

# Lesson 20

## Polynomial Expressions

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### Outline

#### Part A Simplifying and Classifying Polynomials

- Classifying Polynomials in Standard Form
- The Sum or Difference of Polynomial Expressions
- Solving for an Unknown Coefficient

#### Part B Products of Polynomials

- Multiplying Monomials
- Multiplying Binomials 1
- Multiplying Binomials 2
- Multiplying Binomials and Trinomials
- Multiplying a Trinomials
- Solving for an Unknown Coefficient When Multiplying

### Targeted Review

### Vocabulary

- constant
- linear
- quadratic
- cubic
- degree  $n$
- polynomial
- degree
- monomial
- binomial
- trinomial
- leading coefficient



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

## Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Classification is an important part of categorizing numbers and polynomials. This Warm Up is meant to help your student recall the classifications of numbers from Lesson 1 to prepare them for classifying polynomials in this lesson.

## Part A: Simplifying and Classifying Polynomials

### Objectives

In this part of the lesson, you will learn about polynomial expressions.

By the end of this lesson, you will be able to the following:

- ☑ Classify polynomials by degree and number of terms.
- ☑ Write polynomials in standard form.
- ☑ Calculate the sum or difference of polynomial expressions.

### Why?

Being able to classify polynomials gives you a common language to describe algebraic terms that others will understand. Knowing these classifications is also necessary when performing operations with polynomials.

### Warm Up

Describe each number using the following classifications: natural, whole, integers, rational, irrational, real.  $\{N, W, Z, Q, I, R\}$

- 1)  $\frac{2}{3}$  rational, real,  $\{Q, R\}$
- 2)  $-8$  integer, rational, real,  $\{Z, Q, R\}$
- 3)  $0.\overline{123}$  rational, real,  $\{Q, R\}$
- 4)  $0$  whole, integer, rational, real,  $\{W, Z, R, Q\}$

### ▶ Classifying Polynomials in Standard Form

- A polynomial is an expression containing one or more terms.
- Polynomials in standard form are written in descending order by degree.
- Polynomials are classified first by their degree and then by the number of terms in the expression.
- The degree of a polynomial indicates how the polynomial will appear when graphed.
- When classifying polynomials by the number of terms, prefixes are used to describe how many terms are in the expression.

Degree	Classification	Number of Terms	Classification
$x^0$	constant	$x$	<u>mono</u> mial
$x^1$	linear	$x + 3$	<u>bi</u> nomial
$x^2$	quadratic	$x^2 + x + 3$	<u>tri</u> nomial
$x^3$	cubic	$x^3 + x^2 + x + 3$	<u>poly</u> nomial with $n$ -terms
$x^n; n \geq 4$	degree $n$		

- Before you classify a polynomial, combine all like terms and write the expression in standard form.

**Example 1**

**Write the expressions in standard form. Classify by degree and number of terms.**

**Plan** Find the terms with the largest exponent.  
Write the remaining terms in descending order based on the exponent.  
Classify by degree and terms.

**Implement**

- A)  $8x - 2x^3 + 4x^2 + 23$  Standard form:  $-2x^3 + 4x^2 + 8x + 23$   
Classification: cubic polynomial with 4 terms
- B)  $-10 - 7x + x^2$  Standard form:  $x^2 - 7x - 10$   
Classification: **quadratic trinomial**
- A)  $-2$  Classification: **constant monomial**

- The leading coefficient is the number in front of the term with the highest degree. This is the first number when a polynomial is written in standard form.
- In Example 1A, the leading coefficient is -2, the degree is 3, and the constant is 23.

 **Checkpoint**

**Write in standard form. Classify by degree and number of terms.**

$$7x - 2x^2 + 8$$

$$-2x^2 + 7x + 8$$

**quadratic trinomial**

The exponent rule  $x^0 = 1$  is on the Formula Sheet and is also covered in Pre-Algebra.

If your student is curious, a 4th degree polynomial is formally named a quartic polynomial, and a 5th degree polynomial is formally named a quintic polynomial. However, this is not required to know for this course.

 **Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What should you look for first when writing an expression in standard form?

A: *The term with the largest degree (exponent).*

Q: What is the order that polynomials are classified by?

A: *Degree, then number of terms.*

Q: Which expression from Example 1 has the same classification as the Checkpoint?

A: *Expression B.*

**The Sum or Difference of Polynomial Expressions**

- Like terms have the same variables and degree.
- An expression written in simplest form has all like terms combined.
- Polynomial expressions are most commonly simplified horizontally; however, adding expressions vertically allows you to better see the place value of the terms by aligning the terms by degree.

**Example 2**

**Simplify.**

$$(3x^2 + 2x + 1) + (x^3 - x^2 - 8x - 9)$$

**Plan** Combine like terms.  
Write the simplified expression in standard form.

**Implement**

Vertical

$$\begin{array}{r} 3x^2 + 2x + 1 \\ + x^3 - x^2 - 8x - 9 \\ \hline x^3 + 2x^2 - 6x - 8 \end{array}$$

Horizontal

$$(3x^2 + 2x + 1) + (x^3 - x^2 - 8x - 9)$$

$$x^3 + 2x^2 - 6x - 8$$

**Example 3**

**Simplify.**

$$(14x^2 - 22x + 1) - (x^2 + 7x - 11)$$

$$(14x^2 - 22x + 1) + (-x^2 - 7x + 11)$$

$$14x^2 - 22x + 1 - x^2 - 7x + 11$$

$$13x^2 - 29x + 12$$

Remember that only the coefficient changes when adding like terms. The variable and its exponent will remain the same for each term of the expression when adding or subtracting.

**Example 3**

Your student can use the digital manipulatives to build expressions. This can help them see like terms visually when simplifying polynomial expressions.

**Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

**Q:** What is the leading coefficient and degree of your simplified expression?

**A:** The leading coefficient is 1. The degree is 2.

**Checkpoint**

**Simplify.**

$$(3x - 11) + (x^2 - 15x + 30)$$

$$x^2 - 12x + 19$$

### Ⓟ Solving for an Unknown Coefficient

- Problems with an unknown coefficient give the final polynomial, but \_\_\_\_\_ **one of the coefficients** \_\_\_\_\_ is missing from the original expression.
- The key to solving for an unknown coefficient is to look for the \_\_\_\_\_ **degree** \_\_\_\_\_ of the unknown term and work with the terms of the same degree.
- \_\_\_\_\_ **Checking your work** \_\_\_\_\_ tells you right away whether the value you found is the correct coefficient.

#### Example 4

##### Find the value of $R$ .

$$(-3x^2 - x + 5) - (Rx^2 + 27) = -4x^2 - x - 22$$

**Plan** Find the degree of  $R$ . Rewrite the equation using only the terms with the same degree as the unknown term. Solve for  $R$ .

Remember to check that your answer makes sense. Checking your answer is especially important when the value will be substituted into a problem with subtraction signs.

##### Implement

$$(-3x^2 - x + 5) - (Rx^2 + 27) = -4x^2 - x - 22$$

$$-3x^2 - Rx^2 = -4x^2$$

$$-3 - R = -4$$

$$+3 \quad +3$$

$$-R = -1$$

$$R = 1$$

##### Explain

- ◀ Identify the degree of  $R$
- ◀ Rewrite equation with terms of the same degree
- ◀ Simplify out  $x^2$
- ◀ Addition Property of Equality
- ◀ Symmetry Property

##### Check

$$-3x^2 - 1x^2 = -4x^2 \checkmark$$

#### Checkpoint

##### Find the value of $R$ .

$$(9x^2 - 4x + 1) + (3x^2 + Rx - 5) = 12x^2 + 7x - 4$$

$$-4x + Rx = 7x$$

$$-4 + R = 7$$

$$R = 11$$

#### Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What terms should be highlighted or written down to find  $R$ ?

A: *Terms to the first power (linear terms).*

Q: How do you check that the value of  $R$  is correct?

A: *Replace  $R$  with the number you solved for, then check that the left and right sides of the equation are equal.*



## Practice 1



Worked solutions for these problems are located in the Digital Pack.

1–4)

Q: How can you determine if your expression is in standard form?

A: *The terms are written in descending order by degree.*

5) The degree of a polynomial is determined by the largest exponent in the expression. The leading coefficient is the number in front of the term with the largest degree.

Q: What is the difference between a constant and a coefficient?

A: *A constant is a numerical term in a polynomial expression. A coefficient is a number in front of a variable in an expression.*

6) Q: Why are  $x$  and  $x^2$  not like terms?

A: *They do not have the same degree.*

Your student can use the digital manipulatives to build problems 7 and 9.

7) Q: What should the first step in this problem be?

A: *Rewrite the terms in the second set of parentheses by changing their signs to the opposite.*

9) If the coefficient of a term is zero, it is not necessary to write that term as part of the final answer.

10) Q: How are finding the perimeter and simplifying polynomial expressions related?

A: *Finding the perimeter and simplifying expressions are both completed by combining like terms.*

11) Q: What is the degree of the term you need to solve for?

A: 2



## Practice 1

Complete the problems on a separate sheet of paper.

Write the polynomial expression in standard form. Classify by degree and number of terms. Identify the leading coefficient.

- |  |   |
|--|---|
| 1) $-8 + 2x$<br>Standard form: $2x - 8$<br>Classification: <b>linear binomial</b><br>Leading coefficient: 2                          | 2) $2x - 3x^2 - x^3 + 1$<br>Standard form: $-x^3 - 3x^2 + 2x + 1$<br>Classification: <b>cubic polynomial with 4 terms</b><br>Leading coefficient: <b>-1</b> |
| 3) $11x^2 + x + 6$<br>Standard form: $11x^2 + x + 6$<br>Classification: <b>quadratic trinomial</b><br>Leading coefficient: <b>11</b> | 4) $x$<br>Standard form: $x$<br>Classification: <b>linear monomial</b><br>Leading coefficient: <b>1</b>   |

- 5) What determines the degree of a polynomial? What is the leading coefficient?

Simplify. Write in standard form.

- 6)  $(-x + 17 + 23x^3) + (-3x^2 - 1 - x)$   **$20x^3 - 2x + 16$**
- 7)  $x^2 + 3x - 8 - (3x^2 + 6x - 2)$   **$-2x^2 - 3x - 6$**
- 8)  $(x^2 - 16) - (19x^2 + 3x - 2)$   **$-18x^2 - 3x - 14$**
- 9)  $(x^2 - 5x + 8) - (x^2 - 5x + 15)$   **$-7$**

- 10) Find the perimeter of a triangle with the following side lengths:  $x^3 - 8x + 9$ ,  $4x^3 + 10x$ , and  $10x + 8$   **$5x^3 + 12x + 17$  units**

Find the value of  $R$ .

- 11)  $(Rx^2 - 7x - 8) - (6x^2 + 2) = 3x^2 - 7x - 10$   **$R = 9$**
- 12)  $(12x^2 + 11x + 10) + (Rx^2 - 10) = 2x^2 + 11x$   **$R = -10$**

**Mastery Check**

**Show What You Know**

The total perimeter of the Newton family's rectangular yard is  $2x^2 + 10x + 8$  feet. They want to use this measurement to purchase a fence for the yard, but the measurement includes the side of the house where no fence is needed.

- A) If the side of the house is  $(5x + 1)$  feet long, what is the perimeter of the yard excluding this side? Show your work.

$$(2x^2 + 10x + 8) - (5x + 1)$$

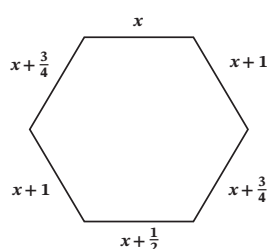
$$2x^2 + 10x + 8 - 5x - 1$$

$$2x^2 + 5x + 7 \text{ ft}$$

- B) Explain your reasoning to support your work in Part A.

**The side of the house,  $5x + 1$ , is subtracted from the total perimeter since it is not needed when fencing in the yard.**

- C) The Newton family also wants to put garden fencing around a garden in their yard. The figure below shows the garden with measurements in feet. Find the perimeter of the garden and classify your expression.



$$P = (x) + (x + 1) + (x + \frac{3}{4}) + (x + \frac{1}{2}) + (x + 1) + (x + \frac{3}{4})$$

$$P = 6x + 2 + \frac{6}{4} + \frac{2}{4}$$

$$P = 6x + 4 \text{ ft}$$

**The perimeter is a linear binomial.**

**Say What You Know**

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

**Mastery Check**

**Show What You Know**

Q: What should you do if you do not want one side included when finding the perimeter of a shape?

A: *Subtract it from the given perimeter.*

**Say What You Know**

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- ☑ Classify polynomials by degree and number of terms.
- ☑ Write polynomials in standard form.
- ☑ Calculate the sum or difference of polynomial expressions.


 **Practice 2**
 **Worked solutions for these problems are located in the Digital Pack.**

- 4) Q How is a term to the zero power usually written?

A: As 1, or not at all.

If needed, your student may choose to build problems 6–10 with the digital manipulatives.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

 **Practice 2**

Complete the problems on a separate sheet of paper.

Write the polynomial expression in standard form. Classify by degree and number of terms. Identify the leading coefficient.

- |  |   |
|--|---|
| <p>1) <math>-3x + 5 + x^2</math><br/>Standard form: <math>x^2 - 3x + 5</math><br/>Classification: <b>quadratic trinomial</b><br/>Leading coefficient: 1</p>                      | <p>2) <math>3x^2 - 16</math><br/>Standard form: <math>3x^2 - 16</math><br/>Classification: <b>quadratic binomial</b><br/>Leading coefficient: 3</p> |
| <p>3) <math>-7x^2 + 37 - 16x</math><br/>Standard form: <math>-7x^2 - 16x + 37</math><br/>Classification: <b>quadratic trinomial</b><br/>Leading coefficient: <math>-7</math></p> |   |

Name the degree, leading coefficient, and constant.

- |  |   |
|--|---|
| <p>4) <math>5x^0</math><br/>Degree: <b>degree 0</b><br/>Leading coefficient: 5<br/>Constant: 5</p> | <p>5) <math>6x + 7x^2 - 22</math><br/>Degree: <b>degree 2</b><br/>Leading coefficient: 7<br/>Constant: <math>-22</math></p> |
|--|---|

Simplify. Write in standard form.

- 6)  $(5x^2 - 2x) + (8x^2 - 11)$   **$13x^2 - 2x - 11$**
- 7)  $(x^2 + 13x + 20) - (x^2 + 22x - 3)$   **$-9x + 23$**
- 8)  $(x^2 - 4x - 7) - (3x^2 - 2x - 1)$   **$-2x^2 - 2x - 6$**
- 9)  $(5x^2 - 11x + 8) + (x^2 + 3x + 2)$   **$6x^2 - 8x + 10$**
- 10)  $(x^3 - x^2 - x - 1) + (3x + 8)$   **$x^3 - x^2 + 2x + 7$**

- 11) Find the perimeter of a triangle with the following side lengths:  $3x^2$ ,  $x^2 + 5x + 19$ ,  $7x - 1$   
 **$4x^2 + 12x + 18$  units**

Find the value of  $R$ .

- 12)  $(Rx + 2) + (x^2 - 9) = x^2 + 5x - 7$   **$R = 5$**
- 13)  $(x^2 + 3x - 3) - (x^2 + Rx - 3) = -5x$   **$R = 8$**

## Part B: Products of Polynomials

## Objectives

In this part of the lesson, you will learn about products of polynomials.

By the end of this lesson, you will be able to do the following:

- ☑ Multiply a monomial by a polynomial using the Distributive Property and exponent rules.
- ☑ Find the product of a binomial and a binomial using the Distributive Property.
- ☑ Find the product of a binomial and a trinomial, as well as a trinomial and a trinomial.

## Why?

What else can you use the Distributive Property for? In this lesson, you will learn how to distribute in various ways and use the Distributive Property with exponent rules to multiply monomials, binomials, and polynomials.



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

## Warm Up

Simplify.

1)  $8(x - 7)$   $8x - 56$

2)  $(5x)(3x)$   $15x^2$

3)  $x \cdot x$   $x^2$

4)  $x + x$   $2x$

## Multiplying Monomials

- To find the **product** of a monomial and a polynomial expression, you must use the exponent rules with the Distributive Property to multiply every term in the expression by the **monomial** term.

## Example 1

Find the area of the rectangle.

$$l = 2x^2 \text{ and } w = x^2 + 4x + 5$$

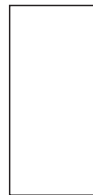
$$A = lw$$

$$A = 2x^2(x^2 + 4x + 5)$$

$$A = 2x^4 + 8x^3 + 10x^2$$

$$2x^4 + 8x^3 + 10x^2 \text{ square units}$$

$$x^2 + 4x + 5 \text{ units}$$



$$2x^2 \text{ units}$$

Remember that the unit label will be in square units for all area problems.

## Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

- 1) Q: What property is being used to simplify this question?

A: *The Distributive Property.*

Q: How do you simplify the coefficients? How do you simplify the exponents?

A: *Multiply the coefficients and add the exponents.*

## Example 1

Your student may choose to build more problems with the digital manipulatives found in the Digital Pack.

**Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

**Q:** When you distribute a monomial, how are coefficients and exponents treated differently?

**A:** Coefficients are multiplied together, and exponents are added when their bases are the same.

**Example 2**

Your student may choose to build more problems with the digital manipulatives found in the Digital Pack.

**Checkpoint**

Find the area of the rectangle. Remember to write your formula and define your variables.

$l = 3x^2$

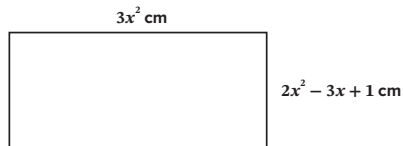
$w = 2x^2 - 3x + 1$

$A = lw$

$A = (3x^2)(2x^2 - 3x + 1)$

$A = 6x^4 - 9x^3 + 3x^2$

$6x^4 - 9x^3 + 3x^2 \text{ cm}^2$



**▶ Multiplying Binomials 1**

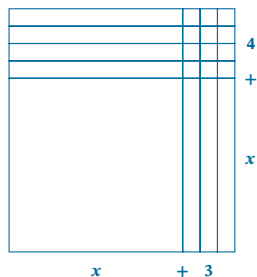
- Multiplying two binomials is an extension of the **Distributive** Property.
- If you start with two linear binomials and multiply them together, the result is usually a **quadratic trinomial**.
- When multiplying binomials (and polynomials), remember to write a **(+) or (-)** between the terms depending on the sign of the product.
- The place value of each term corresponds with its **degree** rather than its number of **units**.

**Example 2**

**Multiply binomials using blocks.**

$(x + 3)(x + 4)$

**Plan** Use blocks to build a rectangle that is  $x + 3$  by  $x + 4$ . Multiply the terms.



**Implement**

Label the vertical side

Label the horizontal side

Multiply the terms:

$x^2 + 3x + 4x + 12$

$x^2 + 7x + 12$

**Explain**

- ◀ Multiply  $x$  by  $x$  (square):  $x^2$
- ◀ Multiply  $x$  by 3 (rectangles):  $3x$
- ◀ Multiply  $x$  by 4 (rectangles):  $4x$
- ◀ Multiply 3 by 4 (units):  $12$
- ◀ Write an expression using all the terms.
- ◀ Simplify

**Example 3****Multiply using vertical multiplication.**

$(x + 3)(x + 4)$

**Implement****Explain**

$x + 3$	
$\times x + 4$	◀ Write the binomials vertically
$4x + 12$	◀ Multiply 4 by 3 and multiply 4 by $x$ : $4x$
$+ x^2 + 3x$	◀ Multiply $x$ by 3 and multiply $x$ by $x$ : $x^2$
$x^2 + 7x + 12$	◀ Combine like terms

**Example 4****Multiply using vertical multiplication.**

$(2x + 7)(x + 5)$

**Implement****Explain**

$2x + 7$	
$\times x + 5$	◀ Write the binomials vertically
$10x + 35$	◀ Multiply 5 by 7 and multiply 5 by $2x$
$+ 2x^2 + 7x$	◀ Multiply $x$ by 7 and multiply $x$ by $2x$
$2x^2 + 17x + 35$	◀ Combine like terms

 **Checkpoint****Multiply using vertical multiplication.**

$(3x + 8)(x + 1)$	
	$3x + 8$
	$\times x + 1$
	$3x + 8$
	$+ 3x^2 + 8x$
	$3x^2 + 11x + 8$

 **Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

Your student may build this using the Digital Manipulatives inside the Digital Toolbox if they would like to see an area model representation.

Q: How should you line up the terms?

A: *By degree*

### ▶ Multiplying Binomials 2

- The split Distributive Property method is a way to multiply binomials by splitting the first binomial into two parts.
- When you distribute, every term from the first expression will be multiplied by every term in the second expression.
- Since the Commutative Property is true for multiplication, either expression can be first.

#### Example 5

Multiply using the split Distributive Property method.

$$(x + 3)(x + 4)$$

**Implement**

$$\bar{x}(x + 4) + 3(x + 4)$$

$$\bar{x}(x + 4) + 3(x + 4)$$

$$x^2 + 4x + 3x + 12$$

$$x^2 + 7x + 12$$

**Explain**

- ◀ Split the first binomial into two terms and write each term next to the second binomial
- ◀ Distribute the terms across the binomial expression
- ◀ Combine like terms
- ◀ This is a quadratic trinomial.

#### Example 6

Find the product.

**A)**  $(2x - 3)(5x + 6)$

$$10x^2 + 12x - 15x - 18$$

$$10x^2 - 3x - 18$$

**B)**  $(7x - 1)(x - 9)$

$$7x^2 - 63x - 1x + 9$$

$$7x^2 - 64x + 9$$

### ☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How many terms will you have after multiplying all terms but before simplifying the expression completely?

A: 4

### ☑ Checkpoint

Find the product

$$(x - 3)(3x + 7)$$

$$3x^2 + 7x - 9x - 21$$

$$3x^2 - 2x - 21$$

### Ⓢ Multiplying Binomials and Trinomials

- binomial · trinomial = a polynomial
- (   2   terms) · (   3   terms) =   6   terms (before simplifying)
- The total number of terms will continue to   increase   as   terms   are added to each polynomial.

#### Example 7

Find the product horizontally.

$$(4x - 5)(x^2 + 2x + 3)$$

Implement

$$(4x - 5)(x^2 + 2x + 3)$$

$$(4x - 5)(x^2 + 2x + 3)$$

$$4x^3 + 8x^2 + 12x - 5x^2 - 10x - 15$$

$$4x^3 + 3x^2 + 2x - 15$$

Explain

◀ Multiply each term in the trinomial by  $4x$

◀ Multiply each term in the trinomial by  $-5$

◀ Combine like terms

◀ This is a cubic polynomial with four terms.

#### Example 8

Find the product using the horizontal hybrid method.

Horizontal multiplication with vertical addition:

Implement

$$(4x - 5)(x^2 + 2x + 3)$$

$$4x^3 + 8x^2 + 12x$$

$$+ \quad -5x^2 - 10x - 15$$

$$4x^3 + 3x^2 + 2x - 15$$

Explain

◀ Distribute by  $4x$  and write the first set of terms

◀ Distribute by  $-5$  and write like terms under one another

◀ Add vertically

#### Example 9

Find the product.

$$(2x + 3)(x^2 + 11x - 4)$$

Implement

$$-2x^3 + 22x^2 - 8x - 3x^2 + 33x - 12$$

$$-2x^3 + 19x^2 + 25x - 12$$

Explain

◀ Distribute  $2x$  and  $3$

◀ Combine like terms

The answer is a cubic polynomial with 4 terms.

**Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What are the two terms being distributed across the trinomial?

A: *The two terms are  $x$  and negative ten.*

Q: What are the degrees of the terms that you should combine to simplify the expression completely?

A: *The degree 2 and degree 1 terms have like terms that should be combined.*

Be sure that your student carefully multiplies terms since there are three terms with negative symbols.

**Checkpoint**

**Find the product. Classify the product.**

$$(x - 10)(x^2 - 2x - 5)$$

$$x^3 - 2x^2 - 5x - 10x^2 + 20x + 50$$

$$x^3 - 12x^2 + 15x + 50$$

**cubic polynomial with 4 terms**

**▶ Multiplying Trinomials**

- trinomial · trinomial = a polynomial
- (   3   terms) · (   3   terms) =   9   terms (before simplifying)
- Remember that by using the   Commutative   Property, you can multiply every term in the second expression by every term in the first expression and get the   same   result.

**Example 10**

**Find the product horizontally and by using the horizontal hybrid method.**

$$(3x^2 - 8x + 6)(2x^2 + x + 1)$$

Horizontally:

**Implement**

$$((3x^2 - 8x + 6)(2x^2 + x + 1))$$

$$(3x^2 - 8x + 6)(2x^2 + x + 1)$$

$$(3x^2 - 8x + 6)(2x^2 + x + 1)$$

$$6x^4 + 3x^3 + 3x^2 - 16x^3 - 8x^2 - 8x + 12x^2 + 6x + 6$$

$$6x^4 - 13x^3 + 7x^2 - 2x + 6$$

**Explain**

- ◀ Multiply all the terms of the second trinomial by  $3x^2$
- ◀ Multiply all the terms of the second trinomial by  $-8x$
- ◀ Multiply all the terms of the second trinomial by  $6$
- ◀ Combine like terms
- ◀ The result is a 4th degree polynomial with 5 terms.

Horizontal hybrid:

**Implement**

$$(3x^2 - 8x + 6)(2x^2 + x + 1)$$

$$\begin{array}{r} 6x^4 + 3x^3 + 3x^2 \\ -16x^3 - 8x^2 - 8x \\ + \quad 12x^2 + 6x + 6 \\ \hline 6x^4 - 13x^3 + 7x^2 - 2x + 6 \end{array}$$

**Explain**

- ◀ Distribute by  $3x^2$  and write the first set of terms
- ◀ Distribute by  $-8x$  and write like terms under one another
- ◀ Distribute by  $6$  and write like terms under one another
- ◀ Add vertically

**Checkpoint**

**Multiply.**

$$(x^2 + 2x - 3)(x^2 - 4x + 5)$$

$$x^4 - 4x^3 + 5x^2 + 2x^3 - 8x^2 + 10x - 3x^2 + 12x - 15$$

$$x^4 - 2x^3 - 6x^2 + 22x - 15$$

 **Solving for an Unknown Coefficient When Multiplying**

- To solve for an unknown coefficient when multiplying, start by multiplying the terms together using the Distributive Property.
- Distributing the terms helps you see where the unknown variable ends up in the expression. Then, you can look at the known and unknown terms and make comparisons.

**Example 11**

Find the value of  $Q$ .

$$(Qx + 4)(9x - 4) = 27x^2 + 24x - 16$$

**Implement**

$$(Qx + 4)(9x - 4) = 27x^2 + 24x - 16$$

$$9Qx^2 - 4Qx + 36x - 16$$

$$27x^2 + 24x - 16$$

$$9Qx^2 = 27x^2$$

$$9Q = 27$$

$$Q = 3$$

**OR**

$$(Qx + 4)(9x - 4) = 27x^2 + 24x - 16$$

$$9Qx^2 - 4Qx + 36x - 16$$

$$27x^2 + 24x - 16$$

$$-4Qx + 36x = 24x$$

$$-4Q + 36 = 24$$

$$-4Q = -12$$

$$Q = 3$$

**Explain**

◀ Distribute on the left side of the equation

◀ Line up the given answer under the previous step

◀ Use degree 2 terms to solve for  $Q$

◀ Distribute on the left side of the equation

◀ Line up the given answer under the previous step

◀ Use degree 1 terms to solve for  $Q$

 **Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

It may be helpful to think about this problem as three smaller Distributive Property problems rather than one giant expression. Have your student cover all terms except the one they distribute in the first expression, so they are only looking at one term.

**Example 12**

Find the value of  $Q$  using the split Distributive Property method and vertical combining.

$$(3x^2 + 2x - 5)(-Qx + 1) = -12x^3 - 5x^2 + 22x - 5$$

**Implement**

$$(3x^2 + 2x - 5)(-Qx + 1) = -12x^3 - 5x^2 + 22x - 5$$

$$\underline{-3Qx^3 - 2Qx^2 + 5Qx} + 3x^2 + 2x - 5$$

$$\underline{-12x^3 - 5x^2 + 22x - 5}$$

$$-3Qx^3 = -12x^3$$

$$-3Q = -12$$

$$Q = 4$$

**Explain**

◀ Distribute on the left side of the equation

◀ Line up the given answer

◀ Use the cubic terms to solve for  $Q$

**Check**

$$(3x^2 + 2x - 5)(-4x + 1)$$

$$-12x^3 - 8x^2 + 20x + 3x^2 + 2x - 5$$

$$-12x^3 - 5x^2 + 22x - 5 \checkmark$$

 **Checkpoint**

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the first step to determining the value of  $Q$ ?

A: *Distributing the terms in the expression on the left side of the equation.*

Your student can choose to compare the degree 2 or degree 1 terms for this problem.

 **Checkpoint**

Find the value of  $Q$ .

$$(2x + 3)(Qx + 8) = 10x^2 + 31x + 24$$

$$\underline{2Qx^2 + 16x + 3Qx + 24}$$

$$\text{OR } 16x + 3Qx = 31x$$

$$\underline{10Qx^2 + 31x + 24}$$

$$16 + 3Q = 31$$


$$2Qx^2 = 10x^2$$

$$3Q = 15$$

$$2Q = 10$$

$$Q = 5$$

$$Q = 5$$

 Practice 1

Complete the problems on a separate sheet of paper.

- 1) Find the area of a rectangle with a length of  $3x$  and a width of  $2x^2 - x + 8$ .  
 $6x^3 - 3x^2 + 24x$  square units
- 2)  $(4x^2 - 5x - 3)(11x^2)$   $44x^4 - 55x^3 - 33x^2$

Simplify.

- |  |  |
|--|--|
| 3) $(x + 3)(x + 1)$ $x^2 + 4x + 3$     | 4) $(4x + 1)(x + 2)$ $4x^2 + 9x + 2$     |
| 5) $(2x + 3)(2x + 1)$ $4x^2 + 8x + 3$  | 6) $(x + 4)(3x + 2)$ $3x^2 + 14x + 8$    |
| 7) $(x - 12)(x - 7)$ $x^2 - 19x + 84$  | 8) $(2x - 7)(4x - 5)$ $8x^2 - 38x + 35$  |
| 9) $(x - 6)(3x - 8)$ $3x^2 - 26x + 48$ | 10) $(5x - 3)(x + 10)$ $5x^2 + 47x - 30$ |
| 11) $(x + 2)(x - 13)$ $x^2 - 11x - 26$ | 12) $(7x - 2)(8x + 3)$ $56x^2 + 5x - 6$  |

- 13) Find the area of a rectangle when the length is seven times  $x$  plus one unit and the width is three less than  $y$  units.  $7xy - 21x + 1y - 3$  units

Multiply. Write answers in standard form.

- 14)  $(3x^2 + 2x + 1)(3x - 8)$   $9x^3 - 18x^2 - 13x - 8$     15)  $(6x + 7)(x^2 + 2x + 3)$   $6x^3 + 19x^2 + 32x + 21$
- 16)  $(2x^2 - x + 6)(x^2 + 5x + 6)$   $2x^4 + 9x^3 + 13x^2 + 24x + 36$

Find the missing value.

- 17)  $(2x - 11)(Qx + 2) = 10x^2 - 51x - 22$     18)  $(4x^2 + Qx + 7)(3x + 5) = 12x^3 + 26x^2 + 31x + 35$   
 $2 \cdot Q = 10$ ;  $Q = 5$      $5Q = 10$ ;  $Q = 2$

- 19) Find the area of a triangle with a base of  $2x^2 - 6x + 18$  feet and a height of  $3x - 1$  feet.  
 $3x^3 - 10x^2 + 30x - 9$  ft<sup>2</sup>



## Practice 1



Worked solutions for these problems are located in the Digital Pack.

- 1) Q: Classify the length, width, and area by number of terms.  
 A: *The length is a monomial, the width is a trinomial, and the area is a trinomial.*

3–12)

Your student may wish to build these expressions using the digital manipulatives found in the Digital Pack.

3–6)

Q: Are the signs of the terms positive, negative, or some of each?

A: *Positive*

7–9)

Q: Why are the constants all negative in the parentheses but all positive when the product is found?

A: *Because a negative multiplied by a negative is a positive.*

- 13) Q: Does the process for multiplying a binomial by a binomial change when there are two variables?

A: *No, it is the same.*

14–16)

Q: When a binomial and a trinomial are multiplied together, how many terms will result before simplifying completely?

A: 6

- 16) Q: How many terms will result before simplifying completely when a trinomial is multiplied by a trinomial?

A: 9

- 17) Remind your student to multiply the terms together first to see which terms they should compare to determine the value of  $Q$ .

## Mastery Check

### Show What You Know

Remind your student to write down or label the length and width of the rectangles in the figure. Defining the variables will help keep the problem organized.

- B)** Make sure your student only combines terms with the same degree. This is critical to remember for parts B, C, and D.

### Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- ☑ Multiply a monomial by a polynomial using the Distributive Property and exponent rules.
- ☑ Find the product of a binomial and a binomial using the Distributive Property.
- ☑ Find the product of a binomial and a trinomial, as well as a trinomial and a trinomial.

## Mastery Check

### Show What You Know

The Layton family determined the dimensions of their house, attached garage, and yard in the given figure. They took the measurements in meters.

- A)** Find the area of the house and the area of the garage.

#### House

$$l = 8x - 3; w = 4x - 1$$

$$A = lw$$

$$A = (8x - 3)(4x - 1)$$

$$A = 32x^2 - 8x - 12x + 3$$

$$A = 32x^2 - 20x + 3 \text{ m}^2$$

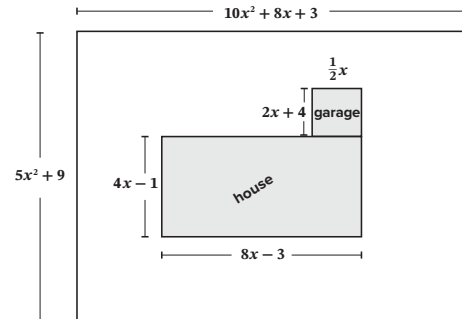
#### Garage

$$l = \frac{1}{2}x; w = 2x + 4$$

$$A = lw$$

$$A = \left(\frac{1}{2}x\right)(2x + 4)$$

$$A = x^2 + 2x \text{ m}^2$$



- B)** Find the combined area of the house and garage.

$$\text{Combined area} = \text{house} + \text{garage}$$

$$(32x^2 - 20x + 3) + (x^2 + 2x)$$

$$33x^2 - 18x + 3 \text{ m}^2$$

- C)** Find the total area of the yard with the house included.

$$l = 10x^2 + 8x + 3; w = 5x^2 + 9$$

$$A = lw$$

$$A = (10x^2 + 8x + 3)(5x^2 + 9)$$

$$A = 50x^4 + 40x^3 + 15x^2 + 90x^2 + 72x + 27$$

$$A = 50x^4 + 40x^3 + 105x^2 + 72x + 27 \text{ m}^2$$

- D)** Find the area of the yard only. (Hint: Use parts B and C to help find this answer)

$$\text{Area of yard only} = \text{part C} - \text{part B}$$

$$A = (50x^4 + 40x^3 + 105x^2 + 72x + 27) - (33x^2 - 18x + 3)$$

$$A = 50x^4 + 40x^3 + 105x^2 + 72x + 27 - 33x^2 + 18x - 3$$

$$A = 50x^4 + 40x^3 + 72x^2 + 90x + 24 \text{ m}^2$$

### Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

## Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

### YES

If your student can answer “yes” to all of these questions, decide if your student is ready to take the Lesson Test.

### NOT YET

If your student cannot answer “yes” to all of these questions, consider having your student complete some of these options:

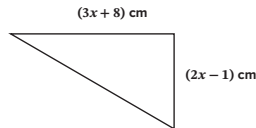
- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Complete the problems on a separate sheet of paper.

For all problems, multiply. Write answers in standard form.

- 1) Find the perimeter of a rectangle with a width of  $5x - 1$  feet and a length of  $x^2 + 3x$  feet.  
 $2x^2 + 16x - 2$  ft
- 2)  $(x - 8)(x + 1)$   $x^2 - 7x - 8$
- 3)  $(2x + 5)(x - 4)$   $2x^2 - 3x - 20$
- 4)  $(3x + 4)(3x + 4)$   $9x^2 + 24x + 16$
- 5)  $(6x - 7)(x + 9)$   $6x^2 + 49x - 63$
- 6)  $(x + 12)(x - 12)$   $x^2 - 144$
- 7)  $(5x - 1)(5x + 1)$   $25x^2 - 1$
- 8)  $(8x - y)(8x + y)$   $64x^2 - y^2$
- 9)  $(2x + 7)(2x - 7)$   $4x^2 - 49$
- 10) Find the area of the triangle.  
 $3x^2 + \frac{13}{2}x - 4$  cm<sup>2</sup>



Find the product.

- 11)  $(x - 4)(3x + 8)$   $3x^2 - 4x - 32$
- 12)  $(5x - 11)(2x - 7)$   $10x^2 - 57x + 77$

Multiply. Answers will have two variables.

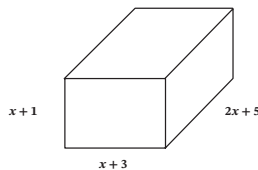
- 13)  $(x + y)(x + y)$   $x^2 + 2xy + y^2$
- 14)  $(2x + 3y)(4x - 5y)$   $8x^2 + 2xy - 15y^2$

- 15) Find the missing value.  
 $(2x - 3)(Qx^2 - 3x - 9) = 10x^3 - 21x^2 - 9x + 27$   $Q = 5$

Multiply.

- 16)  $(x + 2)(11x^2 + 5x - 13)$   
 $11x^3 + 27x^2 - 3x - 26$
- 17)  $(x^2 - 4x + 3)(x^2 + 6x - 7)$   
 $x^4 + 2x^3 - 28x^2 + 46x - 21$
- 18) Find the missing value.  
 $(8x + 7)(Qx - 2) = 24x^2 + 5x - 14$   $Q = 3$
- 19) Find the product.  
 $(x - 2)(15x^2 + 20x - 1)$   $15x^3 - 10x^2 - 41x + 2$

- 20) The formula for the surface area of a rectangular prism is  $SA = 2(lw + lh + wh)$ . Find the surface area of the prism. (Hint: find  $lw$ ,  $lh$ , and  $wh$  first.)  
 $10x^2 + 44x + 46$  square units



Practice 2

Worked solutions for these problems are located in the Digital Pack.

- 13) Q: What is  $x$  times  $x$ ? ( $x^2$ )  
 What is  $x$  times  $y$ ? ( $xy$ )  
 What is  $y$  times  $y$ ? ( $y^2$ )

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

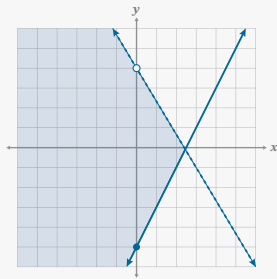
Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

### Targeted Review

 Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

- 3) Finding the value of  $2^{41}$  is not necessary as this cannot be done using mental math.
- 7) Remind your student to change all mixed fractions to improper fractions and then multiply by the LCD. They can also use the strategy they learned in Lesson 2 and clear the fractions from the equation first if they prefer.
- 10) Remember that the y-intercept should have an open point for (0, 4) since it is on a dashed line.



- 12) Distractor Rationale:
  - A) This is the answer if the coefficients are added rather than multiplied.
  - B) This is the answer if the coefficients are added and the exponents are ignored.
  - C) This is the answer if the exponents are ignored.

### Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Simplify.

1)  $(2xy^2)(3xy^2)$     **6**  $x^2y^4$     2)  $(5a^3b^{11}c^2)^2$     **25**  $a^6b^{22}c^4$     3)  $2m^5(2^3m^4n)^5$     **2**  $^{41}m^{25}n^{35}$

- 4) Find the area of a rectangle with a length of  $3xy^5$  units and a width of 11y units.    **A = 33xy<sup>6</sup> square units**
- 5) Find the area of an isosceles triangle with a base and height of 4x feet.    **A = 8x<sup>2</sup> ft<sup>2</sup>**
- 6) Solve. Name the properties used in each step.    7) Solve.  $2\frac{3}{5} = \frac{5}{2}x - 6\frac{1}{5}x$     **x = -\frac{26}{37}**
- 8) Write the verbal model as an algebraic expression, then simplify. *Five times a number less that number, plus three times the number, plus eight.*    **7n + 8**
- 9) If nine gallons of gas cost \$29.25 and five gallons of gas cost \$16.25, what is the cost of one gallon of gas? (Hint: Find the rate of change.)    **One gallon of gas cost \$3.25**
- 10) Graph the system of inequalities.  $y \geq 2x - 5$   $y < -\frac{5}{3}x + 4$

Multiple Choice

- B** 11) Aleesha packed 3 boxes with 5 items each. Krista packed 5 boxes with 3 items each. Their boss made this comparison:  $3(5) = 5(3)$  Determine the property demonstrated.
  - A) Associative Property
  - D) Commutative Property**
  - B) Distributive Property
  - C) Inverse Property
- D** 12) The sides of a rectangle are  $3xy$  and  $2x^2y$ . Find the area of the rectangle.
  - A)  $5x^3y^2$
  - D)  $5xy$
  - E)  $6xy$
  - B)  $6x^3y^2$**

<b>Problem</b>	1	2	3	4	5	6	7	8	9	10	11	12
<b>Lesson Origin</b>	19	19	19	19	19	2	2	2	2	18	1, FS	19