

Lesson 18

More Applications of Linear Systems

Outline

Part A More Applications of Linear Systems

- Geometry Formulas
- Break-Even Problems
- Digit Problems

Part B Applications of Linear Inequalities

- Writing a Linear Inequality
in One Variable
- Writing Systems of Inequalities

Targeted Review

Vocabulary

There are no new vocabulary words for this lesson.



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

- 1) Q: Why is it inaccurate to define perimeter as “the outside of a figure”?

A: *Saying perimeter is the outside of a figure does not explain that it is the distance around the edge of the figure.*

- 2) Q: Where can you find the perimeter formula?

A: *The Formula Sheet*

Q: How are the units for perimeter and area different?

A: *Area uses square units, but perimeter uses units (without exponents).*

Part A: More Applications of Linear Systems

Objectives

In this part of the lesson, you will learn about more applications of linear systems.

By the end of this lesson, you will be able to do the following:

- ☑ Write systems of equations for break-even, formula, and digit word problems and solve them.
- ☑ Explain the solutions to break-even, formula, and digit problems and determine if the solutions are reasonable.

Why?

Suppose you invested a certain amount of money into starting a business. How long would it take for you to earn the money you invested back and make a profit? You can answer this type of question and more by using systems of equations.

Warm Up

- 1) How can you find the perimeter of a figure?

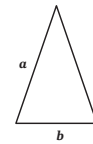
Add the lengths of each side together to find the total linear distance.

- 2) What is the formula for the perimeter of a rectangle?

$$P = 2l + 2w$$

- 3) An isosceles triangle has two sides of equal length and a third side of a different length. Write a formula for the perimeter of an isosceles triangle.

$$P = 2a + b$$



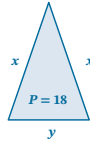
Geometry Formulas

- Perimeter measures the total distance around the edge of an object.

Example 1**Write a system of equations. Solve.**

When two sides of a triangle are the same length you have an isosceles triangle. An isosceles triangle has a perimeter of 18 centimeters. The sum of the two equal sides is 2 more than the remaining side. What are the side lengths of the triangle?

Plan Sketch a triangle.
Define the variables for side length.
Write a system of equations.
Find the side lengths.

**Implement**

x : side of triangle (2), y : side of triangle (1), $P = 18$ cm

$$x + x + y = 18 \rightarrow 2x + y = 18$$

$$x + x = y + 2 \rightarrow 2x = y + 2$$

$$\begin{array}{r} 2x + y = 18 \\ + 2x - y = 2 \\ \hline 4x = 20 \end{array} \quad \begin{array}{r} 5 + 5 + y = 18 \\ y = 8 \end{array}$$

Explain

The sides of the triangle are 5 cm, 5 cm, and 8 cm.

Check

$$2x + y = 18$$

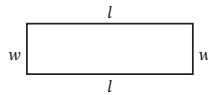
$$2(5) + 10 = 18$$

$$18 = 18 \checkmark$$

Example 2**Write a system of equations. Solve.**

The perimeter of a rectangle is 26 centimeters. Twice the width of a rectangle minus the length is -1 . What are the dimensions of the rectangle?

Plan Define the variables.
Substitute the known facts into the perimeter formula of a rectangle.
Complete the system of equations.
Solve.

**Implement**

l : length, w : width

$$26 = 2l + 2w \quad (26 = 2l + 2w)\left(\frac{1}{2}\right) = 13 = l + w \quad 26 = 2l + 2(4)$$

$$2w - l = -1 \quad + -1 = -l + 2w \quad 26 = 2l + 8$$

$$12 = 3w \quad 18 = 2l$$

$$w = 4 \quad l = 9$$

Check

$$26 = 2(9) + 2(4)$$

$$26 = 18 + 8 \checkmark$$

$$2(4) - 9 = 1 \checkmark$$

Explain

The dimensions of the rectangle are 9 cm by 4 cm.

✓ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What value should you replace P with?

A: 48

Q: What is the width equal to?

A: The width is equal to half the length plus three or $\frac{1}{2}l + 3$.

✓ Checkpoint

Write the system of equations. Do not solve.

The perimeter of a rectangle is 48 centimeters. The width is half of the length plus 3 centimeters.

l : length, w : width

$$48 = 2l + 2w$$

$$w = \frac{1}{2}l + 3$$

▶ Break-Even Problems

- To break-even in business means that you have made back all of the money, or expenses, that were spent to start the business or keep it running.
- Break-even problems can also be used to compare different choices to see what the best deal or outcome is.

Example 3

Write a system of equations. Solve.

Gregg needed to know the break-even point for his sticker business. His expenses totaled \$5,000 plus \$1 per sticker made. He planned to sell each sticker for \$7. How many stickers will Gregg need to sell to break even? (Only whole numbers of stickers can be sold.)

Plan	Implement	
Define your variables.	x : stickers, y : costs	$7x = 5,000 + 1x$
Write a system of equations.	expenses: $y = 5,000 + 1x$	$6x = 5,000$
Solve.	income: $y = 7x$	$x = 833\frac{1}{3}$

Gregg needs to sell 834 stickers to break even. He cannot sell $\frac{1}{3}$ of a sticker, so he needs to round up to the nearest whole number.

Example 4**Write a system of equations. Solve.**

A family is comparing the cost of a graduation party at two different locations. The fire hall charges \$5 per person and a room fee of \$350. The restaurant charges \$10 per person and a room fee of \$225.

- A) How many people would need to attend for the cost of the two locations to be equal?

x: people, y: cost	$5x + 350 = 10x + 225$
Fire hall: $y = 5x + 350$	$125 = 5x$
Restaurant: $y = 10x + 225$	$x = 25$

The locations will be equal if 25 people attend the graduation party.

- B) When is the fire hall a better deal for the family? When is the restaurant a better deal? Provide examples to back up your reasoning.

Fire hall:		Restaurant:	
$x = 20$	$x = 30$	$x = 20$	$x = 30$
$y = 5(20) + 350$	$y = 5(30) + 350$	$y = 10(20) + 225$	$y = 10(30) + 225$
$y = 450$	$y = 500$	$y = 425$	$y = 525$

- The fire hall is a better deal when more than 25 people attend the party.
- The restaurant is a better deal when less than 25 people attend.

Break-even problems are usually written (x, y) because a graph can be made to show the break-even point if necessary.

Checkpoint

Write a system of equations representing the expenses and income for the given scenario. Do not solve.

Pat's Provisions has monthly expenses including \$3 per sandwich sold and \$1,200 in additional expenses. The store sells each sandwich for \$8.50.

(x: sandwich, y: cost)
 expenses: $y = 3x + 1,200$ income: $y = 8.50x$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What are the expenses for the sandwich shop?

A: The expenses are \$1,200 and \$3 per sandwich.

Q: What is the income for the shop? What does it represent?

A: The shop's income is $8.50x$. This represents the price they charge per sandwich.

Digit Problems

- Digit problems require that special attention is paid to the **place-value** of a number.
- The problems in this section will use **tens**, t , and **units**, u .

Make sure to carefully write the variable t . It is not commonly used, but for digit problems, it makes the most sense.

Example 5

Write a system of equations. Solve.

The sum of the digits in a two-digit number is 10. When the value of the digits are reversed, the number is 72 more than the original number. What is the number?

Implement

t : tens, u : units

$$t + u = 10$$

$$t + 10u = 10t + u + 72$$

$$-9t + 9u = 72$$

$$\left(\frac{1}{9}\right)(-9t + 9u = 72) = -t + u = 8$$

$$+t + u = 10$$

$$2u = 18$$

$$u = 9, t = 1$$

Explain

Using mental math and replacing u with 9 in the original equation $t + u = 10$, you find that $t = 1$.

When the digits are placed in the correct place value location:

The number is 19.

Check that the digits are written in the correct order.

Remember, the two numbers that are being solved for create a single two-digit number.

Example 5

Here is another way to think about digit problems when checking your student's work.

$$91 = 90 + 1 = 10(9) + 1$$

This means $t = 9$ and $u = 1$.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What happens to the units digit when the digits are reversed?

A: The units digit is 10 times larger than it should be.

Checkpoint

Write a system of equations. Do not solve.

When the tens digit is subtracted from the units digit the result is 5. The number with the digits reversed is one more than ten times the units.

t : tens, u : units

$$u - t = 5$$

$$t + 10u = 10u + 1$$

 Practice 1

Complete the problems on a separate sheet of paper.

Write a system of equations, define your variables, and solve.

- Two angles that are supplementary total 180 degrees. The smaller angle is half the larger angle less 18. What are the measures of each angle? **The smaller angle measures 48 degrees. The larger angle measures 132 degrees.**
- The perimeter of a rectangle is 48 centimeters. The width is one-third the length. What are the dimensions of the rectangle? **The dimensions of the rectangle are 18 cm by 6 cm.**
- An isosceles triangle has a perimeter of 41 yards. Twice the sum of the sides with different lengths is 52. What is the length of each side of the triangle? **The sides of the triangle are 15 yd, 15 yd, and 11 yd.**
- Two angles that are complementary total 90 degrees. Twice the smaller angle plus three times the larger angle is 240. What is the measure of each angle? **The smaller angle measures 30 degrees. The larger angle measures 60 degrees.**
- The perimeter of a rectangle is 22 inches. The difference between the length and the width is five. What are the dimensions of the rectangle? **The rectangle is 8 inches by 3 inches.**
- What would the area be of the rectangle in Problem 5? Write the formula and solve. **$A = lw$, 24 square inches**
- The sum of two numbers is -7 . The second number is five times the first number less 13. What are the two numbers? **The two numbers are 1 and -8 .**
- The sum of the digits of Joan's age is 16. The sum of the tens digit and twice the units digit is 25. How old is Joan? **Joan is 79 years old.**
- The units digit of a number subtracted from the tens digit is -3 . The value of the digits reversed is twenty times the tens digit plus three. What is the number? **The number is 36.**
- Tanya's T-shirts purchased blank t-shirts for \$3.25 each and screen printing materials for \$180. Tanya sells her designed t-shirts for \$12 each. How many shirts must Tanya's T-shirts sell to break even? **Tanya's T-Shirts must sell 21 t-shirts to break even.**
- Bob's Biscuits spent \$500 on baking supplies. Each biscuit sold at \$2.50 each. How many biscuits will Bob need to sell to break even? **Bob needs to sell 200 biscuits to break even.**
- If Bob increased the selling price to \$3 per biscuit, how many fewer biscuits would he need to sell to break even? **If the price was raised to \$3, Bob would be able to sell 33 fewer biscuits to break even.**

9) t : tens, u : units

$$\begin{aligned} t - u &= -3 \\ t + 10u &= 20t + 3 \\ t &= 3 \\ u &= 6 \end{aligned}$$

Q: If 7 is subtracted from an unknown term, what order should you write the terms in?

A: You should write it as $x - 7$. The 7 will come after the subtraction symbol because it is being taken away from the given term.

10) x : shirts, y : costs

$$\begin{aligned} y &= 12x \\ y &= 3.25x + 180 \\ x &= 20.571 \end{aligned}$$

Q: Suppose Tanya needs to sell 30.11 shirts to break even. Since she cannot sell a fraction of a shirt, what number of shirts should Tanya sell?

A: Tanya needs to sell 31 shirts. If she only sells 30, she will not break even.

11) b : biscuits, c : costs

$$\begin{aligned} c &= 500 \\ c &= 2.50b \\ b &= 200 \end{aligned}$$

 Practice 1

 Worked solutions for these problems are located in the Digital Pack.

You can have your student write the system of equations for each problem but only assign them one of each type to solve. Writing the system of equations should be the focus of the lesson practice, not solving.

1) x : small angle, y : large angle

$$\begin{aligned} x + y &= 180 \\ x &= \frac{1}{2}y - 18 \\ x &= 48 \\ y &= 132 \end{aligned}$$

2) l : length, w : width

$$\begin{aligned} 48 &= 2l + 2w \\ w &= \frac{1}{3}l \\ l &= 18 \\ w &= 6 \end{aligned}$$

Q: How can you determine if your solution is correct?

A: Substitute the values into both equations and see if they both result in true statements.

Q: What must be included when writing the dimensions of a figure?

A: The unit of measure.

3) x : side of triangle, y : side of triangle

$$\begin{aligned} x + x + y &= 41 \\ 2(x + y) &= 52 \\ x &= 15 \\ y &= 11 \end{aligned}$$

4) x : smaller angle, y : larger angle

$$\begin{aligned} x + y &= 90 \\ 2x + 3y &= 240 \\ x &= 30 \\ y &= 60 \end{aligned}$$

5) l : length, w : width

$$\begin{aligned} 22 &= 2l + 2w \\ l - w &= 5 \\ l &= 8 \\ w &= 3 \end{aligned}$$

Q: Would it be reasonable for the solution to include negative values? Explain.

A: No, since you are finding a side length of a rectangle, the numbers cannot be negative.

6) Q: What is the formula for finding the area of a rectangle?

A: $A = lw$

7) Any variables can be chosen to represent the two numbers.

$$\begin{aligned} x &: \text{second number, } y: \text{first number} \\ x + y &= -7 \\ x &= 5y - 13 \\ x &= -8 \\ y &= 1 \end{aligned}$$

8) t : tens, u : units



$$\begin{aligned} t + u &= 16 \\ t + 2u &= 25 \\ t &= 7 \\ u &= 9 \end{aligned}$$

 **Mastery Check**
 **Show What You Know**

- A)** These are suggested variables. Your student can define their variables as they choose.
- B)** Q: Why should you round your answer?
A: *Because you cannot sell a fraction of a meal kit.*
- C)** Q: Quinn told everyone he earned over \$1,000. Why is this statement incorrect?
A: *Because his expenses need to be paid before he can determine how much he earned.*

 **Say What You Know**

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

-  Write systems of equations for break-even, formula, and digit word problems and solve them.
-  Explain the solutions to break-even, formula, and digit problems and determine if the solutions are reasonable.

 **Mastery Check**
 **Show What You Know**

Quinn started a healthy dinner club meal service. The outgoing expenses were \$300 for supplies and \$8 per meal. Quinn sold his kits for \$25 per kit.

- A)** Write a system of equations for expenses and income. Define your variables.

m : meal, c : costs

expenses: $c = 8m + 300$

income: $c = 25m$

- B)** Solve the system from part A for the number of meal kits Quinn needs to sell to break even. Explain your steps.

$$25m = 8m + 300$$

$$17m = 300$$

$$m = 17.647\dots$$

Since both equations were equal to c , substitution can be used by setting the equations equal to one another. Then, m can be solved for. Since a partial meal kit cannot be sold, Quinn needs to sell 18 kits to break even.

- C)** How much will Quinn earn if he sells 42 meal kits? Show your work. (Hint: earnings represent the money left after expenses are deducted.)

$$\text{expenses: } c = 8m + 300; m = 42$$

$$c = 8(42) + 300$$

$$c = 636$$

$$\text{income: } c = 25m; m = 42$$

$$c = 25(42)$$

$$c = 1,050$$

Income – expenses


$$1,050 - 636$$

$$414$$

Quinn will earn \$414 after selling 42 meal kits.

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

 Practice 2

Complete the problems on a separate sheet of paper.

Write a system of equations, define your variables, and solve.

- Two sides of a triangle are of equal length. The remaining side is one-fourth as long as one of the equal sides, less two. If the perimeter of the triangle is 34 inches, what is the length of each side? **The triangle has sides of 16 inches, 16 inches, and 2 inches.**
- Two complementary angles total 90 degrees. The larger angle is three times the smaller angle, plus two. What is the measure of the larger angle? **The larger angle is 68 degrees.**
- Two supplementary angles total 180 degrees. When the smaller is subtracted from one-fourth the larger angle, the result is -55 . What is the difference between the two angles? **The difference between the two angles is 20.**
- The length of a rectangle is five times the width. If the perimeter is 72 feet, what are the dimensions of the rectangle? **The rectangle is 30 ft by 6 ft.**
- Another rectangle has a perimeter of 72 feet. The length is six more than the width. What are the dimensions of this rectangle? **The dimensions of the rectangle are 21 feet by 15 feet.**
- Find the positive difference between the areas of the rectangles in problems 4 and 5.
 $315 - 180 = 135$
- The sum of the digits of a two-digit number is 11. When the digits are reversed, the sum is 27 more than the original number. What is the number? **The number is 47.**
- Gerald is thinking of two numbers. The difference between the numbers is 25. The sum of the numbers is 61. What are the two numbers? **The numbers are 43 and 18.**
- The Corvea family saved \$1,200 for their yearly vacation. The hotel costs \$125 per night and they have \$500 in travel costs. Write a system to determine the number of nights the family can stay at the hotel. Will the family have any money left over? **The family can stay for 5 nights. They will have \$75 left after 5 nights.**
- The Corvea family found another hotel that costs \$80 per night but their travel expenses would be \$650. Would the family be able to stay longer even though the travel expenses increased? **The family can stay for 6 nights instead of 5 nights.**
- Chrissy was overcharged 18 cents when the cashier reversed the digits for her tax owed. If the sum of the digits is 10, what is the correct tax? **The correct tax is 46 cents.**
- Cami's Cleaners charges \$40 for every office they clean. This month they purchased \$2 of supplies per office and spent an additional \$225 on travel costs. How many offices will Cami's Cleaners need to clean to break even this month? **Cami's Cleaners will need to clean 6 offices to break even.**

 Practice 2

 Worked solutions for these problems are located in the Digital Pack.

- x : equal side, y : remaining side
 $2x + y = 34$
 $y = \frac{1}{4}x - 2$
 $x = 16$
 $y = 2$
- x : large angle, y : small angle
 $x + y = 90$
 $x = 3y + 2$
 $x = 68$
 $y = 22$
- x : larger angle, y : smaller angle
 $x + y = 180$
 $\frac{1}{4}x - y = -55$
 $x = 100$
 $y = 80$
- l : length, w : width
 $l = 5w$
 $72 = 2l + 2w$
 $l = 30$
 $w = 6$
- l : length, w : width
 $l = w + 6$
 $72 = 2l + 2w$
 $l = 21$
 $w = 15$
- t : tens, u : units
 $t + u = 11$
 $t + 10u = 10t + u + 27$
 $t = 4$
 $u = 7$
- x : first number, y : second number
 $x - y = 25$
 $x + y = 61$
 $x = 43$
 $y = 18$
- x : nights, y : costs
 $y = 1,200$
 $y = 125x + 500$
 $x = 5.6$
 $y = 1,125$
- t : tens, u : units
 $t + u = 10$
 $t + 10u = 10t + u + 18$
 $t = 4$
 $u = 6$
- x : offices, y : costs
 $y = 40x$
 $y = 2x + 225$
 $x = 5.921$

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

The beginning of Part B is a bridge from previous knowledge to writing and solving a system of linear inequalities. Your student may remember some of this material from Lesson 4, but the vocabulary is important to review.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Your student can use substitution or graphing to determine if any of the given points are solutions to the system of inequalities.

Q: Is it possible to list all of the solutions to this system of inequalities? Explain.

A: *It is not possible to list all solutions to a system of inequalities because the solution contains an infinite number of points.*

Q: What method of solving would allow you to represent all solutions to this system?

A: *Graphing*

Part B: Applications of Linear Inequalities

Objectives

In this part of the lesson, you will learn about applications of linear inequalities.

By the end of this lesson, you will be able to do the following:

- ☑ Identify and apply the following key words to write an inequality from a verbal model:
 - is more than
 - is at least
 - is less than
 - is at most
 - is no more than
 - has a minimum of
 - has a maximum of
- ☑ Write a system of inequalities from a scenario and solve for the solution.
- ☑ Explain what the solution to a system of inequalities means within a given context.

Why?

Your family is planning a vacation. They save a certain amount each month but have a maximum spending limit for the trip. How many days can the trip be? Do you have to spend all of the money saved? Solving systems of linear inequalities will help you answer questions like these.

Warm Up

Determine which ordered pairs, if any, are solutions to the system of inequalities. Write “yes” or “no” for each pair.

$$x + y < 7 \quad \mathbf{1) (0, 5) \text{ no}} \quad \mathbf{2) (-2, 5) \text{ no}} \quad \mathbf{3) (2.33, 6.78) \text{ no}}$$

$$x - 5 > -5$$

▶ Writing a Linear Inequality in One Variable

Symbol	Common Wording	Additional Wording
$>$	is greater than	is more than, is above, is larger than, exceeds
\geq	is greater than or equal to	is at least, minimum, is no smaller/less than
$<$	is less than	is below, is smaller than
\leq	is less than or equal to	is at most, maximum, cannot exceed, is no more/greater than

Example 1

Highlight the phrase that represents the inequality symbol needed. Write the inequality symbolically. Do not solve.

- A) Ethan needs to work **at least** 30 hours this week. $h \geq 30$
- B) Daryl can reformat **no more than** 5 computers a week. $c \leq 5$
- C) Cricket saves \$15 per week for family birthdays throughout the year. She needs a **minimum** of \$500 this year. $15w \geq 500$

Example 2

Write a compound inequality and solve.

Rabina wants to determine the number of hours she would need to work to make a minimum of \$1,200. She makes \$16 per hour. Her boss told her that she could make **no more than** \$2,560 in a given month. What are the possible hours that Rabina can work?

Plan Define your variables. h : hours
Identify key words.
Write the system of inequalities.
Solve.

Implement

$16h \geq 1,200$ AND $16h \leq 2,560$ ◀ Multiplication Property of Equality

$h \geq 75$ AND $h \leq 160$

Explain

Rabina can work at least 75 hours but no more than 160 this month.

$$75 \leq h \leq 160$$

 Checkpoint

Write an inequality. Do not solve.

- A) Pippa wants to earn a minimum of \$1,200 this month. She makes \$15 per hour. h : hour;
 $15h \geq 1,200$
- B) Pippa would prefer to work no more than 70 hours a month. $h \leq 70$

Writing Systems of Inequalities

- When writing a system of inequalities, you will often need more than two inequalities to represent a real life situation.

 Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What symbol would you use for maximum?

A: *Less than or equal to. You cannot go over the given value.*

Q: Why does the word "minimum" require a greater than or equal to symbol?

A: *Because it represents the least amount/value that you can have.*

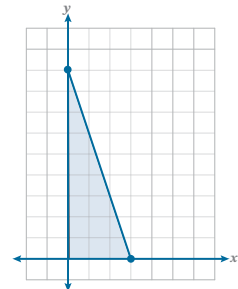
Example 3

Write a system of inequalities. Shade only the solution on the graph.

The Nor family wants to include fruits and vegetables with each lunch this week. Fruit salad, x , is \$6 per pound and carrots, y , are \$2 per pound. They can spend **no more than** \$18 this week on these items.

- A) x : fruit salad, y : carrots fruit: $x \geq 0$ carrots: $y \geq 0$
 spending limit: $6x + 2y \leq 18$

- B) Solve for the x -intercept when y is zero. Solve for the y -intercept when x is zero.
 $(a, 0)$ $(0, b)$
 $6a = 18$ $2b = 18$
 $a = 3$ $b = 9$
 $(3, 0)$ $(0, 9)$



- C) What does the shaded region represent?

The shaded region represents the possible combinations of fruit salad

and carrots that the Nor family can purchase for less than \$18.

- D) Would it be possible for the Nors to purchase 3 pounds of fruit salad and 2 pounds of carrots?

No, (3, 2) is not a possible combination for the Nor family. This point falls outside the shaded region.

- E) Would the family be able to purchase a fraction of a pound of fruit salad and/or carrots?

Yes, the Nor family can purchase any combination of fruit salad and carrots that fall in the shaded region, including a fraction of a pound.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is a vertex?

A: *It is a corner where the edges meet.*

Q: How many inequalities will you need in your system to create a rectangle?

A: *Four.*

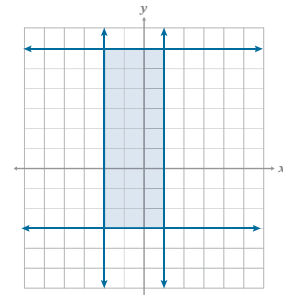
Your student may need to look back at Lesson 11 to recall the equation for a horizontal and vertical line.

Checkpoint

The vertices of a rectangle are $(1, 6)$, $(-2, 6)$, $(-2, -3)$, and $(1, -3)$.

Graph the vertices and write a system of inequalities that will shade the area of the rectangle.

- $y \leq 6$
 $y \geq -3$
 $x \geq -2$
 $x \leq 1$



Practice 1

Complete the problems on a separate sheet of paper.

Identify the phrase that determines the inequality symbol and highlight or underline it. Write a single-variable or compound inequality and solve. Write a sentence to explain your solution within the given context.

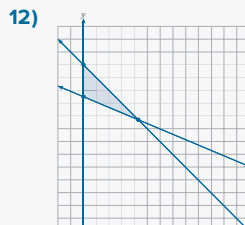
- 1) Khaled could spend no more than \$100 a month on gas. If gas costs \$2.75 per gallon, how many gallons of gas can Khaled purchase? **Khaled can purchase a maximum of 36.36 gallons of gas.**
- 2) Students must have indoor recess at school when the temperature is below 32° or above 93°. Write a compound inequality to show when students cannot have recess outside. When can students have outdoor recess? **n : no recess, $n < 32$ or $n > 93$**
- 3) Giorgi earns a minimum of \$300 per week being paid \$25 per hour. Giorgi can earn no more than \$1,000 each week due to work guidelines. What is the range of time that Giorgi can work in one week? **Giorgi can work between 12 and 40 hours each week.**

Define your variables for the system. Write a system of inequalities. Do not solve.

- 4) The Adams family read a chapter of a book together each night before bed. Depending on the number of pages in the chapter, the family would spend between 15 minutes and 45 minutes reading. The chapters of the books are all at least 2 pages. No chapter is longer than 23 pages.
- 5) The school was hosting a field trip. At least 4 students must attend the field trip. At most 48 students could attend. At least one chaperone must attend for any student trip to occur. **$d \leq 48$, $c \geq 1$, $d \geq 4$**
- 6) Reema works part-time at a local coffee shop. Reema earns \$9 per hour and can work a maximum of 20 hours each week. Reema must work at least 4 hours each week. **$9x \leq 20$, $x \geq 4$**
- 7) Write the system of inequalities. A rectangle has vertices at (4, -2), (4, 3), (-1, 3), and (-1, -2). Graph the vertices, then write a system of inequalities that represents the sides of the rectangle and shades the area of the rectangle. **$y \geq -2$, $y \leq 3$, $x \geq -1$, $x \leq 4$**
- 8) What does the shaded region represent for the rectangle in problem 7?
- 9) Melody practices piano and vocals for a minimum of 12 hours each week. Her combined practice time for both cannot exceed 15 hours a week because of school work. Melody needs to practice piano at least zero hours a week. She also needs to practice vocals at least zero hours each week. Write a system of inequalities to represent all possible practice times where piano is the x -axis and vocals is the y -axis. **p : piano, v : vocals, $p + v \geq 12$, $p \geq 0$, $p + v \leq 15$, $v \geq 0$**
- 10) Graph the solution to problem 9. What does the shaded region represent for Melody?
- 11) Hermine works two part-time jobs while in college. Her catering job (x) pays \$8 per hour, and her tutoring job (y) pays \$20 per hour. Hermine must earn at least \$250 total from both jobs each week, and she can work at most 15 hours per week. Write a system of inequalities to represent all possible ways for her to work both jobs and earn at least \$250.
- 12) Graph the system of inequalities from problem 11. What is the fewest number of tutoring hours Hermine can work? What are two possible combinations of hours that Hermine can work? (Round to the nearest whole hour.) **Hermine needs to work at least 11 hours per week tutoring. Any ordered pair in the shaded region answers this question.**

- 11) Hermine must work greater than or equal to zero hours at both jobs since answers in other quadrants of the graph will not make sense.

x : food-service, y : tutoring
 $x + y \leq 15$
 $8x + 20y \geq 250$
 $x \geq 0$
 $y \geq 0$



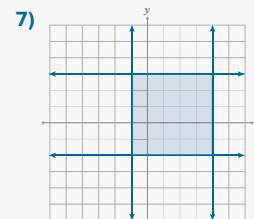
Q: If Hermine needed \$450 instead of \$250 but could not increase her hours, would this be possible? Explain.

A: No, even if she only tutored, at \$20 for 15 hours, she would only make \$300. She would be \$150 short.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

- 1) g : gas; no more than: \leq
 $2.75g \leq 100$
 $g \leq 36.36$
- 2) Students can have outdoor recess when the temperature is greater than 32° but less than 93°. Recess will be held outside when $32 \leq r \leq 93$.
- 3) Q: Will the compound inequality be an AND or OR problem? Explain.
 A: This problem represents an AND problem because there is a range of time that Giorgi can work.
 h : hours; $300 \leq 25x \leq 1,000$
 $300 \leq 25x$ $25x \leq 1,000$
 $12 \leq x$ $x \leq 40$
- 4) p : pages, m : minutes
 $m \geq 15$ $p \geq 2$
 $m \leq 45$ $p \leq 83$
 Your student should have two inequalities relating to the time in minutes and two related to the page length.
- 5) c : chaperone, d : students
 $8c \leq 48$ $d \geq 4$
 $c \geq 1$
- 6) x : hours, y : money earned

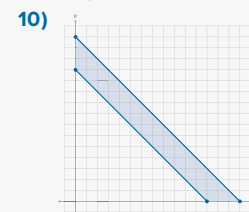


- 7) The shaded region represents all possible ordered pairs that will make the system of inequalities true.
 Q: Why is the shaded region so important when working with systems of inequalities?
 A: The shaded region represents all possible solutions to a system of inequalities and is the only way to represent the infinite number of solutions.

- 9) Remember that both practice times need to be greater than or equal to zero, because it's impossible to practice for less than 0 hours.

Your student can use other variables in their answers, such as x and y .

$x + y \geq 12$
 $x + y \leq 15$
 $x \geq 0$
 $y \geq 0$



The shaded region represents all possible combinations of practice times.

Mastery Check

Show What You Know

C) Q: What do the vertices (or corners) of the enclosed figure represent?

A: *The possible number of hours that the musicians can play in one month.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- ① Identify and apply the following key words to write an inequality from a verbal model:
 - is more than
 - is less than
 - is no more than
 - is at least
 - is at most
 - has a minimum of
 - has a maximum of
- ② Write a system of inequalities from a scenario and solve for the solution.
- ③ Explain what the solution to a system of inequalities means within a given context.

Mastery Check

Show What You Know

A small music shop specializes in creating two Chinese instruments called the xiao and yaogu. At most 9 xiao can be created each month at the music shop. The music shop makes no more than 12 yaogu each month.

- A) Write a system of inequalities that represents the number of each instrument that can be made each month. Remember to include the minimum number that can be made each month as well.

$$\begin{aligned} x &\geq 0 & y &\geq 0 \\ x &\leq 9 & y &\leq 12 \end{aligned}$$

Each month, a group of musicians comes to the music shop to test the new instruments. The musicians are allotted 2 hours of playing time for each xiao and 3 hours of playing time for each yaogu. The number of combined hours the group spends testing all the instruments cannot exceed 42.

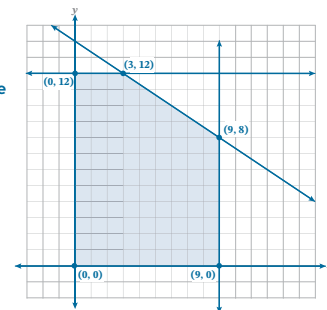
- B) Write an inequality to represent all possible times it takes musicians to test new instruments.

$$2x + 3y \leq 42$$

- C) Graph all inequalities in parts A and B on the same coordinate plane. Mark all intersection points that enclose the shaded region.

The slope and y-intercept from $2x + 3y \leq 42$ are needed to graph all inequalities in part C.

$$\begin{aligned} m &= -\left(\frac{A}{B}\right) = -\left(\frac{2}{3}\right) = -\frac{2}{3} \\ b &= \frac{C}{B} = \frac{42}{3} = 14 \end{aligned}$$



Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Lesson, Unit, and Midterm Assessments

At the end of Unit 3, students can complete the Lesson 18 Test, Unit 3 Test, and the Midterm Exam.

Students should complete the Lesson 18 Test in the same manner as they have in previous lessons.

The next day, before completing the Unit 3 Test, they can review Lessons 15–18, using guided notes, Mastery Checks, and Lesson Tests.

To prepare for the Midterm Exam, students can:

- Review Lesson Tests and Unit Tests. Re-working test problems they missed is a great way to prepare.
- Use Unit dividers to review objectives they have mastered. If needed, they can review guided notes, examples, and Checkpoints for objectives that are less familiar.
- If not already done, complete the Targeted Reviews. The problems come from a variety of lessons, not always in the sequence they learned it, as they will on the Midterm Exam.

Practice 2

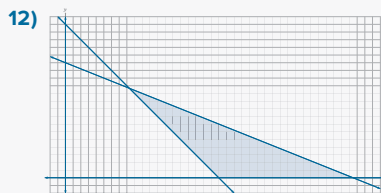
Complete the problems on a separate sheet of paper.

Identify the phrase that determines the inequality symbol and highlight or underline it. Write a single-variable or compound inequality and solve. Write a sentence to explain your solution within the given context.

- To maintain his current class average, Alonso needs to earn at least an 82 but no more than an 86 on his next math test. If each question is worth 2 points and there are 50 questions, what is the range in questions that Alonso can answer correctly and maintain his grade?
- Santiago was using a cab service in the city. The cab service charged \$2 per mile plus a \$1 fee. Santiago could spend no more than \$20 for his ride. What is the maximum distance he can travel in the cab? **Santiago can travel a maximum of 9.5 miles with \$20.**
- Amir plans to walk more than 10,000 steps a day. He walks every morning and logs 4,700 steps. How many more steps does Amir need to walk to meet his goal?

Write a system of inequalities. Do not solve.

- To maximize growing space for a vegetable garden, the Green family determined that the width of their garden box must be greater than 1.5 feet but no more than 3 feet. The maximum length of their garden box is 10.5 feet.
- Camila was working on a science project for her chemistry class. Her teacher told her that if the chemical reaction took less than 2 minutes or more than 5 minutes, she would need to try the experiment again. She knows that each reaction will take at least zero minutes. Write a system of inequalities to represent when the experiment should be reattempted.
- Brian's truck can hold 24 gallons of gas. He never lets his gas tank drop below a quarter full. On average, gas costs \$3.00 in Brian's area, and he spends a maximum of \$150 each month on gas.
- A restaurant is placing a food order for chicken and fish. They must purchase at least 10 whole chickens and at least 6 whole fish. Whole chickens cost \$8 and whole fish cost \$20. The restaurant must spend less than \$400 for this order. Write a system of inequalities where chicken is x and fish is y .
- Graph the system of inequalities from problem 7. If the restaurant knows they must order 11 whole fish for a party, what are three possible chicken orders they can make and stay under budget?
- A distribution warehouse needs to have at least 700 parking spaces for the employee vehicles and the company freight trucks. A car requires 180 square feet of space and a freight truck requires 450 square feet. The total parking lot space is 300,000 square feet. Write a system of inequalities.
- Leonardo and Michelangelo started Turtle's Pizza. Each pepperoni pizza sells for \$15 and costs \$6 to make. Each veggie pizza sells for \$12 and costs \$7 to make. Expenses need to be below \$840, while pizza sales need to be above \$5,000. Write a system of inequalities for sales and expenses.
- Julianna knits scarves and sweaters. Her goal is to have a combination of at least 20 scarves and sweaters to sell at the craft fair. A scarf takes 4 hours to knit and a sweater takes 10 hours to knit. Julianna has 150 hours that she can dedicate to her knitting. Write a system of inequalities where x represents scarves and y represents sweaters.
- Graph the system of inequalities from problem 11. If Julianna can only knit 4 sweaters, will she still be able to reach her goal for the craft fair? Explain. **Julianna will still be able to reach her goal because she has the time to make at least 16 scarves with the hours she has remaining.**



If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

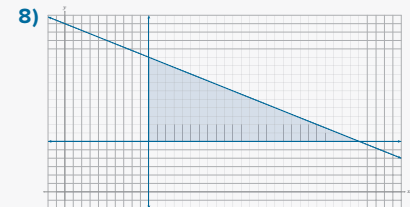
Lesson Test

Lesson Test" note text (global) and change text to "Refer to the Part B Mastery Check instructor note regarding Lesson and Unit Tests and the Midterm Exam.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

- q : question
 $82 \leq 2q < 86$
 $41 \leq q < 43$
 Alonso needs to get 41 or 42 questions correct to maintain his current grade.
- m : mile; no more than \leq
 $2m + 1 \leq 20$
 $2m \leq 19$
 $m \leq 9.5$
- s : steps; more than $>$
 $s + 4,700 > 10,000$
 $s > 5,300$
 Amir needs to walk at least 5,301 steps to meet his goal.
- l : length, w : width
 $w > 1.5$ $l > 0$
 $w \leq 3$ $l \leq 10.5$
 Q: What value must the length be greater than even though it is not mentioned? Explain.
 A: The length must be greater than 0 feet.
- Since this represents a real life scenario, remind your student that the experiment time must be greater than zero minutes. They may need this to write the third inequality.
 m : minutes
 $m < 2$
 $m > 5$
 $m > 0$
- g : gas
 $g \leq 24$
 $g > \frac{1}{4}(24)$
 $3g \leq 150$
- x : chicken, y : fish
 $x \geq 10$
 $y \geq 6$
 $8x + 20y < 400$



Note: Your student's answer should be $10 \leq \text{chicken} \leq 22$, using $y = 11$ fish as the guide. Therefore, their chosen values for chicken orders should be between 10 and 22 or equal to 10 and 22.

- Remember that the values need to be greater than or equal to zero for this real life problem.
 c : car, t : truck
 $180c + 450t \leq 300,000$, $c + t \geq 700$, $c \geq 0$, $t \geq 0$
- x : pepperoni, y : veggie
 $15x + 12y > 5,000$
 $6x + 7y < 840$
- x : scarves, y : sweaters
 $x + y \geq 20$
 $4x + 10y \leq 150$
 $x \geq 0$
 $y \geq 0$

Targeted Review

 Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

1) $n = \text{nickel}, p = \text{penny}$
 $0.05n + 0.01p = 0.97$
 $n + p = 25$

2) h : hot dog, d : drink
 $8h + 3d = 44.50$
 $5h + 7d = 45.75$

3) Sample:
 If a system is graphed and the intersection or solution does not fall on a grid line, it is difficult to accurately determine the solution without solving algebraically.

4) Write the second equation in standard form.

5) Sample:
 The solution to a system of equations is where the lines intersect on the coordinate plane. Another way of saying this is the ordered pair that makes both equations true.

6) (x, y)
 $x = -6$
 $y = \frac{2}{5}$

8)

x	y
-6	8
-3	8
0	14
3	14

Domain: $\{6, -3, 0, 3\}$, Range: $\{8, 14\}$

9) Distractor Rationale:

A) occurs when $(4, 1)$ is used rather than $(1, 4)$ and $2x - y$ is evaluated rather than $y - 2x$.

C) occurs when $2x - y$ is evaluated rather than $y - 2x$.

D) occurs when $(4, 1)$ is used rather than $(1, 4)$.

10) Distractor Rationale:

B) is the x -value of the solution $(-1.5, -2)$

C) and D) are the y -intercepts of the given equation

11) Distractor Rationale:

The first and third points are on a dashed line and do not make both inequalities true.

12) Distractor Rationale:

Your student would select the first answer if they forgot to multiply the sum by $-\frac{5}{6}$

Targeted Review

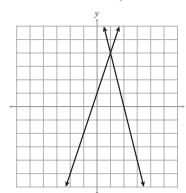
In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Write the system of equations. Do not solve.

- A coin purse contained \$0.97 in pennies and nickels. Twenty-five coins were in the coin purse.
- The Zobel family went to a local baseball game. At first, they ordered eight hot dogs and three drinks for \$44.50. During the 7th inning, they ordered five hot dogs and seven drinks for \$45.75.
- Explain why graphing a system of linear equations in slope-intercept form may not always result in the exact solution.
- Solve the system of equations.
 $x - y = 5$
 $10y + x = 3y + 3x$ **(7, 2)**
- Explain what the solution to a system of equations is in your own words.
- Determine the equations of the vertical and horizontal lines passing through the point $(-6, \frac{2}{5})$.
- Determine the x -intercept for: $5x - 8y = 22$ **($\frac{22}{5}, 0$)**
- Create a table of the relation $\{(-6, 8), (-3, 8), (0, 14), (3, 14)\}$ and name the domain and range for the relation.

Multiple Choice

- B** 9) Use the graph to find the difference between y and $2x$ for the system.
- 
- A** 10) Determine the y -value for the system of equations.
 $y = 2x + 1$
 $y = x - 0.5$
- A) -2
 B) -1.5
 C) -0.5
 D) 1
- 11) Select all solutions that are true for the system of inequalities.
 $y > -\frac{1}{2}x + 3$
 $y \leq x$
- (6, 0)
 (3, 2)
 (2, 2)
 (10.5, -0.5)
- 12) Which of the following is true for the function? Select all that apply.
 $f(x) = -\frac{5}{6}(x + 6)$
- $f(1) = 7$
 $f(2) = -\frac{20}{3}$
 $f(6) = -10$
 $f(-3) = -\frac{5}{2}$

This is the final lesson before the Unit 3 test and the Midterm (Lessons 1–18). Have your student review their Guided Notes, Targeted Review pages, and previous lesson and unit tests to prepare for the Midterm.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	17	17	17	17	15	11	11	7	17	16	15	7