Lesson 16
Solving Systems of Equations Algebraically

Outline

Part A
Solving Systems Using Substitution

- Substitution when Both Equations Have an Isolated Variable
- Substitution: One Variable Isolated
- Substitution: No Variables Isolated

Part B
Solving Systems Using Elimination

- Elimination: Opposite Coefficients
- Elimination: Multiplying by −1
- Elimination: Linear Combinations

Targeted Review

Vocabulary

- linear combinations

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Q: How is a solution to one equation different from the solution to a system?

A: *A solution to one equation lies on one line. The solution to a system must lie on both lines.*

16A EXPLORE

Part A: Solving Systems Using Substitution

Objectives

In this part of the lesson, you will learn about solving systems using substitution.

By the end of this lesson, you will be able to:

- Use substitution to solve a system of equations when both equations have an isolated variable.
- Use substitution to solve a system of equations when one equation has an isolated variable.
- Use substitution to solve a system of equations when none of the equations have an isolated variable.

Why?

Knowing how to solve systems of equations using substitution helps you make connections in algebra. When you use substitution, you can easily see those connections.

Warm Up

Determine if the point is a solution for the given equation.

1) Is $(-1, 5)$ a solution for $y = 3x + 2$? Explain.

No, this is not a solution. When $(-1, 5)$ is substituted into the equation, the equation is not equal on both sides: $5 = 3(-1) + 2, 5 = -3 + 2, 5 \neq -1$

2) Is $(2.5, -6)$ a solution for $2x + 3y = -13$? Explain.

Yes, this is a solution. When $(2.5, -6)$ is substituted into the equation, the equation is equal on both sides: $2(2.5) + 3(-6) = -13, 5 + -18 = -13, -13 = -13$

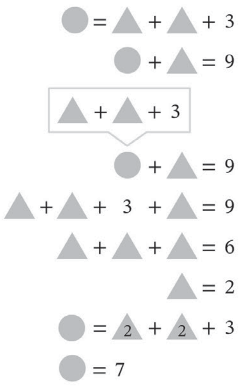
Substitution when Both Equations Have an Isolated Variable

- To solve a system using substitution you must have an **isolated variable**.
- The isolated variable is set equal to an expression. You can use this expression to **replace** the variable anywhere it appears in the second equation.

EXPLORE 16A

Example 1

This picture equation can be written in symbols using c for circles and t for triangles.



$c = 2t + 3$

$c + t = 9$

substitution

$3t + 3 = 9$

$3t = 6$

$t = 2$

$c = 2t + 3$ or $c = 2(2) + 3$

$c = 7$

Solving using substitution follows these general steps:

1) Use an **isolated** variable from one equation to substitute an expression into the remaining equation in the system.

2) Solve the resulting equation for the one **remaining** variable.

3) Use the value of the variable found to make a second substitution into **either** of the original equations to find the value of the second variable.

4) Write the solution to your system as an **ordered pair**.

Example 1
Remember that the solution to a system of equations is the ordered pair that makes *both* equations true.

If a system uses variables other than x and y, the ordered pair is written in alphabetical order.

16A EXPLORE

Example 2

Solve the system of equations using substitution.

$y = 4x - 5$
$y = -2x - 2$

Solution: $\left(\frac{1}{2}, -3\right)$

Plan In a system of equations, the value of the variables must be the same.
$y = y$ (Reflexive Property)
The equations can equal one another using substitution.
Solve for y.
Solve for x.

Implement	Explain
$4x - 5 = -2x - 2$	Substitution
$+2x \quad +2x$	Addition Property of Equality
$+5 \quad +5$	Addition Property of Equality
$\frac{6x}{6} = \frac{3}{6}$	Multiplication Property of Equality
$x = \frac{1}{2}$	

Equation 1	Equation 2
$y = 4x - 5$	$y = -2x - 2$
$y = 4\left(\frac{1}{2}\right) - 5$	$y = -2\left(\frac{1}{2}\right) - 2$
$y = 2 - 5$	$y = -1 - 2$
$y = -3$	$y = -3$

Since the same value was found by checking the equation, the solution is $\left(\frac{1}{2}, -3\right)$.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Make sure that your student writes the answer to the system as an ordered pair.
They may need a reminder now that they are solving systems algebraically and cannot use the graph as a clue that an ordered pair is needed.

Q: Does the ordered pair make both equations in the system true? Explain.

A: *Yes, because when you substitute 9 for x in both equations, the result is 4.*

Checkpoint

Solve the system of equations using substitution.

$y = -x + 13$
$y = x - 5$

$-x + 13 = x - 5$	$y = x - 5$
$13 = 2x - 5$	$y = (9) - 5$
$18 = 2x$	$y = 4$
$x = 9$	

$(9, 4)$

EXPLORE 16A

Substitution: One Variable Isolated

- When one variable is isolated, **substitution** is a common way to solve a system of equations.

Example 3

Solve the system of equations using substitution.

$a = \frac{1}{2}b - 1$
$4a + 3b = 15$

Solution: $\left(\frac{9}{10}, \frac{19}{5}\right)$

Plan Substitute the right side of the first equation into the second equation for a.
Solve for b.
Substitute the value for b in the first equation.
Solve for a.
Substitute the values for a and b into both equations to check.

Implement	Explain	Check
$4\left(\frac{1}{2}b - 1\right) + 3b = 15$	Substitution	$4\left(\frac{9}{10}\right) + 3\left(\frac{19}{5}\right) = 15$
$2b - 4 + 3b = 15$	Distributive Property	$\frac{36}{10} + \frac{57}{5} = 15$
$+4 \quad +4$	Addition Property of Equality	$\frac{18}{5} + \frac{57}{5} = 15$
$\frac{5b}{5} = \frac{19}{5}$	Multiplication Property of Equality	$\frac{75}{5} = 15$ ✓
$b = \frac{19}{5}$		
$a = \frac{1}{2}\left(\frac{19}{5}\right) - 1$	Substitution	
$a = \frac{19}{10} - \frac{10}{10}$	Least Common Denominator	
$a = \frac{9}{10}$		
Solution: $\left(\frac{9}{10}, \frac{19}{5}\right)$	Alphabetically a is before b, so the ordered pair will be (a, b).	

Checkpoint

Solve the system using substitution. Check your work.

$y = 10x - 6$
$5x - 2y = 7$

$\left(\frac{1}{3}, -\frac{8}{3}\right)$

$5x - 2(10x - 6) = 7$
$5x - 20x + 12 = 7$
$-15x = -5$
$x = \frac{5}{15} = \frac{1}{3}$

$y = 10\left(\frac{1}{3}\right) - 6$
$y = \frac{10}{3} - \frac{18}{3}$
$y = -\frac{8}{3}$

Check:
$5\left(\frac{1}{3}\right) - 2\left(-\frac{8}{3}\right) = 7$
$\frac{5}{3} + \frac{16}{3} = 7$
$\frac{21}{3} = 7$ ✓

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Make sure that −2 is being distributed across the expression, not 2.

Q: What is another way to represent y in the system?
A: $(10x - 6)$

Q: How do you check your solution to a system?
A: *Substitute the ordered pair back into both equations.*

16A EXPLORE

Substitution: No Variables Isolated

- If neither equation in a system has an isolated variable, a **single variable** in either equation must be **isolated** in order to solve using substitution.

Example 4

Solve the system of equations using substitution.

$2x - 3y = 4$
$4x + y = -6$

Solution: (−1, −2)

Plan Isolate one of the variables.
Substitute the variable in the other equation.
Solve for the variable.
Then substitute that result into one of the equations.
Solve for the other variable.
Write the solution as an ordered pair.

Implement	Explain
$4x + y = -6$	
$-4x \quad -4x$	Addition Property of Equality
$y = -4x - 6$	
$2x - 3y = 4$	Substitution
$2x - 3(-4x - 6) = 4$	Distributive Property
$2x + 12x + 18 = 4$	
$-18 \quad -18$	Addition Property of Equality
$14x = -14$	Multiplication Property of Equality
$x = -1$	
$4x + y = -6$	
$4(-1) + y = -6$	Substitution
$-4 + y = -6$	Addition Property of Equality
$y = -2$	

EXPLORE 16A

- Remember from Lesson 15, a system of equations can also have **zero solutions** (parallel lines) or **infinite solutions** (coincident lines).
- When solving a system of **parallel** lines algebraically, the variables will simplify out, and the result will be an equation with no solution.

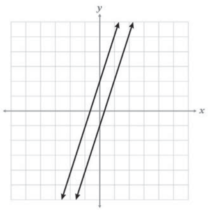
Example 5

Solve the systems of equations using substitution.

Parallel Lines:

$y = 3x - 1$

$y = 3x + 2$



$3x - 1 = 3x + 2$

$-3x \quad -3x$

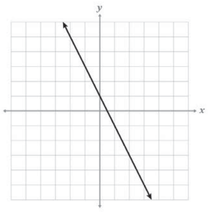
$-1 = 2$

no solution

Coincident Lines:

$2x + y = 1$

$y = -2x + 1$



$2x + (-2x + 1) = 1$

$1 = 1$

infinite solutions on the line

Checkpoint

Solve the system using substitution.

$x - 2y = 7$
$x + y = 4$

Solution: (5, −1)

$x - 2y + 2y = 7 + 2y$

$x = 2y + 7$

$x + y = 4$

$(2y + 7) + y = 4$

$3y + 7 = 4$

$3y = -3$

$y = -1$

$x + y = 4$

$x + (-1) = 4$

$x = 5$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Is y the only variable that you can isolate?

A: *No, you can isolate either variable.*

Q: Does it matter what equation is written first or second? Explain.

A: *No, the order of the equations does not matter and can be switched if needed.*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) Q: How could this problem be written using symbols/algebraically?

A: *Sample:*
All circles could be c, and triangles could be t. The top equation would be $2c + 3t = 25$.

(Your student may use variables instead of shapes if they prefer.)

2) (Your student may use variables instead of shapes if they prefer.)

4) Q: What order do you write an ordered pair in that has variables other than x and y?

A: *Alphabetical order is used. For example,* (a, b).

5) Q: What are the different types of solutions to a system, and what do they look like on a graph?

A: *One solution (an ordered pair), no solutions (parallel lines), and infinite solutions (coincident lines).*

9) Q: What variable makes the most sense to isolate? Explain.

A: *In the first equation, a makes the most sense because its coefficient is 1. It can be isolated by adding* $8b$ *to both sides.*

You can isolate either variable. The suggestion is to try and do this in the fewest number of steps to eliminate the potential for errors.

11) Q: Why would graphing and determining the exact solution be challenging?

A: *Sample:*
The values are fractions, so it would be difficult to approximate these exact values on a graph.

Practice 1

Complete the problems on a separate sheet of paper.

Solve the system of equations using substitution.

1) ● + ● + ▲ + ▲ + ▲ = 25
● = − ▲ + 11
Solution: ● = **8**
▲ = **3**

2) ● + ● + ● + ■ = −2
■ = − ● + 4
Solution: ■ = **7**
● = **−3**

3) $y = 2x - 11$
$y = \frac{1}{2}x + 4$ **(10, 9)**

4) $a = 2b + 5$
$a + b = 6$ $\left(5\frac{2}{3}, \frac{1}{3}\right)$

5) $y = -3x - 2$
$3x + y = -5$ **No solution (parallel lines)**

6) $x + 2y = -4$
$y = x + 1$ **(−2, −1)**

7) $y = \frac{1}{2}x$
$x - 4y = -3$ $\left(3, \frac{3}{2}\right)$

8) $x - 7y = 0$
$3x + 8y = -29$ **(−7, −1)**

9) $a - 8b = 1$
$2a - 10b = 5$ $\left(5, \frac{1}{2}\right)$

10) $3x + y = 3$
$7x + 2y = 1$ **(−5, 18)**

11) $4c - 2d = 0$
$3c - 4d = -2$ $\left(\frac{2}{5}, \frac{4}{5}\right)$

12) $y = 4x - 4$
$x = -\frac{1}{2}y - 2$ **(0, −4)**

MASTERY CHECK 16A

Mastery Check

Show What You Know

Tom is in Algebra class and writes an equation to represent his age relative to his brother's age. Tom's age is twice his brother's age plus one year. The equation below represents Tom (T) and his brother (b).

$$T = 2b + 1$$

A) The total age of Tom and his brother is 19.
Write an equation to represent Tom (T) and his brother (b).

T: Tom, b: brother

$T + b = 19$

B) Write a system of equations to represent the age of Tom and his brother.
Remember to define the variables in the equations.

T: Tom, b: brother

$T = 2b + 1$

$T + b = 19$

C) What are the ages of Tom and his brother? Solve the system of equations from part B.

$T = 2b + 1$

$T + b = 19$

$(2b + 1) + b = 19$

$3b + 1 = 19$

$3b = 18$

$b = 6$

$T = 2(6) + 1$

$T = 13$

Tom is 13 years old, and his brother is 6 years old.

D) Tom and his brother have an Uncle Kevin who is three times Tom's age minus Tom's brother's age. Find the age of Uncle Kevin. Show your work.

K: Kevin

$K = 3T - b$

$K = 3(13) - (6)$

$K = 33$

Uncle Kevin is 33 years old.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

B) Q: Why might using x and y as variables be confusing for this problem?

A: *Sample:*
Because you might not know which variable represents Tom and his brother.

C) Try having your student write their answer in a sentence to help them determine if they answered the question completely.

D) The equation $K = 3T - b$ is optional but may help your student organize the information.

This answer is dependent on the solutions to part C. If your student has an error in part D, you should check their work based on their answers to part C.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Use substitution to solve a system of equations when both equations have an isolated variable.
- Use substitution to solve a system of equations when one equation has an isolated variable.
- Use substitution to solve a system of equations when none of the equations have an isolated variable.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

12) Recall that when a system has no solution, the lines are parallel.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

16A PRACTICE 2

Practice 2

Complete the problems on a separate sheet of paper.

Solve the system of equations using substitution.

1) ■ + ■ + 3 = ▲
■ + ▲ + ▲ = 16

Solution: ■ = 2
▲ = 7

2) $x = \frac{1}{3}y + 2$
$2x - 4y = 5$ $\left(\frac{19}{10}, -\frac{3}{10}\right)$

3) $a = 4b - 10$
$a = 3b + 11$ $(74, 21)$

4) $y = -\frac{5}{2}x - \frac{5}{2}$
$5x + 2y = -5$ **infinite solutions (coincident lines)**

5) $2x + 2y = 15$
$y = 9x + 5$ $\left(\frac{1}{4}, \frac{29}{4}\right)$

6) $m = 6n$
$2m + 8n = -15$ $\left(-\frac{9}{2}, -\frac{3}{4}\right)$

7) $4x + y = 5$
$2x - 3y = 13$ $(2, -3)$

8) $3e - f = -1$
$e - 3f = 1$ $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

9) $2p - q = -3$
$8p - q = -7$ $\left(-\frac{2}{3}, \frac{5}{3}\right)$

10) $50x + 25y = 200$
$x + y = 1$ $(7, -6)$

11) $e + 2f = 8$
$2e - 3f = -19$ $(-2, 5)$

12) $5x - 4y = -1$
$4y + 6 = 5x + 5$ **no solution**

EXPLORE 16B

Part B: Solving Systems Using Elimination

Objectives

In this part of the lesson, you will learn about solving systems using elimination.

By the end of this lesson, you will be able to do the following:

- Use elimination to solve a system of equations when one variable has a set of opposite coefficients.
- Use elimination to solve a system of equations by distributing −1 across an equation.
- Use elimination to solve a system of equations using linear combinations.

Why?

Using elimination to solve systems of equations is a very efficient method. It also helps reduce mistakes compared to other methods because you are eliminating a variable.

Warm Up

Simplify.

1) $3(2x-4)+8x$

$6x-12+8x$

$6x+8x-12$

$14x-12$

2) $-15x+4+y-2+15x$

$-15x+15x+y+4-2$

$y+2$

3) $-(8x-7)+2(4x-3)-1$

$-8x+7+8x-6-1$

$-8x+8x+7-6-1$

0

Elimination: Opposite Coefficients

- When a system of equations is written in **standard form**, the **elimination** method is the most efficient way to solve.
- The Additive Inverse Property, $a+(-a)=0$, is used to eliminate the chosen variable.
- When the coefficients of a variable are opposites, they will simplify out of the system when **added** together.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

1) Q: What property should you use first to simplify the terms?

A: *The Distributive Property.*

2) Q: Why can the expression not be written as one term when simplified?

A: *Only like terms may be combined, and a variable and a constant are not like terms.*

Q: What prevents you from *solving* for y?

A: *There is no equals sign, so you can only simplify.*

3) Q: What property lets you "cancel out" terms when you simplify an expression?

A: *The Additive Inverse Property.*

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What variable will you eliminate first when the equations are added together? Explain.

A: *The variable y will be eliminated using the additive inverse.*

Example 1

Solve the system of equations using elimination.

$$10x - 14y = 28$$
$$-10x + 18y = 4$$

Solution: (14, 8)

Plan	Implement	Explain
Eliminate opposite x terms. Solve for y. Substitute y to solve for x. Check solution.	$10x - 14y = 28$	Elimination
	$+ \; -10x + 18y = 4$	
	$4y = 32$	Multiplication Property of Equality
	$y = 8$	

Check

$10(14) - 14(8) = 28$	$10x - 14y = 28$	
$140 - 112 = 28$ ✓	$10x - 14(8) = 28$	Substitution
$-10(14) + 18(8) = 4$	$10x - 112 = 28$	Addition Property of Equality
$-140 + 144 = 4$ ✓	$10x = 140$	Multiplication Property of Equality
	$x = 14$	

Checkpoint

Solve the system of equations using elimination.

$x + y = 13$

$x - y = 5$

$$x + y = 13$$
$$+\, x - y = 5$$
$$2x = 18$$
$$x = 9$$

$$9 + y = 13$$
$$y = 4$$

$(9, 4)$

Elimination: Multiplying by −1

- The **Multiplication Property of Equality** is key in solving systems using the **elimination** method.
- If the same number is multiplied across **all terms** of an equation, the equation maintains equality.

Example 2

Solve the system of equations using elimination.

$-3x + 8y = -10$
$2x + 8y = -15$
$\downarrow$
$-3x + 8y = -10$
$+2x + 8y = -15$
$x + 16y = -25$ ✗

Solution: $\left(-1, -\frac{13}{8}\right)$

$-3x + 8y = -10$
$(-1)(2x - 8y = -15) = \ + \ -2x - 8y = 15$
$-5x = 5$
$x = -1$

$2(-1) + 8y = -15$
$-2 + 8y = -15$
$8y = -13$
$y = -\frac{13}{8}$

Check

$-3(-1) + 8\left(-\frac{13}{8}\right) = -10$
$3 - 13 = -10$ ✓
$2(-1) + 8\left(-\frac{13}{8}\right) = -15$
$-2 - 13 = -15$ ✓

Checkpoint

Solve the system of equations using elimination. Name the variable to be eliminated.

$2x - 3y = 6$
$7x - 3y = 16$

$\left(2, -\frac{2}{3}\right)$

Eliminate *y*.

$(-1)(2x - 3y = 6)$
$-2x + 3y = -6$
$+7x - 3y = 16$
$5x = 10$
$x = 2$

$2x - 3y = 6$
$2(2) - 3y = 6$
$4 - 3y = 6$
$-3y = 2$
$y = -\frac{2}{3}$

Elimination: Linear Combinations

- When linear combinations are used, **one or more** equations in the system are **multiplied** by a constant.
- Multiplying by a constant allows a variable to be **eliminated** when the equations are **added** together.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

The bottom equation could also be multiplied by -1 to eliminate y.

Q: Will terms with the same sign (e.g., $7x$ and $7x$) ever equal zero when added?
A: *No, if two terms have the same sign, they will not equal 0.*

Q: Should you eliminate x or y? Explain.
A: *You should eliminate y because if one of the equations is multiplied by -1, and the equations are added, y will cancel from the system.*

Steps to solving using elimination:

1) Make sure equations are all written in the **same** form.

2) Choose the **variable** to be eliminated and mark it (highlighter, arrow, etc.).

3) Determine the **least common multiple (LCM)** of the coefficients for the variable to be eliminated.

4) Multiply one or more equations by the **constant** to get the LCM. Make sure that the terms are opposites—one positive and one negative.

5) Add the equations together.

6) Solve for each variable one at a time.

7) Check by **substituting** the solution into both equations.

Example 3

Solve the system using linear combinations.

$8a + 3b = 4$ **(2, −4)**

$5b = -34 + 7a$

$8a + 3b = 4$

$-7a + 5b = -34$

$(-5)(8a + 3b = 4) = -40a - 15b = -20$

$(3)(-7a + 5b = -34) = + -21a + 15b = -102$

$-61a = -122$

$a = \frac{-122}{-61}$

$a = 2$

$8a + 3b = 4$, when $a = 2$

$8(2) + 3b = 4$

$16 + 3b = 4$

$3b = 4 - 16$

$3b = -12$

$b = -\frac{12}{3}$

$b = -4$

Check

$8a + 3b = 4$

$8(2) + 3(-4) = 4$

$16 - 12 = 4$ ✓

$5b = -34 + 7a$

$5(-4) = -34 + 7(2)$

$-20 = -34 + 14$ ✓

Checkpoint

Solve the system using linear combinations.
Name the variable to be eliminated and the least common multiple of the coefficients.

$6x + 2y = 33.50$
$5x + 7y = 45.25$

(4.50, 3.25)

Eliminate x, LCM (5, 6) = 30

$$(5)(6x + 2y = 33.50) = 30x + 10y = 167.50$$
$$(-6)(5x + 7y = 45.25) = -30x - 42y = -271.50$$
$$-32y = -104$$
$$y = 3.25$$

$$6x + 2(3.25) = 33.50$$
$$6x + 6.50 = 33.50$$
$$6x = 27$$
$$x = 4.50$$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Either variable can be eliminated to solve the system. Both are shown here, but only one method will be shown in the practice problems.

Q: What is the LCM (6, 5)?
A: *30*

Q: What is the LCM (2, 7)?
A: *14*

OR

Eliminate y, LCM (2, 7) = 14

$$(7)(6x + 2y = 33.50) = 42x + 14y = 234.50$$
$$(-2)(5x + 7y = 45.25) = -10x - 14y = -90.50$$
$$32x = 144$$
$$x = 4.50$$

$$6(4.50) + 2y = 33.50$$
$$27 + 2y = 33.50$$
$$2y = 6.5$$
$$y = 3.25$$

Practice 1

Worked solutions for these problems are located in the Digital Pack.

Either variable can be eliminated when solving. The worked solutions show one method using elimination. If your student starts by eliminating the other variable and has the same solution, they have correctly solved the system.

Either fraction or decimal answers can be used when not specified.

Q: Why should you determine which variable to eliminate before solving?

A: *You should determine which variable to eliminate first, so you have a plan to solve the problem.*

2) Q: How will the solution be written using the variables u and v?

A: (u, v)

6) Your student may use variables instead of shapes if they prefer.

Q: What would the value of one circle and one triangle be?

A: *If* 2 circles + 2 triangles = 10, *then* 1 circle + 1 triangle = 5.

7) Q: What must be true of the coefficients for a variable to be eliminated?

A: *The coefficients must be opposite values so that they equal zero when added.*

9) Q: Does having a fractional solution mean that your system has been solved incorrectly?

A: *No.*

Q: How can you determine if your solution is correct?

A: *Substitute the solution into both equations to see if the result is a true equation.*

12) Q: What should be completed first to solve using elimination?

A: *The bottom equation should be written in standard form by adding* $\frac{12}{5}c$ *to both sides and multiplying all terms by the denominator, 5.*

Q: What other method could be used to solve this system?

A: *Substitution could also be used since there is an isolated variable, but the directions say to use elimination.*

Practice 1

Solve using elimination. Name the variable you will eliminate before solving the problem.

1) $x + 2y = 2$; $-x - 3y = 0$ — **$(6, -2)$ Eliminate x.**

2) $u + v = 66$; $v = 30 + u$ — **$(18, 48)$ Eliminate u.**

3) $3a - b = 15$; $-3a - 3b = -27$ — **$(6, 3)$ Eliminate a.**

4) $3x - 5y = 11$; $3x + 3y = 3$ — **$(2, -1)$ Eliminate x.**

5) $5x + 12y = 19$; $5x + 6y = 7$ — **$(-1, 2)$ Eliminate x.**

6) ● + ● + ▲ + ▲ = 10; ● − ▲ = 11

Solution: ▲ = **−3**; ● = **8**

Solve each system using the elimination method. Name the variable you will eliminate and the least common multiple of the coefficients.

7) $x - y = 3$; $4x - 3y = -6$ — **$(-15, -18)$ Eliminate y, LCM $(1, 3) = 3$**

8) $x - 2y = 1$; $2x + 3y = 16$ — **$(5, 2)$ Eliminate x, LCM $(1, 2) = 2$**

9) $5x + 6y = -8$; $2x + 3y = -6$ — **$\left(4, -\frac{14}{3}\right)$ Eliminate y, LCM $(6, 3) = 6$**

10) $6x + 7y = 2$; $5x + 8y = 6$ — **$(-2, 2)$ Eliminate x, LCM $(6, 5) = 30$**

11) $4e - 1.5f = 6$; $e + 3.5f = -14$ — **$(0, -4)$ Eliminate e, LCM $(1, 4) = 4$**

12) $7a + 14c = 2$; $a = -\frac{12}{5}c$ — **$\left(\frac{12}{7}, -\frac{5}{7}\right)$ Eliminate a, LCM $(5, 7) = 35$**

Mastery Check

Show What You Know

A) Solve the system.
$4x - 2y = 55$
$x - y = 9$

$(18.5, 9.5)$

Eliminate x.

$(-4)(x - y = 9)$
$-4x + 4y = -36$
$+4x - 2y = 55$
$2y = 19$
$y = \frac{19}{2} = 9.5$

$x - (9.5) = 9$
$x = 18.5$

B) Explain the steps you took to solve the system in part A.

Sample:
First, eliminate the variable x. To do this, multiply the second equation by negative four so that x is eliminated when the equations are added vertically (this is the Additive Inverse Property).

Then isolate y. This gives $y = \frac{19}{2} = 9.5$.

Next, substitute the value of y into the original equation $x - y = 9$ and solve for x. The value of $x = \frac{37}{2} = 18.5$.

C) Use the solution of the system in part A to determine the sum of x and y. Show your work.

$x = 18.5, y = 9.5$
$x + y = 18.5 + 9.5$
28

D) Use the solution of the system in part A to determine the difference of twice y and x. Show your work.

$x = 18.5, y = 9.5$
$2y - x$
$2(9.5) - (18.5)$
$19 - 18.5$
0.5

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Answers can be written as fractions or decimal values. Many students prefer working with decimals.

B) Q: How can you check to see if your answer is correct?
A: *Substitute the ordered pair into both equations.*

C) Q: What does the word "sum" indicate for this problem?
A: *That you should add the numbers together.*

D) Q: What does the word "difference" indicate for this problem?
A: *That the numbers should be subtracted from one another.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words.

- Use elimination to solve a system of equations when one variable has a set of opposite coefficients.
- Use elimination to solve a system of equations by distributing -1 across an equation.
- Use elimination to solve a system of equations using linear combinations.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

Either fraction or decimal answers can be used when not specified.

6) Your student may use variables instead of shapes if they prefer.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Practice 2

Complete the problems on a separate sheet of paper.

Solve each system using the elimination method.

1) $10x + 5y = -35$; $10x + 6y = -34$ — $(-4, 1)$

2) $m - 2n = 25$; $m + 2n = 20$ — $(22.5, -1.25)$

3) $-0.6x + y = -3.5$; $x - y = 2.5$ — $(-2.5, -5)$

4) $x + y = 4$; $x - y = 8$ — $(6, -2)$

5) $4x + y = 4$; $y = -12 + 2x$ — $\left(\frac{8}{3}, -\frac{20}{3}\right)$

6) ■ + ▲ = 18
■ + ■ + ▲ = 22

Solution: ▲ = 14, ■ = 4

Solve using elimination. Name the variable you will eliminate and the least common multiple of the coefficients.

7) $2a - 15b = 7$; $a - 6b = 4$ — $\left(6, \frac{1}{3}\right)$, Eliminate a, LCM $(2, 1) = 2$

8) $x + 3y = 6$; $3x + 4y = 13$ — $(3, 1)$, Eliminate x, LCM $(1, 3) = 3$

9) $3m = 5p + 8$; $3p = 2m + 3$ — $(-39, -25)$, Eliminate m, LCM $(3, 2) = 6$

10) $10.5c + 8.5d = 74$; $c + d = 8$ — $(3, 5)$, Eliminate d, LCM $(1, 8.5) = 8.5$

11) $-4x + 2.5y = -21.75$; $2x + y = 7.5$ — $(4.5, -1.5)$, Eliminate x, LCM $(2, 4) = 4$

12) $16b + 3g = 30$; $22b + 5g = 43$ — $(1.5, 2)$, Eliminate g, LCM $(3, 5) = 15$

TARGETED REVIEW 16

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Solve the system by graphing.

$y = -\frac{1}{2}x + 5$

$3x - 8y = 16$ **(8, 1)**

2) Graph the system of inequalities.

$y < -\frac{1}{2}x + 5$

$3x - 8y < 16$

3) How do you determine the solution to a system of equations when graphing?

4) How do the graphs and solutions of systems of equations and systems of inequalities differ?

5) Solve by clearing the fractions: $\frac{2}{3}(5x + 8) = \frac{1}{15}$ $\boldsymbol{x = -\frac{79}{50}}$

6) Determine which point(s), in the set $\{(-6, 0), (8, -3), (16, -6), (-8, -9)\}$, are possible solutions to the linear equation: $3x - 8y = 48$ **(8, −3) and (−8, −9)**

7) Zeke purchased veggies and dip each day at school as a snack for \$1.75. After 11 days, Zeke had spent \$19.25. Write the equation of the line. Choose the form that makes the most sense and name it.

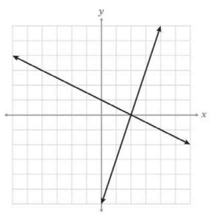
8) The Juarez family used the equation $y = 0.32x + 85$ to represent the total cost of renting a car on their vacation. The family was charged for each mile they drove. Explain the meaning of the slope and the y-intercept in context.

9) Solve for the given variable w: $P = 2l + 2w$

10) Write the equation in standard form: $y - \frac{3}{5} = \frac{8}{3}(x + 9)$

Multiple Choice

__D__ **11)** Determine the solution to the system given the graph.



A) $(0, 1)$

B) $(0, -6)$

C) $(0, 2)$

D) **(2, 0)**

12) Select all possible solutions for systems of equations.

■ **one solution**

■ **no solution**

■ **infinite solutions**

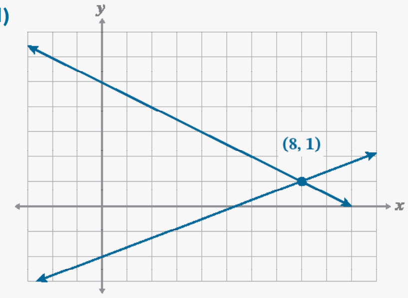
□ all real numbers

Targeted Review

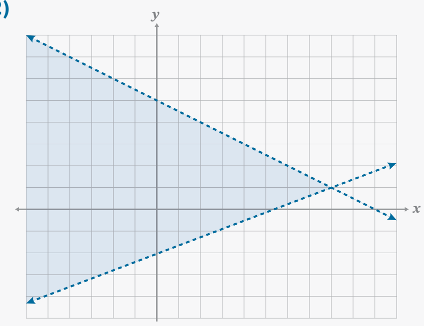
Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

1)



2)



3) Sample:
The ordered pair where the lines intersect is the solution to a system of equations. To check this solution, substitute the ordered pair into both equations to check that they are both true.

4) Sample:
Systems of inequalities need to have solid or dashed lines and also need to have the region where all solutions are true shaded. Systems of equations will always have solid lines and can have zero, one, or infinitely many solutions on the line.

7) Point-slope form, (day, money)
$y - 19.25 = 1.75(x - 11)$

8) $y = 0.32x + 85$

The slope is 0.32, which represents the cost per mile driven.

The y-intercept is 85. This represents an initial fee that the family paid (this could also be insurance).

9) $w = \frac{P}{2} - l$

10) $40x - 15y = -369$

11) Distrator Rationale:

A and B are the y-intercepts for the lines.

C contains the correct values, but the ordered pair is not in the correct order.

12) Distrator Rationale:

All real numbers cannot be a solution to a system because a solution to a system must be a point on the line(s).

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	15	15	15	15	2	7	14	10	2	11	15	15