Lesson 15
Graphing Systems of Linear Equations and Inequalities

Outline

Part A
Graphing Systems of Equations

- Systems and Their Solutions
- Systems of Equations in Slope-Intercept Form
- Systems in a Variety of Forms

Part B
Graphing Systems of Inequalities

- Solutions to Linear Inequalities
- Graphing Linear Inequalities
- Graphing Systems of Linear Inequalities

Targeted Review

Vocabulary

- system of equations
- solution (to a system of equations)
- coincident lines
- solutions (to an inequality)
- system of linear inequalities

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

If needed, have your student use their formula sheet to review the equations for horizontal and vertical lines.

Q: What is the formula for horizontal lines? Does this represent a function?

A: $y = b$
Yes, this is a function.

Q: What is the formula for vertical lines? Does this represent a function?

A: $x = a$
No, this fails the vertical line test, so it is not a function.

Finding the point where the lines intersect provides the solution to a system of linear equations, which is the focus of this lesson.

Specifying "infinite solutions on the line" is important because only ordered pairs on the line will make the answer true. Saying "infinite solutions" alone could be confused as any point on the coordinate plane.

15A EXPLORE

Part A: Graphing Systems of Equations

Objectives

In this part of the lesson, you will learn about graphing systems of equations.

By the end of this lesson, you will be able to do the following:

- Identify and describe the types of possible solutions for a system of equations.
- Graph a system of equations given in slope-intercept form. Then find and explain the solution to the system.
- Graph a system of equations not given in slope-intercept form. Then find and explain the solution to the system.

Why?

What do businesses, engineers, and air traffic controllers have in common? They all use systems of equations. From predicting profits, planning and building cities, or charting safe flight paths, many diverse careers use systems of equations to solve problems.

Warm Up

1) Write the equations of the two horizontal lines.

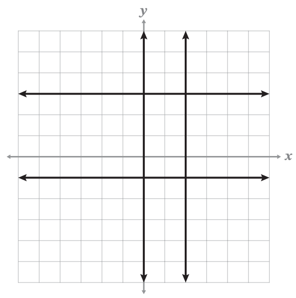
$y = 3, y = -1$

2) Write the equations of the two vertical lines.

$x = 0, x = 2$

3) Name all the points where the lines intersect.

$(2, 3), (2, -1), (0, 3), (0, -1)$



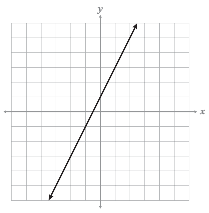
Systems and Their Solutions

- A **system of equations** is two or more equations grouped together.
- The **solution** to a system of linear equations is an ordered pair where the lines of the system intersect.
- Linear systems of equations have three types of solutions: **zero, one, or infinite on the line.**

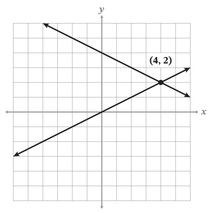
EXPLORE 15A

Example 1

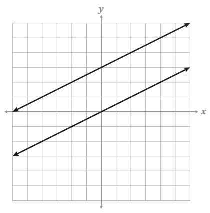
Name the number of solutions for the graphed systems of equations.



infinite solutions on the line



one solution



zero solutions

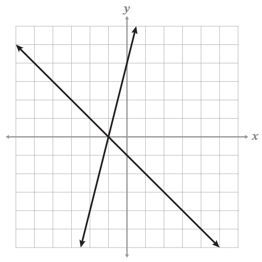
- The graphed equations for systems of linear equations that have one solution will intersect at exactly **one point** on the coordinate plane.
- A system with one solution occurs when lines have **different** slopes.
- Both equations have $(4, 2)$ as a solution; this is the **one solution** to the system. If the ordered pair does not make both equations true, then it is *not* a solution.
- A system with zero solutions is a system of **parallel** lines.
- **Coincident lines** are lines that are graphed exactly on top of one another.
- The solution to a system of coincident lines is any point on the line or infinite solutions **on the line**.

Checkpoint

Write the equations of the lines in the system from the given graph. Then find the solution.

Solution: $(-1, 0)$

System: $y = 4x + 4$
$y = -x - 1$



Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What should you first identify when writing the equation of a line?

A: *The y-intercept and the slope.*

Q: What is the solution to a system when the lines intersect?

A: *The point where the lines cross.*

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can you determine if your solution is correct?

A: *Substitute the ordered pair into both equations. If both are true, the solution is correct.*

15A EXPLORE

Systems of Equations in Slope-Intercept Form

- When graphing a system of equations on a coordinate plane, graph the **lines** across the **entire plane** so that you can find the intersection point.

Example 2

Given the system of equations, draw the graph and identify the solution.

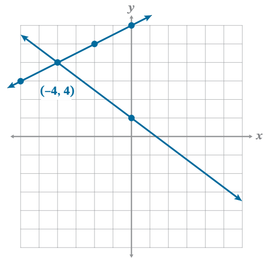
$y = -\frac{3}{4}x + 1$

$y = \frac{1}{2}x + 6$

Solution: (−4, 4)

Plan Graph both lines on the same coordinate plane until the solution is found.
Check the solution algebraically.

Implement



Check

$y = -\frac{3}{4}x + 1$	$y = -\frac{1}{2}x + 6$
$y = -\frac{3}{4}(-4) + 1$	$y = -\frac{1}{2}(-4) + 6$
$y = 3 + 1$	$y = 4$ ✓
$y = 4$ ✓	

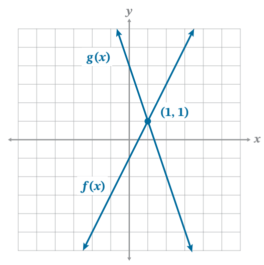
Both equations are true for (−4, 4), which means this is the solution to the system.

Checkpoint

Graph the system of equations and name the solution.

$y = 2x - 1$

$y = -3x + 4$



Solution: **(1, 1)**

EXPLORE 15A

Systems in a Variety of Forms

- When the equations in a system of equations are not in the same form, choose **one form** and rewrite the equations before graphing, or graph each equation from its given form.
- The **most efficient** way to graph equations is to leave them in their given form.

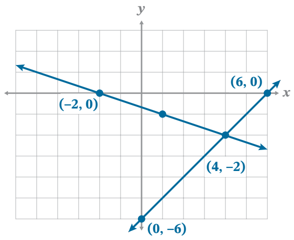
Example 3

Graph the system of equations and name the solution.

$x - y = 6$

$x + 3y = -2$

Solution: (4, −2)



Plan Find the x- and y-intercepts
Graph the lines
Find the solution

Implement

Equation 1: $x - y = 6$

$(x, 0)$

$x - 0 = 6$

$x = 6, (6, 0)$

$(0, y)$

$0 - y = 6$

$-y = 6$

$y = -6, (0, -6)$

Equation 2: $x + 3y = -2$

$(x, 0)$

$x + 3(0) = -2$

$x = -2, (-2, 0)$

$(0, y)$

$0 + 3y = -2$

$3y = -2$

$y = -\frac{2}{3}, \left(0, -\frac{2}{3}\right)$

$m = -\left(\frac{A}{B}\right)$

$m = -\left(\frac{1}{3}\right) = -\frac{1}{3}$

Check

$x - y = 6$

$4 - (-2) = 6$

$6 = 6$ ✓

$x + 3y = -2$

$4 + 3(-2) = -2$

$4 - 6 = -2$

$-2 = -2$ ✓

Your student can also find the solution to a system of linear equations by utilizing technology. However, you should save this until they can demonstrate mastery of graphing systems by hand.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the ordered pair in the second equation?

A: *(−2, 3)*

Q: Is it necessary to write the equation in slope-intercept form before graphing?

A: *No, point-slope form gives enough information to graph the line.*

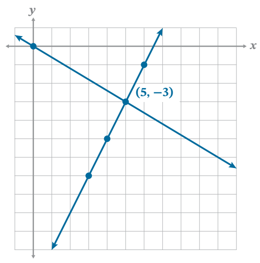
Example 4

Graph the system of equations and name the solution.

$3x + 5y = 0$

$y + 7 = 2(x - 3)$

Solution: (5, −3)



Equation 1: $3x + 5y = 0$

$m = -\left(\frac{A}{B}\right)$

$m = -\frac{3}{5}$

$\frac{C}{B} = 0$

Equation 2: $y + 7 = 2(x - 3)$

$m = 2$

point: (3, −7)

Check

$3x + 5y = 0$

$3(5) + 5(-3) = 0$

$0 = 0$ ✓

$y + 7 = 2(x - 3)$

$-3 + 7 = 2(5 - 3)$

$4 = 2(2)$

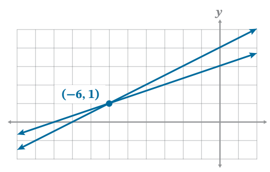
$4 = 4$ ✓

Checkpoint

Graph the system of equations and name the solution.

$y = \frac{1}{3}x + 3$

$y - 3 = \frac{1}{2}(x + 2)$



Solution: (−6, 1)

PRACTICE 1 15A

Practice 1

Complete the problems on a separate sheet of paper.

Fill in the blanks with the correct vocabulary word.

1) Lines that have equal slope are either coincident lines or **parallel** lines.

2) Systems of linear equations can have **zero**, one, or **infinite** solutions.

Determine if (3, −1) is a solution to the given systems. Show your work.

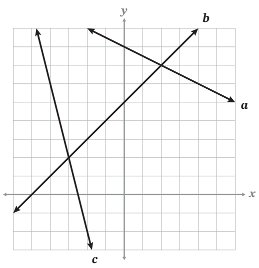
3) $x - y = 4$
$x + y = 3$ **No, (3, −1) is not a solution. It does not make the second equation true.**

4) $2x + 5y = 1$
$y = \frac{1}{2}x - \frac{1}{4}$ **No, (3, −1) is not a solution. It does not make the second equation true.**

Use the graph of lines a, b, and c to find the solution between the given equations.

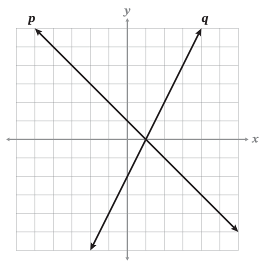
5) line a and line b **(2, 7)**

6) line c and line b **(−3, 2)**



Given the graph of lines p and q, write the equations of the system and find the solution.

7) line p and line q **line p: $y = -x + 1$ line q: $y = 2x - 2$ solution: (1, 0)**



Draw a sketch of the three types of solutions to a system of equations on a separate sheet of paper.

8) zero solutions

9) one solution

10) infinite solutions

Graph the system of equations. Mark and name the solution.

11) $y = -2x + 5$
$y = \frac{5}{2}x + 5$ **(0, 5)**

12) $y = x$
$y = -2x - 3$ **(−1, −1)**

13) $y = \frac{1}{2}x - 1$
$y - 4 = \frac{1}{2}(x - 2)$ **no solution (lines are parallel)**

14) $y + 5 = x + 4$
$2x - 3y = 5$ **(−2, −3)**

15) $2x + y = 8$
$y = x + 5$ **(1, 6)**

16) $x - y = 2$
$y - 2 = x - 4$ **infinite solutions on the line (coincident lines)**

Practice 1

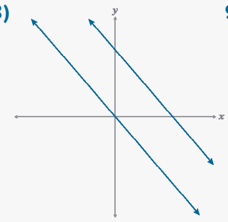
Worked solutions for these problems are located in the Digital Pack.

2) Q: What do coincident lines look like on a graph?
A: *Coincident lines look like one line on a graph because they are the same linear equation.*

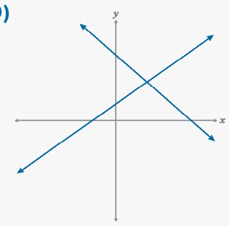
5–6)
Q: What quadrant would you expect line a and line c to intersect in? Explain.
A: *Line a and line c will intersect in Quadrant 2 because they are close to intersecting but cannot be seen on the given graph.*

7) Q: What steps should you take to find the equations given the graph?
A: *Find the y-intercept and the slope of the line and then write in slope-intercept form.*

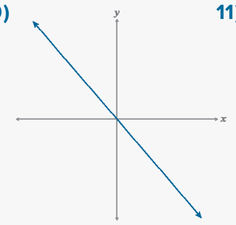
8)



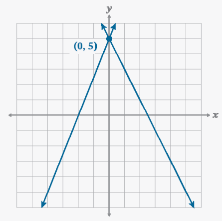
9)



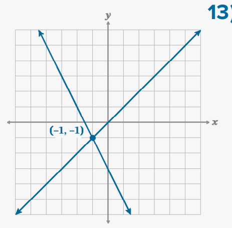
10)



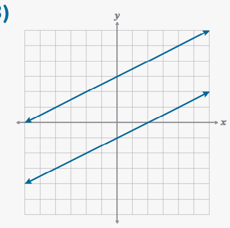
11)



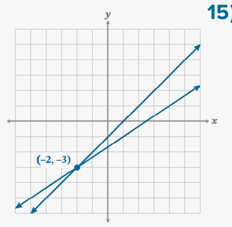
12)



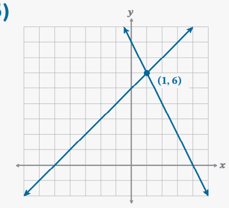
13)



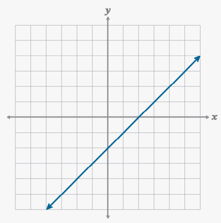
14)



15)



16)



11–16)
Q: Why do you need to check both equations when you check your solution?
A: *The ordered pair may be true for one line and not the other, so both need to be checked.*

Q: Will all solutions to a system be ordered pairs with only integer values? Explain.
A: *No, lines can intersect at points on a graph with rational (fraction/decimal) values, but these values are more difficult to determine from a graph.*

The graphs in this lesson intersect at integer values to make reading the graph and checking solutions from the graph possible. If technology is used, then fractional or decimal solutions can be found.

Mastery Check

Show What You Know

A) Your student may not need to complete or show work to graph the equations given the practice they completed in Unit 2.

Q: What is the slope of each line? Explain.

A: *The slope of both lines is negative one-half. The equation for outgoing flights is given in slope-intercept form, so the slope is already given in the equation. The equation for incoming flights is given in standard form, so the slope can be found using* $m = -\left(\frac{A}{B}\right)$.

B) Q: Why is the slope between the two intersection points equal to one?

A: *Because this is the slope of the line that represents the access road.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Identify and describe the types of possible solutions for a system of equations.
- Graph a system of equations given in slope-intercept form. Then find and explain the solution to the system.
- Graph a system of equations not given in slope-intercept form. Then find and explain the solution to the system.

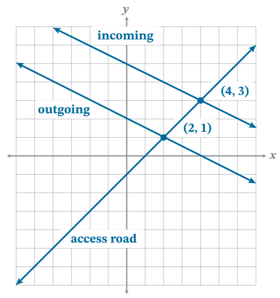
Mastery Check

Show What You Know

Airplane flights between Historyville and Mathtopia run multiple times each day. There is an outgoing flight path as well as an incoming flight path between these two cities. The equations below represent the runways for the airport in Mathtopia.

Outgoing flights: $y = -\frac{1}{2}x + 2$ Incoming flights: $x + 2y = 10$

A) Graph the outgoing and incoming runways as lines on the given coordinate plane. Label the lines as outgoing and incoming.



Outgoing flights: $y = -\frac{1}{2}x + 2$

$m = -\frac{1}{2}, b = 2$

Incoming flights: $x + 2y = 10$

$m = -\left(\frac{A}{B}\right) = -\frac{1}{2}$

$b = \frac{C}{B} = \frac{10}{2} = 5$

or

x-int = (10, 0)

y-int = (0, 5)

A) Describe the relationship between the runways. Why is this important for the airplanes' flight paths?

The runways are parallel. The slopes are equal, but the intercepts are different. This is important because the incoming and outgoing flights need runways that do not cross to avoid any accidents.

B) An access road crosses both runways so that airport employees can enter the runway with baggage to load onto the planes. Graph this equation on the same coordinate plane.

Access Road: $y + 2 = x + 1$

$-2 \quad -2$

$y = x - 1$

$m = 1,\ b = -1$

C) Where does the access road intersect each runway?

Outgoing runway: **(2, 1)** Incoming runway: **(4, 3)**

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Practice 2

Complete the problems on a separate sheet of paper.

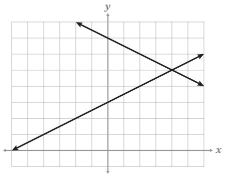
Determine if $\left(-\frac{1}{3}, 4\right)$ is a solution to the given systems. Show your work.

1) Is $\left(-\frac{1}{3}, 4\right)$ a solution for $3x - 2y = -9$ and $3x - 5y = -21$?

2) Is $\left(-\frac{1}{3}, 4\right)$ a solution for $3x + 3y = 11$ and $6x + y = 7$?

Write the system of equations given the graph. Then find the solution.

3)

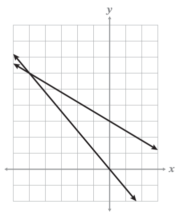


Equation 1: $y = \frac{1}{2}x + 3$

Equation 2: $y = -\frac{1}{2}x + 7$

Solution: $(-5, 6)$

4)



Equation 1: $y = -\frac{3}{5}x + 3$

Equation 2: $y = -\frac{6}{5}x$

Solution: $(-5, 6)$

Match the solution to the system of equations.

C **5)** $2x - 5y = -3$
$2x + 3y = 5$

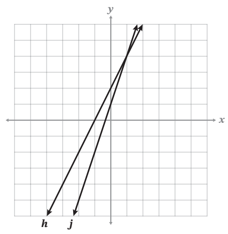
A **6)** $2x + 5y = 3$
$3x - 2y = -5$

B **7)** $x + 3y = -5$
$4x - 3y = -5$

A)	$(-1, 1)$
B)	$(-2, -1)$
C)	$(1, 1)$

Write the system of equations given the graph. Then find the solution.

8)



line h: $y = 2x + 2$

line j: $y = 3x + 1$

Solution: $(1, 4)$

Graph the system of equations. Mark and name the solution.

9) $2x + y = 8$
$y = 2x - 4$ $(3, 2)$

10) $3x + y = 2$
$3x + y = -2$ no solution (parallel lines)

11) $2x - y = 1$
$x - y = -1$ $(2, 3)$

12) $y = 3$
$x = -2$ $(-2, 3)$

13) $x = 2 - 2y$
$2x = -2 - y$ $(-2, 2)$

14) $3y + 2x - 3 = 0$
$6 + 2x + 3y = 0$ no solution (parallel lines)

Practice 2

Worked solutions for these problems are located in the Digital Pack.

1) Yes, $\left(-\frac{1}{3}, 4\right)$ is a solution for the system.

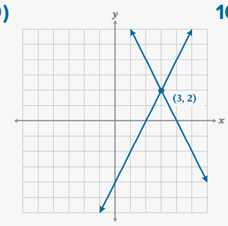
2) No, $\left(-\frac{1}{3}, 4\right)$ is not a solution to the system.

3–4) The order of the equations does not matter as long as both equations have the correct values.

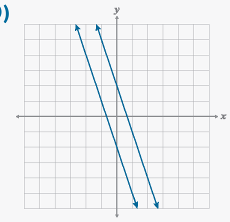
5–7) To match the solution to the system, substitute the ordered pair into each equation. When both equations are true, this is the correct solution.

9–14) Equations in standard form are graphed using the x- and y-intercepts.

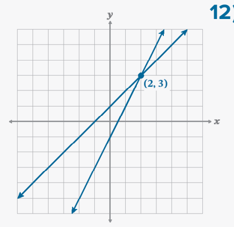
9)



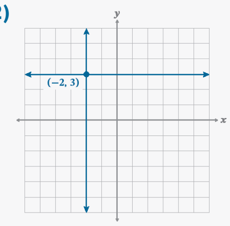
10)



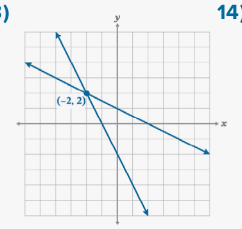
11)



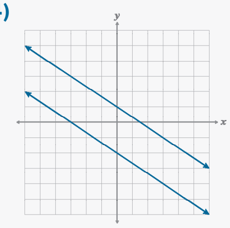
12)



13)



14)



If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Part B: Graphing Systems of Inequalities

Objectives

In this part of the lesson, you will learn about graphing systems of inequalities.

By the end of this lesson, you will be able to do the following:

- Describe what the shaded region on a graph of a linear inequality or a system of linear inequalities represents.
- Graph a linear inequality.
- Graph a system of linear inequalities.

Why?

Being able to read, explain, and create graphs of systems of inequalities will help you better understand how inequalities, equations, and their graphs all relate. This will help you master future topics in algebra.

Warm Up

Solve the inequalities. Graph the solution on a number line.

1) $-\frac{2}{5}x + 7 > 1$

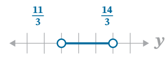
$-7 \quad -7$

$-\frac{2}{5}x > -6$

$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}x\right) > \left(-\frac{5}{2}\right)(-6)$

$x < 15$

2) $0 < 3y - 11 < 3$



$0 < 3y - 11 \qquad 3y - 11 < 3$

$11 < 3y \qquad 3y < 14$

$\frac{11}{3} < y \qquad y < \frac{14}{3}$

$\frac{11}{3} < y < \frac{14}{3}$

3) What does the shaded portion of your number line represent?

The shaded portion of the number line represents all possible solutions for the inequality.

Solutions to Linear Inequalities

- The following rules apply to linear inequalities:
  - The **equal** symbol is replaced by one of the inequality symbols (<, >, ≥, or ≤).
  - A region of the coordinate plane is **shaded**.
  - The inequality will be in one of the **linear** forms.
- The graph of a linear inequality is shaded to show **every possible solution** that will make the inequality **true**.

EXPLORE 15B

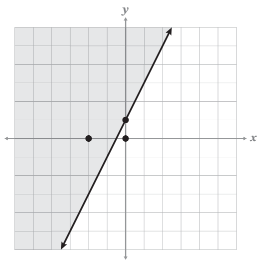
Four Types of Linear Inequalities

Symbol	Wording	Represented Graphically	
>	is greater than	**dashed** ←--------→	**open point** ○
<	is less than	←--------→	○
≥	is greater than or equal to	←——→	●
≤	is less than or equal to	←——→	●

Example 1

Given the graph of the linear inequality, describe the solution.

$y \geq 2x + 1$



First, identify familiar values in the inequality.

$m = 2$

$b = 1$

Pick test points above, on, and below the line to check the inequality algebraically.

Above the line: $(-2, 0)$

$y \geq 2x + 1$

$0 \geq 2(-2) + 1$

$0 \geq -4 + 1$

$0 \geq -3$ **True**

On the line: $(0, 1)$

$y \geq 2x + 1$

$1 \geq 2(0) + 1$

$1 \geq 1$ **True**

Below the line: $(0, 0)$

$y \geq 2x + 1$

$0 \geq 2(0) + 1$

$0 \geq 0 + 1$

$0 \geq 1$ **False**

The solution to the inequality is *all* ordered pairs *on* and *above* the graphed line.

Example 2

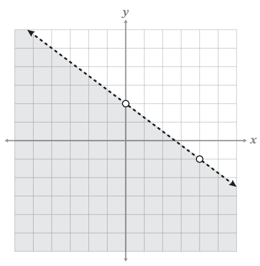
Identify key features of the given graph, then write the linear inequality.

line: **dashed**

shading: **below**

Mark the following on the graph: m, b

linear inequality: $y < -\frac{3}{4}x + 2$



☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What point can you use to determine if your symbol is correct for the inequality you wrote?

A: *Any point in the given shaded region, though (0, 0) is probably the most efficient in this case.*

If your student has a false inequality statement after checking, this usually means the direction of the inequality symbol should change.

Example 3

Add $-3x$ to both sides.

Multiply both sides by the reciprocal of -2.

☑ Checkpoint

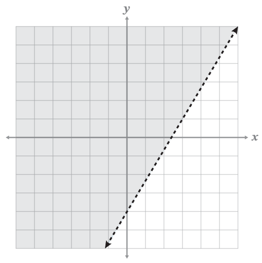
Identify key features of the given graph, then write the linear inequality.

line: solid or dashed
shading: above or below the line

inequality symbol: >

$m = \frac{5}{3}$ $b = -4$

inequality: $y > \frac{5}{3}x - 4$



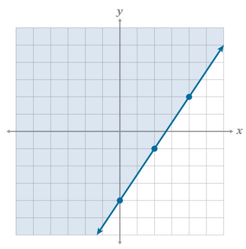
Graphing Linear Inequalities

- The key difference between graphing linear inequalities and equations is determining if the line will be **solid** or **dashed** and then **shading** the region where the solutions lie.

Example 3

Graph the inequality: $3x - 2y \leq 8$

Plan Write in slope-intercept form.
Identify characteristics of the graph.
Graph the inequality.



Implement

$$3x - 2y \leq 8$$
$$-2y \leq -3x + 8$$
$$\left(-\frac{1}{2}\right)(-2y) \leq \left(-\frac{1}{2}\right)(-3x + 8)$$
$$y \geq \frac{3}{2}x - 4$$

Identify the graph characteristics

line: solid or dashed

shading: above or below the line

$b = -4$

$m = \frac{3}{2}$

Explain

◂ The line is solid because there is an =.

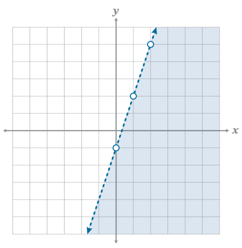
◂ The shading is above the line because the inequality switches directions, and greater numbers are above the line.

◂ The y-intercept is $(0, -4)$ and a closed point.
The slope is $\frac{3}{2}$.

EXPLORE 15B

Example 4

Graph the linear inequality: $y < 3x - 1$



Identify the graph characteristics	Explain
line: solid or dashed	The line is dashed because there is no "=."
shading: above or below the line	The shading is below the line because numbers with less value are negative.
$b = -1$ open or closed point	The y-intercept is $(0, -1)$ and marked with an open point.
$m = 3$	The slope is $\frac{3}{1}$.

Checkpoint

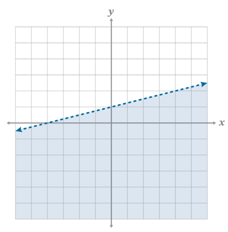
Identify the key information for the linear inequality, then graph it.

$y < \frac{1}{4}x + 1$

line: solid or dashed

shading: above or below the line

$m = \frac{1}{4}$ $\qquad b = 1$



Graphing Systems of Linear Inequalities

- A **system of linear inequalities** is two or more linear inequalities grouped together.
- Graphing a system of linear inequalities follows the same rules as graphing one **linear inequality**.
- The biggest difference is the solutions to a system of linear inequalities are in the shaded **region** of the graph where the linear inequalities **overlap**.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Should open or closed points be used to graph points for this inequality? Explain.

A: *Open points because the inequality does not have an equal bar under it.*

Q: Why should you determine the key features of the graph before you start graphing?

A: *Determining these before graphing will make graphing more efficient.*

Example 5

Graph the system of inequalities.

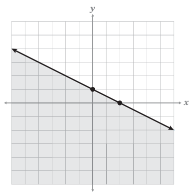
$y \le -\frac{1}{2}x + 1$

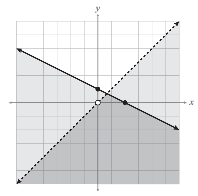
$y < x$

Plan Identify the key information for each inequality.
Graph each inequality.

Implement

Inequality 1: $y \le -\frac{1}{2}x + 1$	Inequality 2: $y < x$
line: solid	line: **dashed**
shading: below	shading: **below**
$m = -\frac{1}{2}$, $b = 1$, closed point	$m =$ **1**, $b =$ **0**, **open** point





The portion of the graph where the shaded regions overlap is the solution to the system of inequalities.

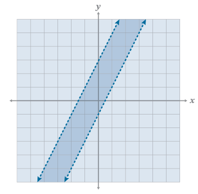
Example 6

Graph the system of inequalities. Name the quadrant(s).

$y < 2x + 3$ — $\mathbf{m = 2, b = 3}$

$2x - y < 1$ — $\mathbf{m = 2, b = -1}$

The solution is in quadrants 1, 2, 3, 4.



Checkpoint

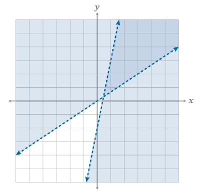
Graph the system of inequalities.

$y < 5x - 2$

$y > \frac{2}{3}x$

Name the quadrant or quadrants where the solution is located.

Quadrant 1



Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: If the solution lies in all four quadrants, does this mean the entire coordinate plane is the solution?

A: *Not necessarily. It means that parts of each quadrant are in the shaded region.*

See Example 6 for a graph that has a shaded solution in all four quadrants.

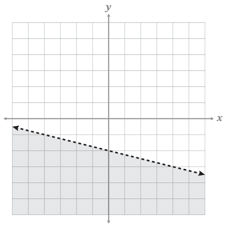
Practice 1

Complete the problems on a separate sheet of paper.

1) Given the graph, identify the slope, y-intercept, if the line is solid or dashed, and if the shading is above or below the line.

$m = -\frac{1}{4}$, $b = -2$

line is dashed
shading is below the line



2) Given $y \geq x$, identify the slope, y-intercept, if the line is solid or dashed, and if the shading is above or below the line. Then graph.

Write the sentences using one of the following: always, sometimes, never. (Answer choices may be used more than once.)

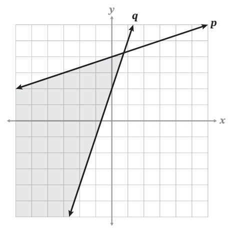
3) Points on the line are **sometimes** solutions to a linear inequality.

4) The shaded region **always** represents all possible solutions.

5) When dividing or multiplying by a negative coefficient, the inequality symbol **always** changes directions.

6) Graph $y \leq -\frac{2}{3}x + 4$

7) Write the system of inequalities given the graph.



$y \leq \frac{1}{3}x + 4$

$y \geq 3x + 2$

Use the inequalities in problem 7 to prove if the following ordered pairs are solutions.

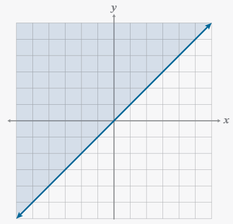
8) $(-2, -4)$ **This is a solution because the point makes both inequalities true and is in the shaded region.**

9) $(4, 1)$ **This is not a solution because *line q* is false, and the point is not in the shaded region.**

Practice 1

Worked solutions for these problems are located in the Digital Pack.

2) line is solid
shading is above the line
$m = 1, b = 0$



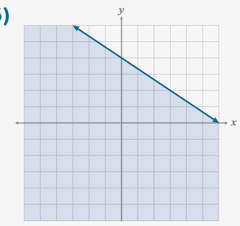
Q: Why should the line be extended to the edge of the given coordinate plane?

A: *The line should be extended to the edge of the graph so that the entire solution region can be shaded correctly.*

3) Q: When would the line be included as part of the possible solutions to an inequality?

A: *The line is part of the possible solutions when ≥ or ≤ is the symbol used.*

6)



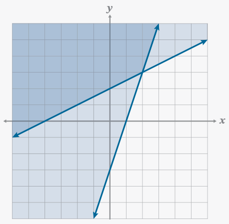
10) Q: What does region B represent?

A: *Ordered pairs that make the inequality for line b true, but not the inequality for line a.*

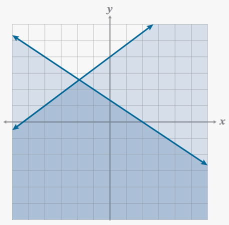
Q: What does region C represent?

A: *Ordered pairs that are not solutions for the inequalities for lines a or b.*

16)



17)

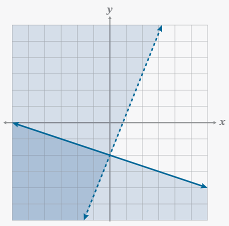


Students will have to find the x-intercept and then another ordered pair using the slope because the y-intercept is a fraction.

Q: How can you graph the inequality written in standard form?

A: *You can find the intercepts and graph both points. However, the y-intercept for this inequality is a fraction, so the slope should be found and plotted across the graph.*

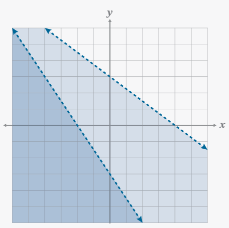
18)



Q: What type of lines are formed when graphing this system of inequalities? Explain.

A: *The lines formed are perpendicular, because the slopes are opposite reciprocals.*

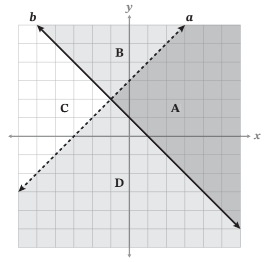
19)



Q: Will the lines on this graph ever intersect? Explain.

A: *Yes, the lines will intersect somewhere in Quadrant 2. The lines are not parallel, so they will eventually intersect, even if that point is not shown on the graph.*

10) Write the system of inequalities given the graph.
$y < x + 3 \quad y \geq -x + 1$



From the graph, determine which region (A, B, C, or D) each point is in and whether it is a solution to the system.

11) (2, 0) **region A, solution**

12) (−2, −2) **region D, not a solution**

13) (0, −3) **region D, not a solution**

14) (5, −2) **region A, solution**

15) (−3, 4) **region B, not a solution**

16) Identify the slope, y-intercept, if the line is solid or dashed, and if the shading is above or below the line. Then graph.

Inequality c: $y \geq 3x - 3$ Inequality d: $y \geq \frac{1}{2}x + 2$

	Inequality c	Inequality d
line	solid	solid
shading	above	above
b	−3	2
m	3	$\frac{1}{2}$

Graph the system of inequalities.

17) $y \leq \frac{3}{4}x + 4$
$2x + 3y \leq 4$

18) $y \leq -\frac{1}{3}x - 2$
$y > 3x - 2$

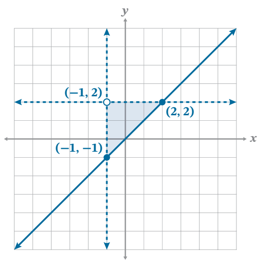
19) $3x + 4y < 12$
$y < -\frac{3}{2}x - 3$

MASTERY CHECK 15B

Mastery Check

Show What You Know

A) Graph the system of inequalities:
$y < 2$
$x > -1$
$y \geq x$



B) What is the shape formed by the three inequalities on the coordinate plane?

Triangle

C) Mark where the lines in part A intersect. Name the three vertices of the figure formed on the coordinate plane.

$(-1, -1), (-1, 2), (2, 2)$

D) Find the area of the figure using a formula from your Formula Sheet. Write the formula, then solve.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(3)(3) = \frac{9}{2}$

4.5 square units

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

C) Q: Is the origin a solution to the graph? Explain.

A: *The origin is a solution because it falls on the solid line and, when solved algebraically, makes the system of inequalities true.*

D) Q: What is the formula for the figure? Remember to use your Formula Sheet.

A: *The area of a triangle is* $A = \frac{1}{2}bh$.

Q: What is the base and the height of the triangle?

A: *Both the base and the height are 3 units.*

Q: How did you find the base and height?

A: *You can find the base and height by counting the vertical and horizontal distance between the vertices (corners).*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Describe what the shaded region on a graph of a linear inequality or a system of linear inequalities represents.
- Graph a linear inequality.
- Graph a system of linear inequalities.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

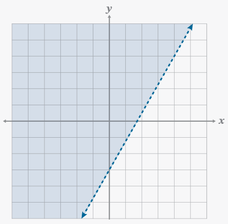
- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

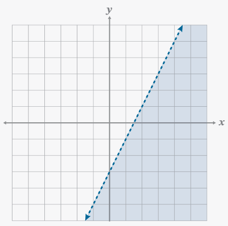
Worked solutions for these problems are located in the Digital Pack.

1) The shaded region represents all of the possible solutions to the inequality. Every ordered pair in the shaded region will make the inequality true.

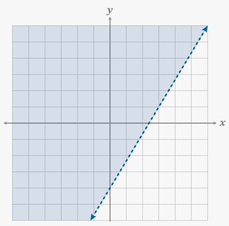
3)



4)

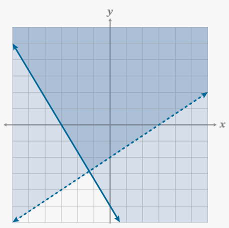


5)

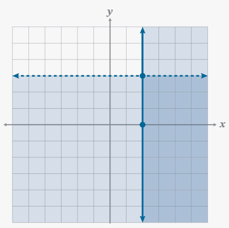


8)

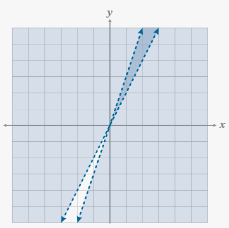
	Inequality m	Inequality n
line	solid	dashed
shading	above	above
b	-5	-2
m	$-\frac{5}{3}$	$\frac{2}{3}$



9)



10)

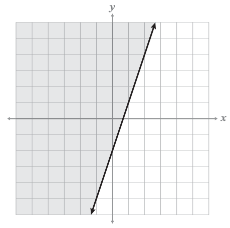


Practice 2

Complete the problems on a separate sheet of paper.

1) What does the shaded region represent for a linear inequality on the coordinate plane?

2) Given the graph, identify the slope, y-intercept, if the line is solid or dashed, and if the shading is above or below the line.



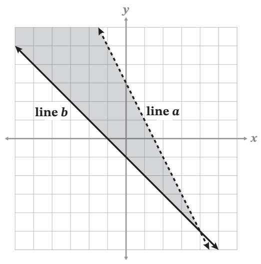
$m = 3, b = -2$

line is solid
shading is above the line

Graph.

3) $y > \frac{7}{4}x - 3$ **4)** $2x - y > 3$ **5)** $y > \frac{5}{3}x - 4$

6) Write the system of inequalities given the graph.



$y < -2x + 3$

$y \geq -x - 1$

7) Explain how you determined which inequality symbol to use for each linear inequality in problem 6.

For inequality a, the line is dashed and the shading is below the y-intercept. This means the symbol must be less than (<). For inequality b, the line is solid and the shaded region is above the y-intercept. This means the symbol must be greater than or equal to (≥).

8) Identify the slope, y-intercept, if the line is solid or dashed, and if the shading is above or below the line. Then graph.
Inequality m: $5x + 3y \geq -15$
Inequality n: $2x - 3y < 6$

Graph the system of inequalities.

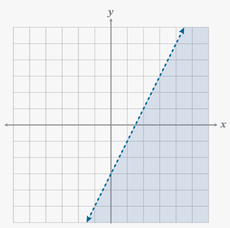
9) $y < 3$
$x \geq 2$

10) $y < 3x$
$y > 2x$

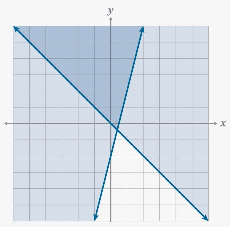
11) $y \geq 4x - 2$
$x + y \geq 0$

12) $y > \frac{5}{4}x - 4$
$y > -\frac{2}{3}x + 5$

11)



12)



If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

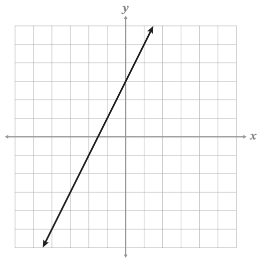
TARGETED REVIEW 15

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Write the equation of the line in slope-intercept form using the graph.



$y = 2x + 3$

2) Write the equation in slope-intercept form: $5x - 11y = 28$

$y = \frac{5}{11}x - \frac{28}{11}$

3) Solve the compound inequality. Graph the solution on a number line.

$6 - 3x \leq -4$ or $2 - x > 4$ $\quad x \geq \frac{10}{3}$ **OR** $x < -2$

Hillary hired someone to paint the outside of her house. The paint cost Hilary $78, and the painter charged $50 per hour to paint the house.

4) Write an equation to find the total cost of painting the house for any number of hours. $y = 50x + 78$

5) If the total bill was $253, how long did the painter work? **The painter worked 3.5 hours.**

6) Solve: $\frac{2}{3}(5x - 2) = \frac{1}{4}(x + 8)$ $\quad x = \frac{40}{37}$

7) Write an equation for the line that is perpendicular to $y = \frac{5}{8}x - 2$ and passes through $(-1, 6)$. $\quad y = -\frac{8}{5}x + \frac{22}{5}$

8) Mary-Jo was planning her graduation party at a hall. She was told that 40 people would cost $600 and 54 people would cost $768. What formula is the caterer using to give Mary-Jo the prices? Explain what the slope and y-intercept mean in the context of the problem. $\quad y = 12x + 120$

9) Guy's Grocery recorded the number of customers they had over five days. Then Guy did some calculations and determined the following statistics.

Day 1: 1,000	Day 2: 870	Day 3: 900	Day 4: 100	Day 5: 910
mean = 756	mode = none	median = 900	range = 900	

What is the best measure to determine the typical number of customers? Explain.

10) Solve the compound inequality: $\frac{5}{3} < \frac{1}{5}x - 3 \leq 9$

$\frac{70}{3} < x \leq 60$

11) Find the x- and y-intercepts as ordered pairs: $15x + 13y = -30$

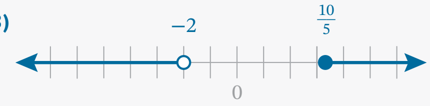
x-intercept $(-2, 0)$ $\quad$ **y-intercept** $\left(0, -\frac{30}{13}\right)$

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

3)



8) The slope is 12. This means the cost is $12 per person. The y-intercept is $(0, 120)$. This means there is a fee (to rent the hall, or maybe for the food).

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13
Lesson Origin	9	11	4	9	9	2	12	10	6	4	11	13	10

9) The median would be the best measure because it is not skewed by the very low number of customers on day 4.

12) If your student chose “All students scored above 85%,” they probably missed the student that studied 0 hours.

If your student chose “The exact average for the class was 85%,” they may think you can find the average by looking at a scatter plot; however, the exact values of the ordered pairs are unknown and the scale of the graph is not precise enough to determine this.

13) Distractor Rationale:

A) The cost is not mentioned for this problem.

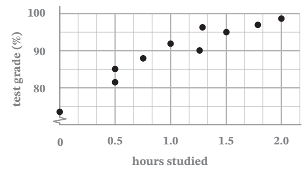
C) The cost is not mentioned for this problem.

D) Nine hundred represents the full propane tank. More propane is not purchased when only 75.8 gallons have been used.

TARGETED REVIEW 15

Multiple Choice

12) Select the two sentences that are true based on the scatterplot.



- ■ **As study time decreases, test grades decrease.**
- ☐ All students scored above 85%.
- ☐ The exact average for the class was 85%.
- ■ **Students who studied more earned higher scores.**

B **13)** Kara documented propane delivery and usage for her family. She tracked the number of gallons the family used monthly and knew that their propane tank held 900 gallons of propane when full. The equation of the line of best fit is shown below.

$$y = -75.8x + 900$$

Which statement correctly describes the slope of the equation in the context of the situation?

A) Each month, Kara spent $900.

E) **Each month, Kara's family used about 75.8 gallons of propane.**

B) With 900 gallons of propane, Kara's family spends $75.80.

C) For every 75.8 gallons used, the family buys 900 more gallons.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13
Lesson Origin	9	11	4	9	9	2	12	10	6	4	11	13	10