4(x-10)^2}$

3) $-3x^3 - 24x^2 - 36x$ $\mathbf{-3x(x+6)(x+2)}$

4) $2b^2 - 9b + 9$ $\mathbf{(b-3)(2b-3)}$

5) $54x^4 - 13x^2 - 9$ $\mathbf{(x+1)(x-1)(2x+3)(2x-3)}$

6) $5x^5 - x^4 - 6x^3$ $\mathbf{x^3(5x-6)(x+1)}$

7) $4x^2 + 22x + 10$ $\mathbf{2(2x+1)(x+5)}$

8) $2x^2 + 7x + 7$ **cannot be factored**

9) $24x^2 + 39x - 18$ $\mathbf{3(8x-3)(x+2)}$

10) $6x^2 - 7x - 3$ $\mathbf{(2x-3)(3x+1)}$

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11) A rectangular prism has a volume of $2x^2 + \frac{20}{7}x + \frac{50}{49}$ cubic inches. Find the side lengths by factoring. **The side lengths of the prism are 2 inches on one side and $\left(x + \frac{5}{7}\right)$ inches on two sides.**

12) $x^4 - 4x^2 - 9x^2y^2 + 36y^2$ $\mathbf{(x-3y)(x+3y)(x-2)(x+2)}$

EXPLORE 24B

Part B: Solving Polynomials Equations Equal to Zero

Objectives

In this part of the lesson, you will learn about solving polynomial equations equal to zero.

By the end of this lesson, you will be able to do the following:

- Explain what the zeros of a quadratic equation represent.
- Solve quadratic equations equal to zero by factoring.

Why?

Factoring is only one method of solving quadratic equations. But knowing how to solve an equation using factoring will give you a solid foundation as you learn other methods in this course and Algebra 2.

Warm Up

Solve for x.

1) $5x - 3 = 0$

$5x - 3 + 3 = 0 + 3$

$5x = 3$

$\left(\frac{1}{5}\right)(5x) = \left(\frac{1}{5}\right)(3)$

$x = \frac{3}{5}$

2) $2x + 11 = 0$

$2x - 11 + 11 = 0 + 11$

$2x = 11$

$\left(\frac{1}{2}\right)(2x) = \left(\frac{1}{2}\right)(11)$

$x = \frac{11}{2}$

The Zero-Product Property and Factoring

- The solution to a polynomial equation is any value that makes the equation **true**.
  - Quadratic equations often have **two** solutions.
- The **Zero-Product Property** allows you to solve polynomial equations by factoring.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

24B EXPLORE

- The Zero-Product Property says that if the product of two factors is zero, then **at least one** of the factors is also zero.
  - If ab = 0, then $a = 0$, or $b = 0$, or $a = b = 0$.
  - This allows you to set each factor equal to **zero** and solve for the **variable** in each equation.
- The number of times the variable appears in the factored expression is usually the number of **solutions** you will have for the equation.

Example 1

Solve using the Zero-Product Property.

$(x + 5)(x - 4) = 0$

Plan Set each binomial factor equal to zero. Solve each equation.

> Most polynomial equations can be solved in one or two steps using the zero-product property. You may be able to find the solutions using mental math. If so, be sure to always check the signs of your terms.

Implement

$x + 5 = 0$ $\quad x - 4 = 0$

$-5 \quad -5$ $\quad +4 \quad +4$

$x = -5$ $\quad x = 4$

$x = -5, 4$

Check

$x = -5$

$(-5 + 5)(-5 - 4) = 0$

$(0)(-9) = 0$

$0 = 0$

$x = 4$

$(4 + 5)(4 - 4) = 0$

$(9)(0) = 0$

$0 = 0$

Example 2

Solve using the Zero-Product Property.

$2x(9x - 4)(x + 7) = 0$

Plan Set each factor equal to zero. Solve for x.

Implement

$2x = 0$

$\frac{1}{2}(3x) = \frac{1}{2}(0)$

$x = 0$

$9x - 4 = 0$

$+4 \quad +4$

$9x = 4$

$\frac{9x}{9} = \frac{4}{9}$

$x = \frac{4}{9}$

$x + 7 = 0$

$-7 \quad -7$

$x = -7$

EXPLORE 24B

Checkpoint

Solve using the Zero-Product Property.

$20(x-5)(2x+3)=0$

$x-5=0,\ 2x+3=0$

$x=5,\ -\frac{3}{2}$

Solving Polynomial Equations Equal to Zero

- To solve quadratic equations using factoring, follow these steps:
  - Factor the expression on the non-zero side of the equation using all of the factoring rules learned throughout this unit.
  - Then, set each factored expression equal to zero.
  - Finally, solve for all possible solutions.
- Note that if you graphed these solutions, they would represent the x-intercepts.

Example 3

Solve.

$4x^2-16x=0$

Plan Factor out the GCF.
Set all factors equal to zero.
Solve for all possible solutions.

Implement

$4x(x-4)=0$

$4x=0 \qquad x-4=0$

$x=0 \qquad x=4$

$x=0,\ 4$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can you check to make sure your solutions are correct?

A: *Substitute the solutions one at a time to see if the equation equals zero on both sides.*

This problem has two solutions because the GCF does not contain a variable as it did in Example 2.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How do you know when you need to find solutions in addition to factoring?

A: *There is an equals sign or an equation instead of an expression.*

Your student can factor using any method they prefer. The worked solutions will continue to show factoring by grouping.

24B EXPLORE

Example 4

Solve.

$10x^2 - 35x - 150 = 0$

$5(2x^2 - 7x - 30) = 0$

$5(2x + 5)(x - 6) = 0$

$5 = 0$

This is not true. This is not a possible solution.

$2x + 5 = 0$

$2x = -5$

$x = -\frac{5}{2}$

$x - 6 = 0$

$x = 6$

$x = -\frac{5}{2}, 6$

Remember:

- Factor using your preferred method, but always check that your factors multiply back to the original expression.
- If there is an equals symbol, find all values that make the equation true after factoring.
- If you make a mistake, try again. Factoring always includes some trial and error.

Checkpoint

Solve.

$2x^2 + 13x + 6 = 0$

$(2x^2 + 12x) + (1x + 6)$

$2(x + 6) + 1(x + 6)$

$(x + 6)(2x + 1) = 0$

$x + 6 = 0, 2x + 1 = 0$

$x = -6, -\frac{1}{2}$

PRACTICE 1 24B

Practice 1

Complete the problems on a separate sheet of paper.

Solve using the Zero-Product Property.

1) $(x+1)(x-72)=0$ $\quad \mathbf{x = -1, 72}$

2) $(x-10)(x+10)=0$ $\quad \mathbf{x = 10, -10 = \pm 10}$

3) $x(x-18)(x+3)=0$ $\quad \mathbf{x = 0, 18, -3}$

4) $(2x-7)(5x+11)=0$ $\quad \mathbf{x = \frac{7}{2}, -\frac{11}{5}}$

5) $3x(x+1)(9x-14)=0$ $\quad \mathbf{x = 0, -1, \frac{14}{9}}$

6) $(4x-5)(6x+1)(x+2)=0$ $\quad \mathbf{x = \frac{5}{4}, -\frac{1}{6}, -2}$

Solve by factoring.

7) $x^2 - 19x - 66 = 0$ $\quad \mathbf{x = 22, -3}$

8) $x^2 - 144 = 0$ $\quad \mathbf{x = \pm 12}$

9) $x^2 + 28x + 196 = 0$ $\quad \mathbf{x = -14}$

10) $4x^2 + 17x + 15 = 0$ $\quad \mathbf{x = -\frac{5}{4}, -3}$

11) $-3x^2 + 5x + 2 = 0$ $\quad \mathbf{x = -\frac{1}{3}, 2}$

12) $5x^3 + 29x^2 - 6x = 0$ $\quad \mathbf{x = 0, -6, \frac{1}{5}}$

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) Q: If you have the same values but the signs are opposite, is your answer still correct? Explain.

A: *No, your answer is incorrect because the opposite of a number is not the same as the number itself.*

3) Q: How can you determine the number of answers an equation will likely have?

A: *You can determine the number of answers by counting how many times the variable occurs in the factored equation.*

4) Q: When the variable has a coefficient, what type of answer is usually the result?

A: *A fraction.*

Remind your student that their answers do not need to be written as mixed numbers.

6) Q: Why do problems 5 and 6 have three solutions?

A: *There are three x's to solve for.*

9) A perfect trinomial square will have the same solution repeated twice.

Q: Why does this equation only have one answer?

A: *Because the* $x + 14 = 0$ *repeats twice.*

11) Q: When the leading coefficient is negative, what should you factor out first?

A: *Negative one (–1)*

12) Q: How can you tell when the GCF will contain a variable?

A: *All of the terms before factoring include a variable.*

Mastery Check

Show What You Know

A) Have your student place the values they believe will be true and then solve for x. If they need to try again, rewrite a new problem rather than erasing so they can see their previous attempts.

C) Your student may be able to solve for the values by finding the opposite of the constant divided by the coefficient of the variable.

Q: What are the steps you used to factor this equation?

A: *Sample:*
First, factor out the GCF, then put $(x+)$ in both sets of parentheses. Find factors that make the equation true and solve for the values of x.

Mastery Check

Show What You Know

For parts A and B, use the set of numbers {1, 2, 3, 4, 5, 6, 7, 8, 9}. Each value can only be used once.

A) Determine the correct coefficients and constants that will result in the given solutions.

$$(\boxed{3}x+\boxed{7})(\boxed{9}x+\boxed{8})=0$$

$x=-\frac{7}{3}$ $\quad$ $x=-\frac{8}{9}$

$(3x+7)(9x+8)=0$

B) Determine the correct coefficients and constants that will result in the given solutions.

$$(\boxed{5}x-\boxed{2})(\boxed{1}x+\boxed{4})=0$$

$x=\frac{2}{5}$ $\quad$ $x=-4$

$(5x-2)$
$(1x+4)=0$

C) Two students solved the equation $x^3+30x^2+81x=0$. Mark said that there were three solutions to the equation, but Steven said there were two solutions. Who is correct? Prove your answer by showing your work.

$x(x^2+30x+81)=0$

$x(x+27)(x+3)=0$

$x=0, x+27=0, x+3=0$

$x=0, -27, -3$

Mark is correct. There are three solutions.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Explain what the zeros of a quadratic equation represent.
- Solve quadratic equations equal to zero by factoring.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

PRACTICE 2 24B

Practice 2

Complete the problems on a separate sheet of paper.

Solve using the Zero-Product Property.

1) $(x-9)(x+3)=0$ $\mathbf{x=9,-3}$

2) $(x-6)^2=0$ $\mathbf{x=6}$

3) $x(x+13)(x+3)=0$ $\mathbf{x=0,-13,-3}$

4) $(7x+1)(14x+9)=0$ $\mathbf{x=-\frac{1}{7},-\frac{9}{14}}$

5) $-x(15x+2)(3x-8)=0$ $\mathbf{x=0,-\frac{2}{15},\frac{8}{3}}$

6) $6x(8x-17)(5x+2)=0$ $\mathbf{x=0,\frac{17}{8},-\frac{2}{5}}$

Solve by factoring.

7) $x^2+5x-14=0$ $\mathbf{x=-7,2}$

8) $9x^2-36=0$ $\mathbf{x=\pm 2}$

9) $2x^2-x-36=0$ $\mathbf{x=\frac{9}{2},-4}$

10) $2x^3+10x^2+12x=0$ $\mathbf{x=0,-3,-2}$

11) $8x^2-22x+15=0$ $\mathbf{x=\frac{3}{2},\frac{5}{4}}$

12) $35x^3+38x^2+8x=0$ $\mathbf{x=0,-\frac{2}{7},-\frac{4}{5}}$

Practice 2

Worked solutions for these problems are located in the Digital Pack.

2) A binomial square will have the same solution repeated twice.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

10) $x + y = 4$
$x - y = -18$
The two numbers are -7 and 11

11) Distractor Rationale:
- A) The given expression is not a perfect square trinomial. The middle term would need to be $36x$.
- C) This will result in a middle term of $111x$.
- D) This will result in a middle term of $60x$.

12) Distractor Rationale:
- A) There cannot be 2 addition symbols if one of the terms is negative.
- B) There cannot be 2 subtraction symbols if the last term is negative.

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Factor.

1) $x^2 + 22x - 75$ $\mathbf{(x - 3)(x + 25)}$

2) $25x^2 + 15x + 2$ $\mathbf{(5x + 2)(5x + 1)}$

3) Factor. State whether the given expression is a difference of two squares or a perfect square trinomial.
$36z^2 - 60z + 25$
$\mathbf{(6z - 5)^2}$ **perfect square trinomial**

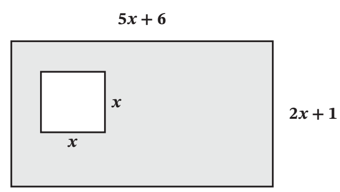
4) Factor. State whether the given expression is a difference of two squares or a perfect square trinomial.
$\frac{4}{9}r^2 - \frac{16}{25}$
$\left(\frac{2}{3}r + \frac{4}{5}\right)\left(\frac{2}{3}r - \frac{4}{5}\right)$ **difference of two squares**

Use the given figure.

5) Find the area of the white square.
$\mathbf{x^2}$ **square units**

6) Find the area of the large rectangle.
$\mathbf{10x^2 + 17x + 6}$ **square units**

7) Find the area of the shaded region only.
$A_{\text{shaded region}} = \mathbf{9x^2 + 17x + 6}$ **square units**



Simplify.

8) $(2xy^3)^3 \cdot 4x^8$ $\mathbf{32x^{11}y^9}$

9) $3x^2 \cdot 2xy \cdot 5x^3y$ $\mathbf{30x^6y^2}$

10) The sum of two integers is four. The difference between the integers is -18. Write and solve a system of equations to find the two numbers.

Multiple Choice

__**B**__ **11)** Factor
$4x^2 + 39x + 81$

- **A)** $(2x + 9)^2$
- **B)** $\mathbf{(4x + 27)(x + 3)}$
- **C)** $(4x + 3)(x + 27)$
- **D)** $(2x + 3)(2x + 27)$

__**C**__ **12)** When a trinomial is in the form of $x^2 + bx - c$, the factored answer will ______________.

- **A)** always have two addition symbols.
- **B)** always have two subtraction symbols.
- **C)** **always have one addition and one subtraction symbol.**
- **D)** cannot be determined.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	23	23	22	22	19	20	20	19	19	17	22	23