Lesson 22
Patterns in Factoring

Outline

Part A
Patterns in Factoring

- Factors and Products
- Sign Patterns in Polynomials

Part B
Factoring Special Products

- Difference of Two Squares
- Perfect Square Trinomials

Targeted Review

Vocabulary

- perfect square
- perfect square trinomial
- difference of two squares

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

3) Q: What is the result of a negative term plus a negative term?
A: *The result is a negative term.*

Q: What is the result of a positive term plus a positive term?
A: *The result is a positive term.*

4) Q: What is the result of a positive term plus a negative term or a negative plus a positive?
A: *The result depends on the sign of the larger term.*

Part A: Patterns in Factoring

Objectives

In this part of the lesson, you will learn about patterns in factoring.

By the end of this lesson, you will be able to do the following:

- Find the factors of a product that have a given sum or difference.
- Analyze the sign patterns of trinomials.

Why?

Finding sign patterns and understanding how terms work together will help you factor and check whether your factors are correct in future lessons.

Warm Up

1) What are factors?

Factors are the whole number parts of a term that are multiplied together to form the product.

2) What is a product?

A product is the result of multiplying factors.

3) When you multiply two factors with the same sign, what is the sign of the product?

The product will be a positive term.

4) When you multiply one positive factor and one negative factor, what is the sign of the product?

The product will be a negative term.

Factors and Products

- Identifying the **relationship** between factors and products is an important step toward factoring polynomials.
- To find the connections between a product and its factors, list all possible **factor pairs** for each given number.
  - Begin with **one** multiplied by the given number, then work your way up until you have found all the factors.

EXPLORE 22A

Example 1

Name the factors that make each statement true.

A) Factors of 24 that add to 14.

$24 = (1 \cdot 24) = (2 \cdot 12) = (3 \cdot 8) = (4 \cdot 6)$

$2 \cdot 12 = 24$

$2 + 12 = 14$

The factors of 24 that add to 14 are 2 and 12.

B) Factors of 24 that subtract to -5.

$3 \cdot 8 = 24$

$3 - 8 = -5$

The factors of 24 that subtract to -5 are 3 and 8.

C) Factors of 24 that add to -11.

$-8 \cdot -3 = 24$

$-8 + (-3) = -11$

The factors of 24 that add to -11 are -8 and -3.

Remember that when terms are subtracted, the order is important. The Commutative Property is not true for subtraction.

Checkpoint

The product of two factors is 48. When these factors are subtracted, the result is -13. What are the factors?

$48 = 1 \cdot 48 = 2 \cdot 24 = 3 \cdot 16 = 4 \cdot 12 = 6 \cdot 8$

$3 - 16 = -13$

Sign Patterns in Polynomials

- When you multiply binomial factors, **sign patterns** emerge.
- The signs of the **factors** determine the sign pattern in the **product**.
  - Sign patterns when multiplying two binomial factors:
    - When the signs of the factors are the **same**, the last term is always **positive**.
    - When the signs of the factors are **different**, the last term is always **negative**.
    - The sign of the middle term depends on how the factors work together.

Example 1

Your student may choose to build more problems with the digital manipulatives found in the Digital Pack.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can a negative number occur when the factors are all positive?

A: *When you subtract a larger number from a smaller number, the result is a negative value.*

Remind your student that the order of terms is especially important when subtracting since the Commutative Property is not true for subtraction.

Example 2

Your student may choose to build more problems with the digital manipulatives found in the Digital Pack.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Why would the factors 2 and 5 (the other factor pair for 10) be incorrect for this problem?

A: *The factors 2 and 5 will not add or subtract to 9 or −11.*

- The inverse of these patterns is true when factoring.
  - Sign patterns when factoring a trinomial:
    - If the last term is **positive**, you need **two** addition or **two** subtraction symbols in your factors.
    - If the last term is **negative**, you need **one** addition and **one** subtraction symbol in your factors.
      - The **middle** term determines which factor should be positive and which should be negative.

Example 2

Determine the signs needed to factor the trinomial.

$x^2 + 7x + 6$	$x^2 - 7x + 6$	$x^2 + 5x - 6$	$x^2 - 5x - 6$
All terms positive	−, + pattern	End term (−)	End term (−)
two (+)	**two (−)**	**(+) and (−)**	**(+) and (−)**
$(x + 1)(x + 6)$	$(x - 1)(x - 6)$	$(x - 1)(x + 6)$	$(x + 1)(x - 6)$

- Sign Pattern:
  - When the last term is negative but the middle term is positive, the greater factor has the **addition** symbol.
  - When the last term is negative and the middle term is negative, the greater factor has the **subtraction** symbol.

Checkpoint

Factor the given trinomials using sign patterns and the factors 10 and 1.

$x^2 + 9x - 10$	$x^2 - 11x + 10$
$\mathbf{(x + 10)(x - 1)}$	$\mathbf{(x - 10)(x - 1)}$

PRACTICE 1 22A

Practice 1

Complete problems on a separate sheet of paper.

Find the factors for each question. Show your work.

1) What two numbers multiply to 6 and add to 7?

2) What two numbers add to 18 and multiply to 45?

3) When two factors of −24 are added, the result is 2. What are the numbers?

4) When two factors of 81 are added, the result is 30. What are the numbers?

5) The sum of two factors of −72 is −1. What are the factors?

6) The product of two factors is 18. The sum of the same two factors is 11. What are the factors?

Find the product using sign patterns and mental math.

7)	$(x+6)(x+8)$	$(x-6)(x-8)$	$(x+6)(x-8)$	$(x-6)(x+8)$
	$\mathbf{x^2+14x+48}$	$\mathbf{x^2-14x+48}$	$\mathbf{x^2-2x-48}$	$\mathbf{x^2+2x-48}$
8)	$(x-4)(x+12)$	$(x-4)(x-12)$	$(x+4)(x-12)$	$(x+4)(x+12)$
	$\mathbf{x^2+8x-48}$	$\mathbf{x^2-16x+48}$	$\mathbf{x^2-8x-48}$	$\mathbf{x^2+16x+48}$
9)	$(x-3)(x-16)$	$(x-3)(x+16)$	$(x+3)(x+16)$	$(x+3)(x-16)$
	$\mathbf{x^2-19x+48}$	$\mathbf{x^2+13x-48}$	$\mathbf{x^2+19x+48}$	$\mathbf{x^2-13x-48}$

Factor.

10) Use sign patterns and the factors 24 and 2 to factor the expressions.

$x^2+26x+48$	$x^2-26x+48$	$x^2+22x-48$	$x^2-22x-48$
$\mathbf{(x+2)(x+24)}$	$\mathbf{(x-2)(x-24)}$	$\mathbf{(x+24)(x-2)}$	$\mathbf{(x-24)(x+2)}$

11) Use sign patterns and the factors 3 and 1 to factor the expressions.

x^2+2x-3	x^2+4x+3	x^2-2x-3	x^2-4x+3
$\mathbf{(x-1)(x+3)}$	$\mathbf{(x+1)(x+3)}$	$\mathbf{(x+1)(x-3)}$	$\mathbf{(x-1)(x-3)}$

12) Factor.

$x^2+20x+36$	$x^2-16x-36$	$x^2+16x-36$	$x^2-20x+36$
$\mathbf{(x+2)(x+18)}$	$\mathbf{(x-2)(x+18)}$	$\mathbf{(x-2)(x+18)}$	$\mathbf{(x-2)(x-18)}$

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) $1 \cdot 6 = 6$
$1 + 6 = 7$

2) $15 \cdot 3 = 45$
$15 + 3 = 18$

Q: Why is it important to read the questions carefully and not just look for the numbers?

A: *Because the words around the numbers tell you how to solve the problem (e.g., add or subtract).*

3) $6 \cdot -4 = -24$
$6 + (-4) = 2$

4) $27 \cdot 3 = 81$
$27 + 3 = 30$

Q: What strategy can you use when you cannot determine the factor pair right away?

A: *List all of the factor pairs starting with 1.*

5) $8 \cdot -9 = -72$
$8 + (-9) = -1$

6) $2 \cdot 9 = 18$
$2 + 9 = 11$

7–9)

Encourage your student to use mental math to find the product.

If needed, your student may choose to build these problems with the digital manipulatives found in the Digital Pack.

7) Q: How can you find the middle term using mental math?

A: *Multiply the inner and outer terms, then find the sum.*

10) Q: In problems 7 through 10, the last term is positive or negative 48. How is it possible that the last term is the same, but the factors and trinomials are different?

A: *Different factor pairs and sign patterns allow for many different trinomials.*

Mastery Check

Show What You Know

B–D)

Understanding the concepts covered in parts B–D will help your student check their work in the remaining lessons in this unit. Knowing the sign patterns of trinomials is one of the keys to factoring them.

Remind your student that they can check their work by distributing their factors to see if the products are the same.

B) Examples:

$x^2 - 7x - 8 = (x - 8)(x + 1)$

$x^2 + 7x - 8 = (x + 8)(x - 1)$

C) Example:

$x^2 - 9x + 8 = (x - 8)(x - 1)$

D) Examples:

$x^2 + 9x + 8 = (x + 1)(x + 8)$

$x^2 - 9x + 8 = (x - 8)(x - 1)$

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Find the factors of a product that have a given sum or difference.
- Analyze the sign patterns of trinomials.

Mastery Check

Show What You Know

A) Factor the given expressions.

$x^2 + 9x + 8$	$x^2 - 9x + 8$	$x^2 - 7x - 8$	$x^2 + 7x - 8$
$(x + 1)(x + 8)$	$(x - 8)(x - 1)$	$(x - 8)(x + 1)$	$(x + 8)(x - 1)$

For parts B–D, use part A to help you answer the questions.

B) Is it possible for the last term of a trinomial to be negative when the given binomials have one subtraction symbol (–) and one addition symbol (+)? Explain.

Yes, the last term in a trinomial can only be negative when there is one of each symbol because a positive term multiplied by a negative term results in a negative term.

C) When the leading coefficient is one, can the middle term of a trinomial be negative if both of its factors have subtraction symbols?

Yes, when both factors use subtraction symbols, the middle term will always be negative. (The last term will always be positive.)

D) When the last term of a trinomial is positive, what are the possible signs of the factors? Explain.

When the last term is positive, the binomial factors will have two addition or subtraction symbols.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

PRACTICE 2 22A

Practice 2

Complete problems on a separate sheet of paper.

Find the factors for each question. Show your work.

1) When factor(s) of −36 are added, the result is 0. What are the factors?
2) What two numbers multiply to 15 and add to 8?
3) When two factors of 64 are subtracted, the result is −30. What are the numbers?
4) The sum of the two factors of 132 is 23. What are the factors?
5) The product of two factors is 100. The sum of the same factors is 29. What are the factors?
6) The product of two factors is −100. The sum of the same factors is 15. What are the factors?

Find the product using sign patterns and mental math.

7)	$(x+2)(x-5)$	$(x-2)(x+5)$	$(x-2)(x-5)$	$(x+2)(x+5)$
	$\mathbf{x^2-3x-10}$	$\mathbf{x^2+3x-10}$	$\mathbf{x^2-7x+10}$	$\mathbf{x^2+7x+10}$
8)	$(x+3)(x+7)$	$(x+3)(x-7)$	$(x-3)(x-7)$	$(x-3)(x+7)$
	$\mathbf{x^2+10x+21}$	$\mathbf{x^2-4x-21}$	$\mathbf{x^2-10x+21}$	$\mathbf{x^2+4x-21}$

Factor.

9)	x^2+4x-5	x^2-6x+5	x^2-4x-5	x^2+6x+5
	$\mathbf{(x-1)(x+5)}$	$\mathbf{(x-1)(x-5)}$	$\mathbf{(x+1)(x-5)}$	$\mathbf{(x+1)(x+5)}$
10)	$x^2+2x-15$	$x^2+8x+15$	$x^2-2x-15$	$x^2-8x+15$
	$\mathbf{(x-3)(x+5)}$	$\mathbf{(x+3)(x+5)}$	$\mathbf{(x+3)(x-5)}$	$\mathbf{(x-3)(x-5)}$
11)	$x^2-14x-15$	$x^2+14x-15$	$x^2+16x+15$	$x^2-16x+15$
	$\mathbf{(x+1)(x-15)}$	$\mathbf{(x-1)(x+15)}$	$\mathbf{(x+1)(x+15)}$	$\mathbf{(x-1)(x-15)}$
12)	$x^2+21x-72$	$x^2+27x+72$	$x^2-21x-72$	$x^2-27x+72$
	$\mathbf{(x-3)(x+24)}$	$\mathbf{(x+3)(x+24)}$	$\mathbf{(x+3)(x-24)}$	$\mathbf{(x-3)(x-24)}$

Practice 2

Worked solutions for these problems are located in the Digital Pack.

1) $6 \cdot -6 = -36$
$6 + (-6) = 0$

2) $3 \cdot 5 = 15$
$3 + 5 = 8$

3) $2 \cdot 32 = 64$
$2 - 32 = -30$

4) $11 \cdot 12 = 132$
$11 + 12 = 23$

5) $4 \cdot 25 = 100$
$4 + 25 = 29$

6) $20 \cdot (-5) = -100$
$20 + (-5) = 15$

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

1–2)

Q: How are problems 1 and 2 different?

A: *The signs in problem 1 are both subtraction symbols, but problem 2 contains one addition and one subtraction symbol.*

Q: What happens to the middle term in problem 2?

A: *It simplifies out of the expression.*

4) Knowing the square of a fraction will be helpful for some of the practice problems.

Your student may choose to build problems 1–3 with the digital manipulatives found in the Digital Pack.

Part B: Factoring Special Products

Objectives

In this part of the lesson, you will learn about factoring special products.

By the end of this lesson, you will be able to do the following:

- Factor special products including the difference of two squares (DOTS) and trinomial squares (TriSq).

Why?

Some polynomials show up so much in algebra that they have been given their own names. Learning about these special products will deepen your understanding of the patterns contained within polynomial expressions and equations.

Warm Up

Find the product. (Your answers should be a quadratic binomial or trinomial.)

1) $(x-9)(x-9)$

$x^2 - 9x - 9x + 81$

$x^2 - 18x + 81$

2) $(x-9)(x+9)$

$x^2 + 9x - 9x - 81$

$x^2 - 81$

3) **A perfect square is the product of a number times itself. List the perfect squares from 1 through 12.**

1^2	**1**	4^2	**16**	7^2	**49**	10^2	**100**
2^2	**4**	5^2	**25**	8^2	**64**	11^2	**121**
3^2	**9**	6^2	**36**	9^2	**81**	12^2	**144**

4) **Find the square of the given fractions.**

$\left(\frac{1}{2}\right)^2$ $\frac{1}{4}$ $\left(\frac{1}{3}\right)^2$ $\frac{1}{9}$ $\left(\frac{2}{3}\right)^2$ $\frac{4}{9}$ $\left(\frac{1}{10}\right)^2$ $\frac{1}{100}$

EXPLORE 22B

Difference of Two Squares

- A ___perfect square___ is the product of a term multiplied by itself.
- The ___difference of two squares___ is a special type of quadratic binomial where both terms are perfect squares that are subtracted from one another.
- A difference of two squares expression has the following features:
  - It has only ___two___ terms.
  - The last term is ___negative___.
  - There is no ___middle___ term.
  - The sign pattern in the binomial factors of these expressions must be ___+/−___ or ___−/+___.
- Remember to factor out the ___GCF___ first whenever possible.

Example 1

Factor.

$81x^2 - 16$

Implement	Explain
$(? + ?)(? - ?)$	The binomial factors for the expression have a (+)(−) sign pattern since the last term is negative.
$(9x + 4)(9x - 4)$	The factors are the square root of each of the terms.

Check

$(9x + 4)(9x - 4)$	The inner and outer terms are additive inverses. They simplify out of the expression, so the factored answer is correct.

Example 2

Factor.

$x^2 - 64$

Plan	Implement
Draw parentheses and write in the signs. Think, "What times what is x^2?" Think, "What times itself is 64?"	$(x - 8)(x + 8)$

Example 2

Your student may choose to build more problems with the digital manipulatives found in the Digital Pack.

22B EXPLORE

Example 3

Factor.

$\frac{3}{100}x^3 - 3x$

Plan

Factor out the GCF to make perfect squares.
Draw parentheses and write in the signs.
Find the factors.

Implement

$3x\left(\frac{1}{100}x^2 - 1\right)$

$3x\left(\frac{1}{10}x - 1\right)\left(\frac{1}{10}x + 1\right)$

Check

$3x\left(\frac{1}{10}x - 1\right)\left(\frac{1}{10}x + 1\right)$

$3x\left(\frac{1}{10}x - 1\right)\left(\frac{1}{10}x + 1\right)$

$3x\left(\frac{1}{100}x^2 - 1\right)$

$\frac{3}{100}x^3 - 3x$ ✓

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the square root of $4x^2$? What is the square root of $9y^2$?

A: *The square root of $4x^2$ is $2x$, and the square root of $9y^2$ is $3y$.*

Since both variables are raised to the second power, the terms are all perfect squares.

Have your student check their work by distributing the terms back together. Doing this will help them determine if their answers are correct, even using mental math.

Checkpoint

Factor

$4x^2 - 9y^2$

$(2x - 3y)(2x + 3y)$

Perfect Square Trinomials

- Perfect square trinomials:
  - The first term is a perfect square.
  - The last term is a **perfect square**.
  - The middle term is **double** the product of the terms within the binomial factors.
- The factored answer is often written as a binomial **squared**.
  - The **exponent** outside of the parentheses tells you how many times to write the terms inside the parentheses.

EXPLORE 22B

Example 4

Factor.

$x^2 + 2ax + a^2$

Implement	Explain
$(x + a)(x + a)$	◂ The first and last terms are perfect squares (raised to the second power/ coefficients of 1).
$(x + a)^2$	◂ Trinomial has + symbols, so binomial factors have + symbols. Identical pairs of binomials can be written with an exponent.

Check

$(x + a)(x + a)$

Example 5

Factor.

$25x^2 - 60x + 36$

Plan

Identify the sign pattern.
Identify the square root of the first and last terms.
Find the factors.
Verify the perfect square.

Check

$(5x - 6)(5x - 6)$

Implement	Explain
$(\quad - \quad)(\quad - \quad)$	◂ A −, + sign pattern means both binomial factors will have − symbols.
$(5x - 6)(5x - 6)$	◂ The factors' terms are square roots of the product's first and last terms.
$(5x - 6)^2$	◂ Identical pairs of binomials can be written with an exponent.

☑ Checkpoint

Factor.

A) $x^2 + 6x + 9$

$(x + 3)(x + 3)$ or $(x + 3)^2$

B) $100x^2 - 60x + 9$

$(10x - 3)(10x - 3)$ or $(10x - 3)^2$

Example 5

Your student may choose to build more problems with the digital manipulatives found in the Digital Pack.

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Your student can write the expression as two binomials or one binomial squared. However, they should understand that these are the same mathematically.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

If needed, your student may choose to build some of these problems with the digital manipulatives found in the Digital Pack.

2) Q: How does knowing the sign patterns help you determine the binomial factors?

A: *When the last term is negative, the binomial factors will have one addition and one subtraction symbol.*

6) Q: What number remains when the entire term is factored out?

A: *1*

Q: Why is the problem not the difference of two squares until after the GCF is factored out?

A: *Because the terms are not perfect squares.*

7–12)

Answers will be written as a binomial squared for the remainder of the lesson. If your student writes out the solution as two binomials, this is also correct.

7) Q: How does the middle term relate to the factors in a perfect square trinomial?

A: *The middle term is twice the product of the terms within the factors.*

10) Q: Are all even numbers perfect squares?

A: *No, many even numbers are not perfect squares (e.g., 2, 8, 12, etc.).*

12) Q: Why is it important to first factor out the greatest common factor?

A: *The terms are not perfect squares until the GCF is factored out.*

Practice 1

Complete problems on a separate sheet of paper.

Factor the difference of two squares.

1) $x^2 - 1$ $(x - 1)(x + 1)$

2) $9x^2 - 4$ $(3x - 2)(3x + 2)$

3) $\frac{1}{4}x^2 - 25$ $\left(\frac{1}{2}x + 5\right)\left(\frac{1}{2}x - 5\right)$

4) $16x^2 - 121y^2$ $(4x - 11y)(4x + 11y)$

5) $x^2 - 81$ $(x + 9)(x - 9)$

6) $10x^2 - 10$
Hint: Factor out the GCF first.
$10(x + 1)(x - 1)$

Factor the perfect square trinomials.

7) $x^2 + 2xy + y^2$ $(x + y)^2$

8) $x^2 - 8x + 16$ $(x - 4)^2$

9) $49x^2 - 28x + 4$ $(7x - 2)^2$

10) $x^2 + 12x + 36$ $(x + 6)^2$

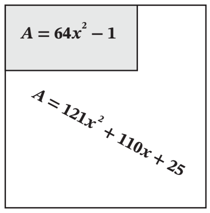
11) $16x^2 + 24x + 9$ $(4x + 3)^2$

12) $3x^2 - 12x + 12$
Hint: Find the GCF first.
$3(x - 2)^2$

Mastery Check

Show What You Know

A) A rectangular garden within a square property is shown in the given figure. The area of the garden is given in feet. What are the side lengths of the rectangular garden?



$64x^2 - 1$

$(8x - 1)(8x + 1)$

The sides of the garden are the $(8x - 1)$ feet and $(8x + 1)$ feet.

B) What are the sides of the square property with a total area of $121x^2 + 110x + 25$ square feet?

$121x^2 + 110x + 25$

$(11x + 5)(11x + 5)$

Each side of the square property is $(11x + 5)$ feet long.

C) How do you determine if an expression is the difference of two squares?

An expression is the difference of two squares when both terms are perfect squares and subtracted from one another.

D) Can a perfect square trinomial ever have the sign pattern (+), (−)? Explain.

No, because the factored expressions have to be the same. When the last term is negative, there will be one of each symbol.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Q: Why is it not possible for the difference of two squares to represent a square?

A: *Squares have four equal sides. However, the factors that create the difference of two squares are not the same, so they cannot represent equal side lengths.*

B) Q: How can you determine if your factored answer is correct?

A: *Multiply the binomials back together using the Distributive Property.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Factor special products including the difference of two squares (DOTS) and trinomial squares (TriSq).

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Practice 2

Complete problems on a separate sheet of paper.

Factor.

1) $36x^2 - 25$ $(6x+5)(6x-5)$

2) $100x^2 - y^2$ $(10x-y)(10x+y)$

3) $\frac{1}{9}x^2 - \frac{1}{4}$ $\left(\frac{1}{3}x-\frac{1}{2}\right)\left(\frac{1}{3}x+\frac{1}{2}\right)$

4) $25x^2 - 64$ $(5x-8)(5x+8)$

Factor. (Hint: Find the GCF first.)

5) $\frac{2}{9}x^2 - 50$ $2\left(\frac{1}{3}x-5\right)\left(\frac{1}{3}x+5\right)$

6) $7x^3 - 28x$ $7x(x-2)(x+2)$

Factor the perfect square trinomials.

7) $x^2 - 12xy + 36y^2$ $(x-6y)^2$

8) $64x^2 - 16x + 1$ $(8x-1)^2$

9) $x^2 + 14x + 49$ $(x+7)^2$

10) $\frac{1}{4}x^2 + x + 1$ $\left(\frac{1}{2}x+1\right)^2$

Factor. (Hint: Find the GCF first.)

11) $12x^2 - 24x + 12$ $12(x-1)^2$

12) $\frac{4}{3}x^2 - \frac{28}{3}x + \frac{49}{3}$ $\frac{1}{3}(2x-7)^2$

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete problems on a separate sheet of paper.

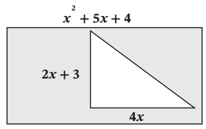
Find the product.

1) $(8x + 7)(5x + 3)$ $\mathbf{40x^2 + 59x + 21}$

2) $12x^3(4x + y)$ $\mathbf{48x^4 + 12x^3y}$

3) What is the relationship between factoring and the Distributive Property?

Use the figure to answer.



4) Find the area of the rectangle from the given figure.

$\mathbf{3x^3 + 17x^2 + 22x + 8}$ **square units**

5) Find the area of the triangle from the given figure.

$\mathbf{4x^2 + 6x}$ **square units**

6) Find the area of the shaded region only. (Hint: rectangle's area − triangle's area)

$\mathbf{3x^3 + 13x^2 + 16x + 8}$ **square units**

7) Factor by grouping.

$10x^2y - 15xy - 2x + 3$ $\mathbf{(5xy - 1)(2x - 3)}$

Simplify.

8) $(7x^2y^5)^3 \cdot x^{-1}y^3$ $\mathbf{7^3x^5y^{18}}$

9) $2x^5y^4 \cdot 6xy^8 \cdot 2xy^{-2}$ $\mathbf{24x^7y^{10}}$

10) Graph the equation. Name the x- and y-intercepts as ordered pairs.

$4x + 3y = -6$

x-intercept: $\left(-\frac{3}{2}, 0\right)$ y-intercept: $\mathbf{(0, -2)}$

11) Write and solve a system of equations.

A rectangle's length is three feet longer than its width. The perimeter of the rectangle is four and a half times the width. What are the dimensions of the rectangle? What is the perimeter of the rectangle?

Multiple Choice

B **12)** Factor.

$2x^3 + 2x^2 + x$

A) $x(2x^2 + 2x)$

B) $\mathbf{x(2x^2 + 2x + 1)}$

C) $2x(x^2 + x + 1)$

D) x

A **13)** Factor.

$10xy - 10y + 27x - 27$

A) $\mathbf{(x - 1)(10y + 27)}$

B) $10y(x - 1) + 27(x - 1)$

C) $(x + 1)(10y - 27)$

D) not factorable

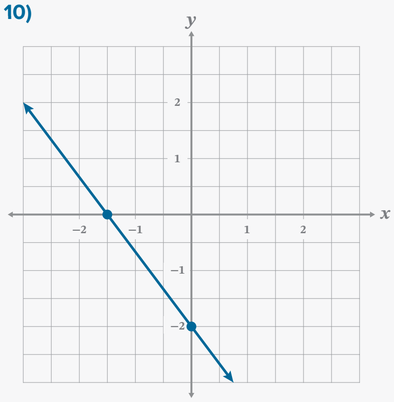
Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

3) Factoring and the Distributive Property are inverses of one another.

10)



11) The rectangle is 15 units by 12 units, and the perimeter is 54 units.

12) Distractor Rationale:

A) This answer removes the final term instead of factoring out x and leaving 1.

C) This has 2 as part of the GCF, but it is not common among all terms.

D) This lists the correct GCF but does not include the complete answer.

13) Distractor Rationale:

B) This answer is the middle step but not the final answer.

C) This answer has the addition and subtraction symbols reversed.

D) The problem is factorable.

Problem	1–2	3	4–6	7	8–9	10	11	12	13
Lesson Origin	20	21	20	21	19	11	18	21	21