Lesson 17
Applications of Linear Systems

Outline

Part A
Efficiently Solving Systems

- Efficiently Solving Systems
- Extending Solutions

Part B
Applications of Linear Systems

- How Much or How Many
- Coins
- Wind and Water

Targeted Review

Vocabulary

There are no new vocabulary words for this lesson.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge

Q: Why is it important to know how to solve a system using any method?

A: *Different methods might be better depending on the directions and type of equations.*

Q: Why is it important to know what the graph of a system looks like, even when you are not graphing?

A: *Knowing what the graph of a system looks like can help you determine if the solution makes sense for the given system.*

Part A: Efficiently Solving Systems

Objectives

In this part of the lesson, you will learn about efficiently solving systems.

By the end of this lesson, you will be able to do the following:

- Choose the best method to solve a system of linear equations and justify your choice.

Why?

What is the most efficient way to solve a system of linear equations? Can you find the answer in fewer steps? Deciding the most efficient method to solve will help you become a stronger math student for this and future math courses.

Warm Up

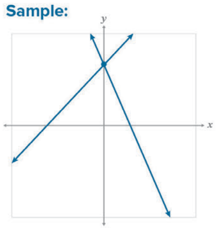
1) What are the three methods you have learned to solve a system?

Graphing
Substitution
Elimination

2) Name the three types of solutions to a system. Make a sketch of each.

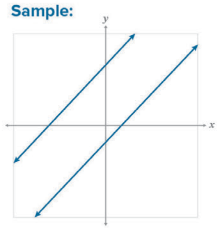
One solution (intersecting lines)

Sample:



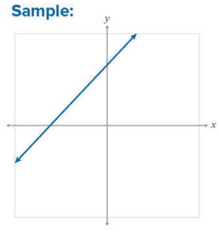
No solution (∥ lines)

Sample:



Infinite solutions on the line (same line)

Sample:



Efficiently Solving Systems

- If a problem does not specify which method to use to solve the system of equations, you must determine which method is most **efficient** by **analyzing** the equations.

EXPLORE 17A

Example 1

Solve the system of equations using graphing and one other method. Explain which method is more efficient and why.

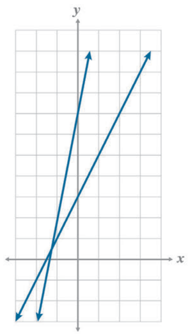
$y = 2x + 3$

$y = 5x + 7$ **Solution:** $\left(-\frac{4}{3}, \frac{1}{3}\right)$

Method: **substitution**

$$2x + 3 = 5x + 7$$
$$-3x = 4$$
$$x = -\frac{4}{3}$$

$$y = 2\left(-\frac{4}{3}\right) + 3$$
$$y = -\frac{8}{3} + \frac{9}{3}$$
$$y = \frac{1}{3}$$



Explain

- In this case, substitution was more **efficient** than graphing because an **exact solution** could not be found by graphing alone.
- Substitution resulted in a **more accurate** solution since the solution contained fractions.
- Substitution was more efficient than elimination because a **variable** was already **isolated** and could be quickly substituted into the other equation.

Checkpoint

Given the following system, explain what method would be most efficient to use and why. (Do not solve the system.)

$2x - 7y = 14$

$3x + 5y = 15$

Elimination would be the most efficient method since both equations are in standard form and none of the variables have a coefficient of one.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Would it be possible to graph this system without writing the equations in slope-intercept form? Explain.

A: *Yes, the system could be graphed using the x- and y-intercepts. However, it is possible the solution falls between grid lines which would make finding the solution by graphing difficult.*

Example 2

For problems like this, the solution to the system is only a partial answer. Your student must finish evaluating the expression to answer the problem completely.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Why is it important to write the solution before answering other parts of the problem?

A: *It is important to write the solution first so you can use the correct values when answering the rest of the problem.*

Your student can use mental math to solve B–D if they choose.

Extending Solutions

- Sometimes, you will be asked a question that cannot be answered until **the system has been solved**.
- First, solve the **system** using any method. Then, use the **values** found to complete the problem.

Example 2

Determine the quotient of y and x for the system of equations.

$y = 6x - 5$

$2x - 4y = -13$

Plan Solve for x and y using substitution.
Calculate the quotient.

Implement

$2x - 4(6x - 5) = -13$

$2x - 24x + 20 = -13$

$-22x = -33$

$x = \frac{-33}{-22}$

$x = \frac{3}{2}$

$y = 6\left(\frac{3}{2}\right) - 5$

$y = 9 - 5$

$y = 4$

$\left(\frac{3}{2}, 4\right)$

$\frac{y}{x} = \frac{4}{\frac{3}{2}}$

$\frac{y}{x} = (4)\left(\frac{2}{3}\right)$

$\frac{y}{x} = \frac{8}{3}$

Substitution

The quotient of y and x for the system of equations is $\frac{8}{3}$

Checkpoint

A) Name the solution. $(-2, 4)$

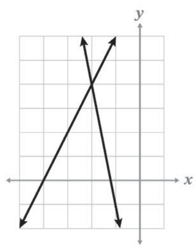
B) Determine the difference between y and x.

$y - x \quad 4 - (-2) = 6$

C) Determine the quotient of y and x.

$\frac{y}{x} \quad \frac{4}{-2} = -2$

D) Determine $3x + y$. $3(-2) + 4 = -2$



PRACTICE 1 17A

Practice 1

Complete the problems on a separate sheet of paper.

1) Graph.
$y = \frac{1}{2}x - 2$
$y = 2x - 15$ **(7, −1)**

2) Solve the system from problem 1 using a different method.

3) Look at problems 1 and 2. Which method of solving (graphing or your chosen method) was more efficient for the system? Explain.

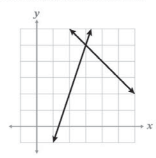
Use the most appropriate method to solve the system of equations.

4) $2x + 3y = 0$
$-5x + 3y = -21$ **(3, −2)**

5) $y = 2x + 8$
$y - 4 = x + 2$ **(−2, 4)**

6) Compare the method(s) used to solve problems 4 and 5. What method did you use to solve each problem? Why did you choose that method?

7) Given the graph of the system, name the solution.
Then, determine $2x - y$.



Match the system to the correct solution.

D **8)** $x + 3y = 37.5$
$4y - x = 39.5$

A **9)** $2x + 3y = 6$
$x = -\frac{3}{4}y$

A) $(-3, 4)$
B) $(11, 4.5)$
C) $(4, -3)$
D) $(4.5, 11)$

10) Use the solution to problem 8 to determine: $x - y$. **−6.5**

11) Use the solution to problem 9 to determine: $|x| - 3y$. **−9**

Explain when it is most efficient to use each method of solving a system.

12) Graphing

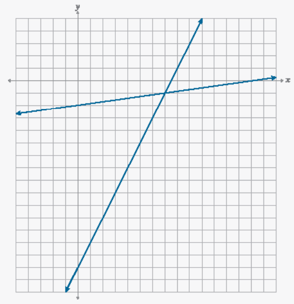
13) Substitution

14) Elimination

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1)



3) Problem 1:
Substitution was more efficient because a variable was already isolated.

Problem 2:
Graphing was more efficient because the slope was a fraction, and it is quicker to graph than clearing the fractions in the equations.

Not every system will have a method that is more efficient than another. As long as your student can explain their reasoning, they have successfully completed the problem.

4–5)
The method that your student chooses to solve may not match the key. It is fine if they solve using a different method as long as the solution is the same and they can explain the reasoning behind their chosen method.

6) Problem 4:
Elimination was used because the equations were both in standard form.

Problem 5:
Substitution was used because the first equation already had an isolated variable.

7) $(3, 5)$ and $2x - y$ is 1

8–9)
Your student could solve these problems by substituting the values given in A–D into the equations until both equations are true. Another option is to solve each equation and then select the correct answer choice.

Q: How can you determine the solution without solving?

A: *Use the answer choices and substitute them into both equations. If both equations are true, the correct solution has been found.*

11) Q: What is the notation around the *x*-value? What does it mean?

A: *The x-value is inside absolute value brackets. This means that the value of x will be positive.*

12–14)
Your student may respond with different reasons. This will help you understand the reasoning behind which method they choose when solving.

12) Sample:
Graphing is most efficient when equations are in slope-intercept form and when the slope is a fraction, but may not result in an integer solution.

13) Sample:
Substitution is most efficient when the equation has a variable that is already isolated or can be quickly isolated.

14) Sample:
Elimination is most efficient when the equations are in standard form or have terms that are aligned vertically and can be eliminated by multiplying one or both equations by a constant.

Mastery Check

Show What You Know

B) Q: What is the solution to the system?

A: *Your student should be able to tell you the x-coordinate is 4, but the y-coordinate cannot be determined exactly without solving algebraically.*

Q: How close was your estimate to the exact answer solved algebraically?

A: *Answers will vary.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Choose the best method to solve a system of linear equations and justify your choice.

Mastery Check

Show What You Know

A city was overlaid onto the coordinate plane. Traffic lights are being installed at the intersection of Pine Street and Mulberry Street.

The equations of two intersecting streets are given below.

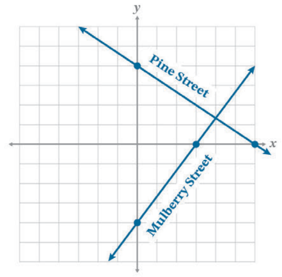
Pine Street: $2x + 3y = 12$

Mulberry Street: $-4x + 3y = -12$

A) Graph the equations using the x- and y-intercepts to show the streets and the intersection.

Pine Street:
$2a = 12$
$a = 3$
$3b = 12$
$b = 4$

Mulberry Street:
$-4a = -12$
$a = 3$
$3b = -12$
$b = -4$



B) The city planner wants the exact location where the streets intersect. Explain why this graph will not give the exact location.

Sample:
The exact intersection point cannot be found from the graph because it is not on exact grid lines.

C) Solve the system using another method to find where the traffic lights will be located.

$2x + 3y = 12$
$-4x + 3y = 12$
$(-1)(-4x + 3y = -12) \quad = \quad 4x - 3y = 12$
$+\ 2x + 3y = 12$
$6x = 24$
$x = 4$

$2(4) + 3y = 12$
$8 + 3y = 12$
$3y = 4$
$y = \frac{4}{3}$

$\left(4, \frac{4}{3}\right)$

Pine Street and Mulberry Street intersect at $\left(4, \frac{4}{3}\right)$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

PRACTICE 2 17A

Practice 2

Complete the problems on a separate sheet of paper.

1) Explain why graphing *would not* be the best approach to solving the system.

$y = 10x + \frac{8}{5}$
$y = -5x - \frac{7}{5}$

Solve using any method. State your chosen method before solving.

2) $y = 10x + \frac{8}{5}$
$y = -5x - \frac{7}{5}$ $\left(-\frac{1}{5}, -\frac{2}{5}\right)$

3) $2x + y = 8$
$y = -2x - 11$ no solution

4) How can you show what problem 3 *looks like* to prove that your solution is correct?

5) Solve.
$c - d = 0$
$c + 4d = 200$ $(40, 40)$

6) What method did you use to solve problem 5? Why did you choose that method?

Match the most efficient method of solving to the system.
Answer choices can be used more than once.

C 7) $5x - y = 58$
$8x - y = 97$

B 8) $y = -2x - 3$
$4x + y = -5$

C 9) $4x - 2y = 55$
$x - y = 9$

A) Graphing
B) Substitution
C) Elimination

10) Solve.
$5x - y = 58$
$8x - y = 97$ $(13, 7)$

11) Determine the difference between y and x from problem 10. -6

12) Solve.
$y = -2x - 3$
$4x + y = -5$ $(-1, -1)$

Practice 2

Worked solutions for these problems are located in the Digital Pack.

1) Graphing this system would be challenging because the y-intercepts are not integers. Unless you rescaled the graph to count by fifths, it would be difficult to accurately graph and find the intersection.

4) Sample:
You can show what a solution looks like by graphing. For problem 3, there would be a set of parallel lines on the graph, which proves there is no solution to the system.

6) Sample 1:
Elimination was used because the equations are in standard form. Multiplying one of the equations by four will eliminate d from the equations.

Sample 2:
Substitution was used because the top equation can quickly be rewritten as $c = d$. Then, c can be replaced by d, and the value of each variable can be found.

7–9)
It is fine if your student's answers vary, as long as they can justify their choices. For example, a student might choose substitution for problem 9 instead of elimination.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge

Q: How can you determine when you should use an equation rather than an expression?

A: *The word "is" tells you to use an equals sign.*

Part B: Applications of Linear Systems

Objectives

In this part of the lesson, you will learn about applications of linear systems.

By the end of this lesson, you will be able to do the following:

- Write systems of equations for how much and how many, coin, and wind and water word problems and solve them.
- Explain the solutions to how much and how many, coin, and wind and water problems and determine if the solutions are reasonable.

Why?

You have a bill and know the number of items you purchased but what was the individual cost per item? Being able to apply linear systems to real-world problems will help you solve situations like this one and many more.

Warm Up

Translate the words into an algebraic expression or equation.

1) Nine times some number less two. $9x - 2$

2) Two times the sum of x and y is eleven. $2(x + y) = 11$

3) The quotient of a number and three is seven $\frac{x}{3} = 7$

How Much or How Many

- Which type of word problem gets its name from the question being asked?

  How many or how much

- When solving these types of problems, remember the following:
  - You need at least two **equations** to solve a system.
  - Both **variables** need to be defined.
  - Any **method** can be used to solve for the correct solution.

EXPLORE 17B

Example 1

Use the given information to define your variables, write a system of equations, and solve.

A math test is worth a total of 100 points and contains 24 questions. The test includes some worked solution questions worth 5 points each and some multiple-choice questions worth 4 points each. How many of each question type are on the test?

Plan Define your variables. m: **multiple choice questions**
Write the equations. w: **worked solution questions**
Solve.

Implement

$4m + 5w = 100$ = 100 points

$m + w = 24$ = 24 questions

$(-4)(m + w = 24)$ = $-4m - 4w = -96$

$+\ 4m + 5w = 100$

$w = 4$

$m + w = 24$

$m + (4) = 24$

$m = 20$

Explain

There are 20 multiple choice questions and 4 worked solution questions.

> Remember to write your answer in a short sentence. Try saying the answer out loud to help you determine if the answer is possible and to help you find any errors.

Example 2

Use the given information to define your variables, write a system of equations, and solve.

The Humanitarian Club is selling two different types of bracelets for a charity. A bracelet with 4 red beads and 12 blue beads costs $40. A bracelet with 8 red beads and 8 blue beads costs $32. How much does each color of bead cost?

Plan Define your variables.
Write the equations.
Solve.

Implement

r: **red beads** b: **blue beads**

$4r + 12b$ = $40 bracelets

$8r + 8b$ = $32 bracelets

$4r + 12b = 40$

$8r + 8b = 32$

$(-2)(4r + 12b = 40)$ = $-8r - 24b = -80$

$+\ 8r + 8b = 32$

$-16b = -48$

$b = 3$

$8r + 8(3) = 32$

$8r + 24 = 32$

$8r = 8$

$r = 1$

Explain

The red beads cost $1, and the blue beads cost $3.

Example 2

Make sure your student pays careful attention to the directions to see if they only need to write the system of equations or solve the system as well.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What are the two totals in the system? Which total do the remaining values belong with? Explain.

A: *The totals are 38 vehicles and $336. The individual costs, $8 and $12, belong with the total amount of money, $336, because all of the costs should be grouped together.*

You and your student can do some hands-on problems with coins. Set up a jar with two types of coins. Give your student the total value and the total number of coins. Then, have your student solve the problem algebraically. Check by taking the coins out and counting to see if they are correct.

17B EXPLORE

Checkpoint

A car wash was held for a local charity. A total of 38 cars (c) and trucks (t) were washed, and $336 was raised. If it cost $8 to wash a car and $12 to wash a truck, how many cars were washed? How many trucks were washed?

$8c + 12t = 336$

$(c + t = 38)(-8) =$

$$\begin{aligned} 8c + 12t &= 336 \\ + \quad -8c - 8t &= -304 \\ \hline 4t &= 32 \\ t &= 8 \end{aligned}$$

$c + 8 = 38$

$c = 30$

Thirty cars and eight trucks were washed.

Coins

- Word problems that mention **coins** are a special type of "how much/how many" problem.
- For coin problems, you must give extra attention to **place value**.
- When solving coin problems, write all monetary values in terms of **dollars**.
    - Nickel: **$0.05**
    - Dime: **$0.10**
    - Quarter: **$0.25**

Example 3

Define your variables, write a system of equations, and solve.

A coin jar contained $6.30 in nickels and quarters. If there were 26 coins, how many quarters were in the jar?

n: **nickels (0.05)** q: **quarters (0.25)**

This problem has two total values: **$6.30** and **26 coins**.

$n + q = 26$

$0.05n + 0.25q = 6.30$

$$\begin{aligned} (-0.05)(n + q = 26) = -0.05n - 0.05q &= -1.30 \\ + \; 0.05n + 0.25q &= 6.30 \\ \hline 0.20q &= 5.00 \\ q &= 25 \end{aligned}$$

The jar has 25 quarters.

> You could also clear the decimals before solving by multiplying by the LCD, in this case 100.

EXPLORE 17B

Checkpoint

A coin jar contained $1.20 in nickels and dimes.
If there were 18 coins, how many nickels and dimes were in the jar?

n: nickel (0.05), d: dime (0.10)

$n + d = 18$

$0.05n + 0.10d = 1.20$

$$(-0.10)(n + d = 18) = -0.10n - 0.10d = 1.80$$
$$+\ 0.05n + 0.10d = 1.20$$
$$-0.05n = -0.60$$
$$n = 12$$

$n + d = 18$

$(12) + d = 18$

$d = 6$

There are 12 nickels and 6 dimes in the jar.

Wind and Water

- Wind and water problems usually involve solving for the **rate**, **distance**, or **time** of a plane in the wind or a boat in the water.
- The formula for wind and water problems is $d = tr$.
- The rate, r, is the **sum or difference** of the speed of the plane and the wind or the boat and the water.

> You may often see the formula $d = rt$. The order of rate and time does not affect the final values, but placing time first for this type of problem can make it simpler to distribute the values.

- Wind and water words used to describe rates (r):

r: rate	airplane	boat/kayak/canoe	effect
$p - w$	headwind	upstream (against the current)	slows down (plane/boat)
$p + w$	tailwind	downstream (with the current)	speeds up (plane/boat)

- Time increments will be in terms of **hours** since the speed of something is usually miles per hour (mph) or kilometers per hour (km/h).

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Some problems will ask students to find the number of one type of coin, and some problems will ask students to find the number of both types of coins. Remind your student to read the directions carefully.

Q: What are the two totals given in the problem?

A: *$1.20 and 18 coins.*

Q: How will you solve this system?

A: *Sample by eliminating dimes from the system first.*

There are many ways to solve a system. The work shown demonstrates eliminating d first.

17B EXPLORE

Example 4

Write a system of equations and use it to answer the given questions.

When flying into a headwind, a plane took 75 minutes to travel 450 km. With a tailwind, the same trip was 15 minutes shorter. What was the speed of the plane and the airspeed?

Define variables:
p: plane, w: wind

Convert all time to hours:
$75 \text{ min} = 1\frac{1}{4} \text{ hr} = \frac{5}{4} \text{ hr}$
$75 - 15 = 60 \text{ min} = 1 \text{hr}$

A) Create the system of equations using known facts:

d	t	rate
450	$\frac{5}{4}$	$(p - w)$ headwind
450	1	$(p + w)$ tailwind

↓ ↓ ↓

$450 = \frac{5}{4}(p - w)$ The equation for the trip in a headwind.

$450 = 1(p + w)$ The equation for the trip in a tailwind.

Note that the distance is the same in both equations because the only difference between trips was the rate and the time.

$$\left(\frac{4}{5}\right) 450 = \left(\frac{4}{5}\right)\left(\frac{5}{4}(p - w)\right)$$

$$\begin{aligned} 360 &= p - w \\ + 450 &= p + w \\ \hline 810 &= 2p \\ p &= 405 \end{aligned}$$

$$\begin{aligned} 450 &= p + w \\ 450 &= (405) + w \\ w &= 45 \end{aligned}$$

The plane's speed was 405 km/h, and the wind speed was 45 km/h.

B) What is the rate when the plane has a tailwind?

$r = p + w$ in a tailwind
$r = 405 + 45 = 450$ **km/h**

C) What is the rate when the plane encounters a headwind?

$r = p - w$ in a headwind
$r = 405 - 45 = 360$ **km/h**

D) What is the speed of the plane in still air (no wind)?

$r = 405$ **km/h**

EXPLORE 17B

Example 5

Write a system of equations and use it to answer the given questions.

The Chu family went on a canoe tour of Beautiful Canyon. The twelve-mile trip took four hours to travel upstream and 1 hour and 20 minutes to travel downstream. How fast was the family paddling? How fast was the water on the day of their trip?

p: canoe, w: water

1 hour 20 minutes = **$1\frac{1}{3}$ hours = $\frac{4}{3}$ hours**

$d = tr$

d	t	rate
12	**4**	**$(p - w)$** heading upstream
12	**$\frac{4}{3}$**	**$(p + w)$** heading downstream

$12 = 4(p - w)$

$12 = \frac{4}{3}(p + w)$

$\left(\frac{1}{4}\right)(12 = 4(p - w)) = \quad 3 = p - w$

$\left(\frac{3}{4}\right)(12 = \frac{4}{3}(p + w)) = +\ 9 = p + w$

$12 = 2p$

$p = 6$

$9 = 6 + w$

$w = 3$

The Chu family paddled 6 mph, and the current was 3 mph.

☑ Checkpoint

A plane flew 450 km/h. The wind speed was 25 km/h.

A) What is the speed of the plane in still air?
450 km/h

B) How fast would the plane be traveling with a tailwind?
$450 + 25 = 475$ km/h

C) How fast would the plane be traveling with a headwind?
$450 - 25 = 425$ km/h

D) What is the flight time with a tail wind for a distance of 1,900 km?
4 hours $\frac{1{,}900 \text{ km}}{(475 \text{ km/h})} = 4$ hr

E) What is the flight time for a plane with a headwind traveling 850 km?
2 hours $\frac{850 \text{ km}}{(425 \text{ km/h})} = 2$ hr

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Does a tailwind make the plane move faster or slower?

A: *A tailwind makes the plane move faster through the air because it is pushing it along.*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) t: tomatoes, g: garlic

$8.5t + 0.5g = 26.75$

$12t = 30$

$t = 2.50,\ g = 11.00$

Q: Why is it helpful to use a variable that starts with the same letter as what you are solving for?

A: *Using the same letter for the variable helps you identify what is being solved for.*

Since your student has already mastered solving systems of equations in various forms, they could practice by writing the system of equations and defining variables rather than solving every question.

2) x, y: grandmothers

$x + y = 167$

$x - y = 7$

Q: Why would negative numbers not make sense for the solution?

A: *Since you are solving for a person's age, the numbers should be positive values.*

3) j: jeans, h: shirt

$2j + 5h = 98$

$3j + 7h = 141$

$j + h = \$31$

$h = 12, j = 19$

It will cost $31 to purchase a pair of jeans and a shirt.

4) f: floor, m: mezzanine

$f + m = 525$

$m = 2f$

There were 175 floor seats and 350 mezzanine seats sold.

Q: Why do your values need to be whole numbers for this problem?

A: *You cannot sell a fraction of a seat for a concert.*

5) M: Mark, S: Steven

$M + S = 3{,}974$

$S - M = 6$

Q: Why would Mark's birth year be subtracted from Steven's if Steven was born after Mark?

A: *Since years increase as time does, Steven will have a larger birth year because he is the younger brother.*

6) x: 2-point questions

y: 10-point questions

$2x + 10y = 100$

$x + y = 30$

7) Q: Why does traveling upstream result in a different speed than downstream?

A: *Upstream means you are paddling against the current and would be slowed down. Downstream gives an extra push to the paddler.*

9) The total trip will take 6 hours. It will take 5 hours to travel upstream at a rate of 1 km/h. It will take 1 hour to travel 5 km back downstream: $5 + 1 = 6$ hours.

12) The total trip will take 8 hours. It will take 6 hours to travel upstream at a rate of 3 mph. It will take 2 hours to travel 18 miles back downstream: $6 + 2 = 8$

Q: Why might the speed of the airplane or the wind need to be rounded?

A: *Speed is usually measured in whole number values.*

13) d: dime, n: nickel

$d + n = 30$

$0.10d + 0.05n = 2.10$

Q: What are the two totals in the problem?

A: *The totals are 30 coins and $2.10.*

Q: If you do not want decimals in your equation(s), what can you multiply by when you have coin problems?

A: *Since coins are out of 100, multiply by 100 to clear the decimals from the equation(s).*

14) r: letter, p: postcard stamp

$0.55r + 0.35p = 38.00$

$p = r - 20$

Calia bought 50 letter stamps and 30 postcard stamps

15) e: ten-dollar bill

w: twenty-dollar bill

$e + w = 20$

$10e + 20w = 330$

Q: Why will the equations not require any decimals in problem 15?

A: *Dollar bills represent whole number increments.*

Practice 1

Complete the problems on a separate sheet of paper.

Write the system of equations, define your variables, and solve.

1) Bailey was at the farmers' market. She wanted to buy tomatoes and garlic to make pasta sauce. At the first farm stand, she purchased 8.5 pounds of tomatoes and 0.5 pounds of garlic for $26.75. At the second farm stand, she purchased 12 pounds of tomatoes for $30. What was the price per pound for tomatoes? What was the price per pound for garlic? (Assume the price per pound of tomatoes was the same at both stands.) **The tomatoes were $2.50/pound, and the garlic was $11/pound.**

2) The sum of the ages of Balthazar's two grandmothers is 167. The difference between their ages is 7. How old are Balthazar's grandmothers? **The ages of Balthazar's grandmothers are 80 and 87.**

3) Xavier purchased two pairs of jeans and five shirts for $98. Yoel purchased three pairs of jeans and seven shirts for $141. What would the cost be for one pair of jeans and one shirt?

4) The music hall sells two ticket levels for concerts, floor and mezzanine. A total of 525 seats were sold for the Saturday night show. There were twice as many mezzanine seats as floor seats. How many floor tickets were sold? How many mezzanine tickets were sold?

5) When the birth years of Mark and Steven are added, the sum is 3,974. Steven was born 6 years after Mark. What year was each person born? **Mark was born in 1984, and Steven was born in 1990.**

6) A math test worth 100 points and containing 30 questions was assigned to the class. Some questions were worth 2 points, and some were worth 10 points. How many questions of each type were on the test? **There were 5 questions worth 10 points each and 25 questions worth 2 point each.**

Malia rows her kayak at a rate of 3 km/h, and the current is 2 km/h.

7) How fast would Malia be able to move upstream? **$3 - 2 = 1$ km/h**

8) How fast would Malia be able to move downstream? **$3 + 2 = 5$ km/h**

9) How long will it take to travel 5 kilometers upstream and back? Explain.

A crew team was rowing 6 mph in a waterway with a current of 3 mph.

10) How fast would the team be able to move upstream? **$6 - 3 = 3$ mph**

11) How fast would the team be able to move downstream? **$6 + 3 = 9$ mph**

12) How long will it take to travel 18 miles upstream and back? Explain.

Write the system of equations, define the variables, and solve.

13) Josef collected dimes and nickels. He had 30 coins that totaled $2.10. How many of each coin does Josef have? **There are 12 dimes and 18 nickels.**

14) When Calia went to the post office, the price of a stamp for a letter was $0.55, and the price of a stamp for a postcard was $0.35. If Calia bought twenty fewer postcard stamps than letter stamps and spent $38 total, how many of each type of stamp did Calia buy?

15) Logan has a total of 20 bills in his wallet. He has ten-dollar bills and twenty-dollar bills totaling $330. How many of each bill does Logan have? **Logan has 7 ten-dollar bills and 13 twenty-dollar bills.**

16) An airplane traveling 2,400 miles from Philadelphia to Los Angeles takes 6 hours. The return trip takes 5.5 hours. What is the speed of the plane? What is the average speed of the wind? Round to the nearest unit. **The plane's speed is about 418 mph, and the speed of the wind is 18 mph.**

MASTERY CHECK 17B

Mastery Check

Show What You Know

The combined age of Sara and Elizabeth was 72. If Elizabeth's age were doubled, she would be 30 years older than Sara.

A) Define your variables and write a system of equations to represent the given scenario.

S: Sara, E: Elizabeth

$S + E = 72$

$2E = S + 30$

B) What are the ages of each person? Use your system from part A to solve.

$$\begin{aligned} S + E &= 72 \\ + -S + 2E &= 30 \\ \hline 3E &= 102 \\ E &= 34 \end{aligned} \qquad \begin{aligned} S + (34) &= 72 \\ S &= 38 \end{aligned}$$

Sara is 38 years old. Elizabeth is 34 years old.

C) Sara and Elizabeth emptied their pockets of all their loose change and found 13 coins. They only had dimes and quarters totaling $2.65. Define your variables and write a system of equations representing the coins Sara and Elizabeth have. Do not solve.

d: dimes q: quarters

$d + q = 13$

$0.10d + 0.25q = 2.65$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Your student may need to use "a" for Sara rather than "s" to avoid confusing "s" and the number 5.

B) Q: Why might elimination be more efficient than substitution for this problem?

A: *There is not an isolated variable in the problem. If you subtract Sara's age from both sides of the second equation, it will be eliminated when the equations are added together.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Write systems of equations for how much and how many, coins, and wind and water word problems and solve them.
- Explain the solutions to how much and how many, coins, and wind and water system problems and determine if the solutions are reasonable.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

If your student is not sure how to begin, start by asking them what type of word problem they are working with: how much, how many, coins, or wind and water.

Your student can practice writing the systems of equations rather than solving them, or only solve a few problems.

1) x: multiple-choice questions
y: open-response questions
$x = 3y$
$x + y = 16$

2) r: Tori, h: Thomas
$r - h = 3$
$\frac{1}{3}r - h = -5$

3) a: apples, p: spinach
$5a + 6p = 37.95$
$3a + 5p = 28.72$

4) b: brush, p: paint
$2b + 5p = \$52.25$
$b + 7p = \$58.75$

5) b: brother, r: sister
$b + b = 4$
$b = 2b - 2$

6) a: adult, c: children
$25a + 16c = 219$
$c = a + 6$

Kelly Miller was a scientist, mathematician, and author. He attended Howard University and Johns Hopkins University and made many notable accomplishments throughout his education and career.

7) d: dimes, n: nickels
$d + n = 18$
$0.10d + 0.05n = 1.25$

14) f: five-dollar bill, n: one-dollar bill
$f = 6n - 2$
$5f + 1n = 83$

15) d: dime, h: half-dollar
$0.10d + 0.50h = 4.40$
$0.50h - 0.10d = 2.60$

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

17B PRACTICE 2

Practice 2

Complete the problems on a separate sheet of paper.

Write the system of equations, define your variables, and solve.

1) A math test has sixteen questions made of multiple-choice questions and open-response questions. There were three times as many multiple-choice questions as open-response questions. How many multiple-choice questions and open-response questions were each on the test? **There were 12 multiple-choice questions and 4 open-response questions.**

2) The difference between the ages of Tori and Thomas is three years. One-third of Tori's age minus Thomas' age is equal to negative five years. How old are Tori and Thomas? **Tori is 12 years old, and Thomas is 9 years old.**

3) The Simensons went to the store to purchase food for school lunches. On their first trip, they purchased 5 pounds of apples and 6 bags of spinach for a total of \$37.95. For their next trip, they purchased 5 bags of spinach and 3 pounds of apples for \$28.72. Assuming the prices were the same for each trip, what is the cost of a pound of apples? What is the cost of a bag of spinach? **One pound of apples costs \$2.49, and one bag of spinach costs \$4.25.**

4) An artist needed more supplies to finish a project. They purchased two brushes and five tubes of paint for \$52.25. Then, they purchased one brush and seven tubes of paint for \$58.75. What was the cost of a tube of paint for the project? **One tube of paint is \$7.25.**

5) Magda has a younger brother and younger sister. The sum of their ages is four. Her brother is twice her sister's age minus two. How old are Magda's siblings? **Magda's brother and sister are both two years old. They are twins.**

6) The Kelly family and the Miller family went to brunch. Adult meals cost \$25, and meals for children under 12 were \$16. The two families spent a total of \$219. If six more children than adults attended, how many meals were purchased for children? **There were 9 meals purchased for children.**

7) Ella and Claire had a combination of 18 dimes and nickels totaling \$1.25. How many of each coin did the girls have? **Ella and Claire have 7 dimes and 11 nickels.**

A plane flew 575 mph. The wind speed was 35 mph.

8) How fast would the plane be traveling with a tailwind? **575 + 35 = 610 mph**

9) How fast would the plane be traveling with a headwind? **575 − 35 = 540 mph**

10) What is the flight time with a tail wind for a distance of 1,525 miles? **2.5 hours**

11) What is the flight time for a plane with a headwind traveling 1,755 miles? **3.25 hours**

Write the system of equations, define the variables, then solve.

12) A park ranger traveled across a lake and back in his boat. He traveled 32 kilometers in one direction, which took 4 hours. His return trip was the same distance and time. What was the speed of his boat? What was the speed of the water? **The boat traveled at a speed of 8 km per hour. The water had a speed of 0 km per hour.**

13) A plane trip from New York to London takes 7 hours. The return flight takes approximately 8 hours to fly the same distance of 3,500 miles. What is the speed of the plane? **The plane is traveling 468.75 mph.**

14) Fabian was counting his commission for doing chores for the past three months. He was paid in five-dollar bills and one-dollar bills. Fabian earned \$83. The number of five-dollar bills was six times the number of one-dollar bills, less two. How many of each bill does Fabian have? **Fabian has 16 five-dollar bills and 3 one-dollar bills.**

15) Burt has \$4.40 in dimes and half-dollar coins. The difference in value between half-dollars and dimes is \$2.60. How many of each coin does Burt have? **here are 9 dimes and 7 half-dollars.**

TARGETED REVIEW 17

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Graph the system of inequalities.
$y > 2x - 3$
$y \le x + 4$

2) Name the quadrant or quadrants where the solution to the system in problem 1 is located.
The solution to the system of inequalities is in all four quadrants.

3) Solve the system of equations.
$x + y = 3$
$x = -\frac{1}{3}y - 2$ $\left(-\frac{9}{2}, \frac{15}{2}\right)$

4) Solve the system of equations.
$7x + 2y = 12$
$7x + 4y = 3$ $(3, -4.5)$

Classify the numbers by their most specific name.

5) $(-7)(-2.3)$: **rational**
6) $\frac{1}{2}\sqrt{2}$: **irrational**
7) $83 - 83$: **whole**
8) $-26 + 3$: **integer**

natural	**whole**
integer	**rational**
irrational	**real**

Classify the equations of lines given in the box as parallel, perpendicular, or neither.

9) $f(x)$ and $g(x)$ **neither**
10) $g(x)$ and $j(x)$ **perpendicular**
11) $h(x)$ and $j(x)$ **neither**
12) $f(x)$ and $h(x)$ **parallel**
13) $g(x)$ and $h(x)$ **neither**

$f(x) = \frac{3}{2}x + 7$
$g(x) = -\frac{3}{2}x - 3$
$h(x) = \frac{3}{2}x + 1$
$j(x) = \frac{2}{3}x - 3$

Convert minutes into hours or fractions of an hour.
Write fractions greater than one as an improper fraction.

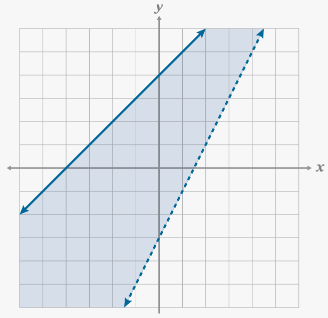
14) 45 minutes: $\frac{3}{4}$ **hour**
15) 100 minutes: $\frac{5}{3}$ **hour**
16) 42 minutes: $\frac{7}{10}$ **hour**
17) 75 minutes: $\frac{5}{4}$ **hour**

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

1)



Problem	1	2	3	4	5–8	9–13	14–17	18	19	20	21	22
Lesson Origin	15	15	16	16	1	12	5	1, 2	10	11	15	16

18) $4(9 - x) + 6x = 20$
$36 - 4x + 6x = 20$ Distributive Property
$36 + 2x = 20$ Combine like terms
$2x = -16$ Addition Property of Equality
$x = -8$ Multiplication Property of Equality

19) $d(t) = 55t$. A slope of 55 represents the speed of 55 mph. The y-intercept is 0 because at the start of Saul's trip, he had not traveled anywhere yet.

21) Distractor Rationale:
A) is a point in Quadrant II.
C) is a point in Quadrant IV.
D) is a point in Quadrant I.

22) Distractor Rationale:
A) This is the product of the constants in the given equations
B) This is the difference of x and y for $(7, 11)$.
C) This is the sum of x and y for $(7, 11)$.

18) Solve. Justify your steps with the properties.

$4(9 - x) + 6x = 20$

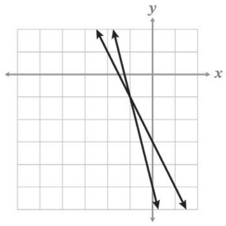
19) Saul was going on a road trip. He started his trip at his house and drove 55 miles per hour. He used the equation $d(t) = 55t$ to find how many hours, t, it would take to travel various distances, $d(t)$. Explain the meaning of the slope and the y-intercept from the word problem.

20) Find the x- and y-intercepts for the equation: $3x - 15y = 45$

(15, 0) **(0, −3)**

Multiple choice

B **21)** Name the solution to the system of equations.



A) $(-1, 1)$

B) $(-1, -1)$

C) $(1, -1)$

D) $(1, 1)$

D **22)** Find the product of x and y using the solution to the system of linear equations.

$y = 2x - 3$
$y = x + 4$

A) -12

B) -4

C) 18

D) 77

Problem	1	2	3	4	5–8	9–13	14–17	18	19	20	21	22
Lesson Origin	15	15	16	16	1	12	5	1, 2	10	11	15	16