Lesson 14
Types of Functions and Arithmetic Sequences

Outline

Part A
Continuous and Discrete Functions

- Interval Notation
- Continuous or Discrete Functions
- Functions in Various Forms

Part B
Arithmetic Sequences

- Arithmetic Sequences

Targeted Review

Vocabulary

- interval
- interval notation
- discrete function
- continuous function
- arithmetic sequence
- terms of the sequence

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

The equations of both graphs would be the same, but the solutions are different. Any real number for x would be a possible solution for Graph A, while only integer values are true for Graph B. This means that all of the solutions in Graph B are true for Graph A, but not all solutions in Graph A will be true for Graph B.

Remember to write the domain and range in numerical order regardless of the order of the ordered pairs in the relation.

14A EXPLORE

Part A: Continuous and Discrete Functions

Objectives

In this part of the lesson, you will learn about continuous and discrete functions.

By the end of this lesson, you will be able to:

- Use interval notation to define the domain and range of functions.
- Decide if a function is discrete or continuous and explain the difference.
- Choose the most appropriate form of the equation of a line for a given scenario.

Why?

Will any point, integer or decimal, on a line be true? Can only whole numbers answer the question? Knowing if a situation is discrete or continuous will allow you to determine if your answers make sense for a real life scenario.

Warm Up

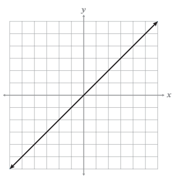
Use the relation to answer the following questions.
{(1, 3) (4, −2), (9, 1), (3, 4), (5, 3)}

1) Name the domain of the relation. **Domain: {1, 3, 4, 5, 9}**

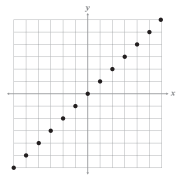
2) Name the range of the relation. **Range: {−2, 1, 3, 4}**

3) Describe the similarities and differences between the two graphs.

Graph A



Graph B



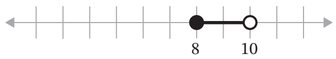
Similarities:
They have the same x- and y-intercept, (0, 0). The slope is the same. They are both functions. All of the ordered pairs in Graph B are also ordered pairs of Graph A.

Differences:
Graph A is a line. Graph B is a set of ordered pairs.

EXPLORE 14A

Interval Notation

- An **interval** is the set of all values between two numbers.
- **Interval notation** is one way to express every element in an interval.
- If the boundary point is **included**, a closed point is used on the graph and a bracket is used in interval notation.
- If the boundary point is **not included**, an open point is used on the graph and a parenthesis is used in interval notation.
- This number line represented in interval notation is:



$[8, 10)$

- The boundary that represents all real numbers is: $(-\infty, \infty)$
- Use **interval notation** to represent the domain and range of a **function**.

Example 1

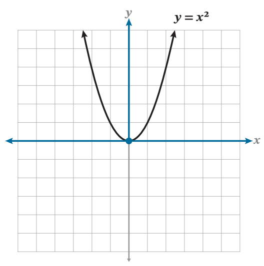
For the function, $y = x^2$, write the domain and range in interval notation.

domain	range
x-value	y-values
$\mathbb{R}$	$y \geq 0$

Using interval notation:

domain: $(-\infty, \infty)$

range: $[0, \infty)$



This graph is in Quadrants **I and II**.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

The equation for this graph is $x^2 + (y + 1)^2 = 9$.

It may help your student find the domain and range by marking the graph, so they can better determine where the boundary points are. Brackets should be used because all of the points are included.

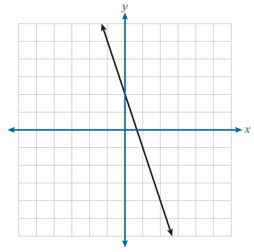
14A EXPLORE

Example 2

For the function $y = -3x + 2$, represent the domain and range in interval notation.

x-coordinate: $\mathbb{R}$

y-coordinates: $\mathbb{R}$

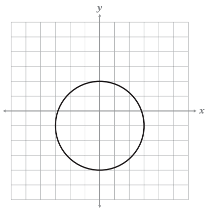


domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

Checkpoint

Provide the domain and range in interval notation for the graph below. Explain whether the graph is a function.



domain: $[-3, 3]$

range: $[-4, 2]$

This graph is not a function because it fails the vertical line test. The x-values repeat in multiple locations.

Continuous or Discrete Functions

- A **continuous function** has continuous domain elements or a domain written as a single interval.
- The graph of a continuous function will be a **smooth curve**.
- **Discrete functions** have a domain made up of distinct elements that can be plotted as separate points.
- The graph of a discrete function is created from a set of **unique points**.

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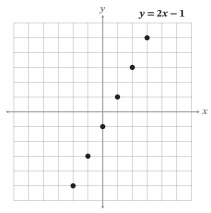
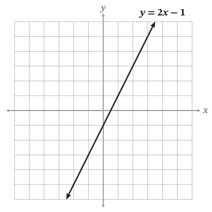
EXPLORE 14A

- This graph represents a **discrete** function.
  - domain: $\{-2, -1, 0, 1, 2\}$
  - range: $\{-5, -3, -1, 1, 3\}$



- This graph represents a **continuous** function.
  - domain: $(-\infty, \infty)$
  - range: $(-\infty, \infty)$



Now that you know there are different types of functions, even within linear graphs, you will be able to more accurately portray the information.

- For both discrete and continuous **functions**, the independent values are associated with the **domain** and the dependent values are associated with the **range**.

Example 3

Randy was going to deliver a van to a family in Oklahoma. The following function shows the number of gallons, $g(m)$, for a given number of miles driven, m.

$$g(m) = \frac{m}{20}$$

A) If Randy was going to travel 1,374 miles to deliver the van, how many gallons of gasoline would he need?

(miles driven, gallons of gas used) $g(1{,}374) = \frac{1{,}374}{20}$

$m = 1{,}374$ miles **$g(1{,}374) = 68.7$ gallons**

B) If he was given a gas card for 14 gallons of fuel, how many miles could he expect to drive using the gas card?

$g(m) = 14$ gallons $14 = \frac{m}{20}$

$m = (14)(20) =$ 280 miles

Explain

This is a **continuous** function, because it is possible to determine the amount of gas used for any number of miles, and gasoline is used continuously while driving.

The domain is $0 \le m \le 1{,}374$, or the interval **$[0, 1{,}374]$**.

The range is $0 < g(m) \le 68.7$, or the interval **$[0, 68.7]$**.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the biggest visual difference between the graphs of discrete and continuous functions?

A: *The graph of a discrete function is a set of ordered pairs while a continuous function has a smooth graph.*

14A EXPLORE

Example 4

At East-West Printers, the cost to print a company logo on an 8×10 inch sticker is $7.25.

A) Write the given equation, $c = 7.25p$, in function notation. Use the function to find how much it costs to print a logo a maximum of 13 times.

Cost (c) is determined by the number of logos printed (p)

(independent, dependent) **(p, c)**

Function Notation: $c(p) = 7.25p$

To print 13 logos: $c(13) = 7.25(13)$

$= \$94.25$

B) Name the domain and range, and what each represent.

Domain: $\{0, 1, 2, 3, \ldots, 13\}$. The domain represents the **number of logos**.

Range: $\{0, 7.25, 14.50, 21.75, \ldots, 94.25\}$.

The range represents the **cost per logo**.

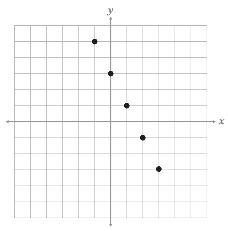
C) Does this represent a discrete or continuous function? Explain.

The logo company would not get paid for printing part of a logo. Since the logo company only gets paid for printing a **counted number** of logos, the function is **discrete**.

In a function, ellipses are used in the domain and range to show that the pattern will continue without having to list all of the terms for the domain and range.

Checkpoint

Is the graph of the following function continuous or discrete? Explain.



This function is discrete because the graph shows a set of unique ordered pairs that are not connected.

EXPLORE 14A

Functions in Various Forms

- When no linear form is specified, it is up to **you** to determine the most efficient way of writing a linear equation.
- Besides choosing the best linear form to represent information, you will also need to determine if a **function** will be discrete or continuous.

Example 5

Write an equation using the most appropriate form of a line (slope-intercept, point-slope, or standard) for the given scenario. Explain why the graph would be discrete or continuous.

John was working to buy a car. He earned $12 per hour, and he already had some money that his mom had saved to help him. After working 25 hours, John had $2,800 available to purchase a car.

Implement

Rate of change: $12/hour **(hour, money)**

Specific point: (25, 2,800) $y - 2800 = 12(x - 25)$

Best form of line: **Point-slope form**

Explain

This is a continuous function because John can earn money for working part of an hour.

Example 6

Write an equation using the most appropriate form of a line for the given situation. Explain why the graph would be discrete or continuous.

David needed to purchase lumber for a building project. He needed 4 × 4 pieces that cost $8 and 2 × 4 pieces that cost $5. He had $45 to spend on wood.

Implement

Cost of two unknowns: $8 per 4 × 4 piece and $5 per 2 × 4 piece

Total cost: $45 $8x + 5y = 45$

Form of line: Standard form
(4 × 4 cost, 2 × 4 cost)

Explain

This is a discrete function because David must purchase whole pieces of lumber from the store.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can you determine if a scenario is discrete or continuous?

A: *If you cannot have a fraction of the independent variable, it tends to be discrete.*

14A EXPLORE

Checkpoint

Laynee is a driver for a food delivery service. She earned $60 after making 15 deliveries and $88 after 22 deliveries. Explain why the graph would be discrete or continuous. Then write a linear equation in the most appropriate form.

(delivery, money)
(15, 60) and (22, 88)
$m = \frac{88 - 60}{22 - 15} = \frac{28}{7} = 4$

Point-slope: $y - 60 = 4(x - 15)$ **OR slope-intercept:** $y = 4x$

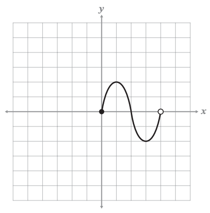
This is a discrete function because Laynee cannot make a fraction of a delivery because customers want their entire food order.

Practice 1

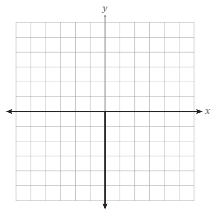
Complete the problems on a separate sheet of paper.

Use the domain and range intervals below to choose the graph that best matches the interval, and fill in the blank with the corresponding letter. Then, write the domain and range for each graph in interval notation.

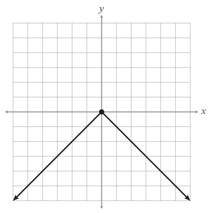
C **1)**



A)



A **2)**



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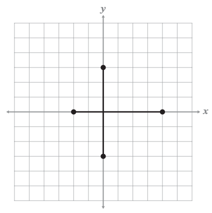
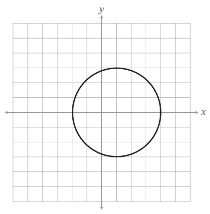
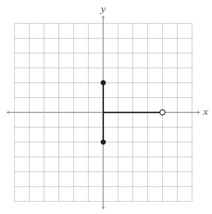


B **3)**



C)



4) Are all of these graphs functions? Explain.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) domain: $[0, 4)$
range: $[-2, 2]$

2) domain: $(-\infty, \infty)$
range: $(-\infty, 0]$

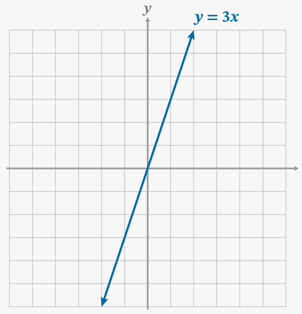
3) domain: $[-2, 4]$
range: $[-3, 3]$

4) No, Graph 3 fails the vertical line test so it is not a function. Graphs 1 and 2 are functions because they pass the vertical line test. This refers back to Lesson 7.

Q: What does an open point mean when using interval notation?

A: *Parentheses should be used because the value is not included.*

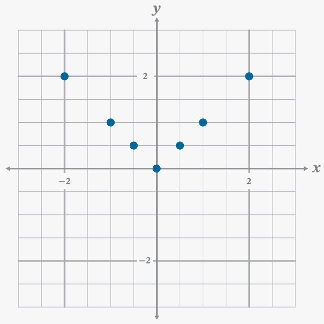
5)



Q: Is this a continuous or discrete graph?

A: *continuous*

6)



domain: $\left\{-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2\right\}$

range: $\left\{0, \frac{1}{2}, 1, 2\right\}$

This represents the equation $y = |x|$.

Q: Is this a continuous or discrete graph?

A: *discrete*

7) The domain is continuous over the interval.

8) The domain is a set of points. The graph will also be a set of points.

9) The domain is time, and time is continuous.

10) Plants are a counted value.

11) slope-intercept form

$y = \frac{1}{2}x + 2$

This is a continuous function because snow can be measured at any time and can be any fraction of an inch in height.

12) Point-slope form

$y - 42 = \frac{3}{2}(x - 28)$

This is a discrete function because the whole box needs to be packed before it can be shipped.

Q: Why is it not necessary to write this in slope-intercept form?

A: *The problem does not specifically say it is needed and the y-intercept is not mentioned.*

13) Slope-intercept form

$y = 49x + 175$

This is a continuous function because the shop can charge for part of an hour to complete the repairs.

14–15) Your student can sketch graph of the scenario to help determine if it is discrete or continuous.

14A PRACTICE 1

Create a graph from the equation or table of values. Then, write the domain and range for the function in interval notation or set notation.

5) $y = 3x$ **domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$**

6)

x	y
-2	2
-1	1
$-\frac{1}{2}$	$\frac{1}{2}$
0	0
$\frac{1}{2}$	$\frac{1}{2}$
1	1
2	2

Determine if the function is discrete or continuous.

7) $g(x) = 3x - 2$ with a domain of $(-\infty, \infty)$. **Continuous**

8) $h(x) = -x - 4$ with the domain $\{-1, 0, 1, 2, 3\}$. **Discrete**

9) The temperature at a given time of day. **Continuous**

10) The number of sprouted plants in a garden in the month of May. **Discrete**

Write the equation for the line. Choose the best form of the line to write for the given scenarios. Explain how you know the function is discrete or continuous.

11) There were two inches of snow on the ground. Then, during the snowstorm, snow was accumulating at half an inch per hour.

12) Alana was packing boxes to be shipped. Before lunch, she packed 8 boxes in 12 minutes. After lunch, she packed 42 boxes in 28 minutes.

13) The auto repair shop in town charges $175 to analyze the problem for a vehicle. After having his car analyzed, Jimmy was charged $49 per hour to make the repairs.

14) Hector was given $30 this month in spending money. He decided that he would purchase a coffee every day that would cost $3. After how many days and cups of coffee will Hector run out of money? Explain.

15) Grapes cost $4 per pound and apples cost $3 per pound. Kervon had a total of $24 to spend on fruit.

14) Slope-intercept form

$y = -3x + 30$

This is a discrete function because he is spending an exact amount each day, and you cannot buy a part of a cup of coffee.

15) standard form

$4g + 3a = 24$

Q: Is it possible to purchase a fraction of a pound of grapes or apples?

A: *yes*

Q: Does this mean that the graph is discrete or continuous for this interval?

A: *continuous*

This is a continuous function because you can purchase any combination of apples and grapes that fall on the graphed line.

MASTERY CHECK 14A

Mastery Check

Show What You Know

You are making an energy snack mix and need to have a total of 33 grams of protein in each snack bag. Bridge Ultra Energy mix has 6 grams of protein per ounce and granola has 3 grams of protein per ounce.

A) Write the equation of the line. Name the form you used.

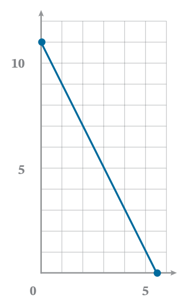
Standard Form
(bridge mix, granola)
$6x + 3y = 33$

B) Explain why you believe this is a discrete or continuous function.

This is a continuous function because any value is a possible value for the domain including fractional values. And the domain can be expressed as an interval.

C) Graph your equation from Part A.
Name the domain and range.

domain: [0, 5.5]
range: [0, 11]



D) Explain the intercepts and how the graph can help determine possible combinations for the snack bags.

The x-intercept (5.5, 0) means that 5.5 oz of bridge mix and 0 oz of granola are in the snack bag. The y-intercept (0, 11) means that 0 oz of bridge mix and 11 oz of granola are in the snack bag.

The points on the line represent the possible combinations of bridge mix and granola that will result in exactly 33 grams of protein per snack bag.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

Your student may write the equation in another form as long as it is equivalent to the equation in standard form.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Use interval notation to define the domain and range of functions.
- Decide if a function is discrete or continuous and explain the difference.
- Choose the most appropriate form of the equation of a line for a given scenario.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

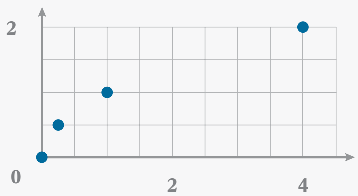
1) domain: $[-2, 2]$
range: $[-1, 3]$

2) domain: $(-\infty, 0]$
range: $(-\infty, \infty)$

3) domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$

4) No, Graph B and C fail the vertical line test so they are not functions. Graph A is a function because it passes the vertical line test.

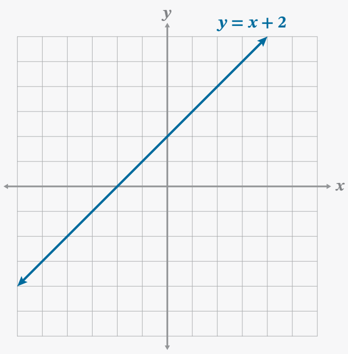
5) This is the table for the graph $y = \sqrt{x}$.



domain: $\left\{0, \frac{1}{4}, 1, 4\right\}$

range: $\left\{0, \frac{1}{2}, 1, 2\right\}$

6)



domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$

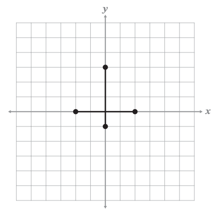
14A PRACTICE 2

Practice 2

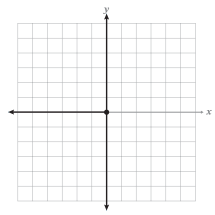
Complete the problems on a separate sheet of paper.

Use the domain and range intervals below to choose the graph that best matches the interval, and fill in the blank with the corresponding letter. Then, write the domain and range for each graph in interval notation.

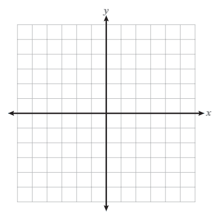
B **1)**



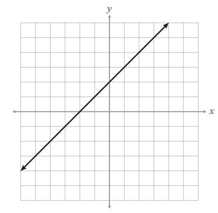
C **2)**



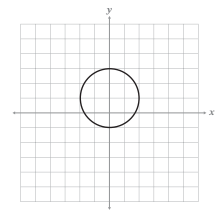
A **3)**



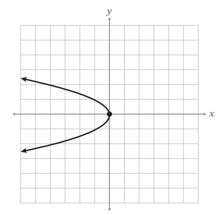
A)



B)



C)



4) Are graphs A, B, and C all functions? Explain.

PRACTICE 2 14A

Create a graph from the equation or table of values. Then, write the domain and range for the function in interval notation or set notation.

5)

x	y
0	0
$\frac{1}{4}$	$\frac{1}{2}$
1	1
4	2

6) $y = x + 2$

Determine if the function is discrete or continuous.

7) $h(x) = -\frac{1}{2}x - 6$ with the domain $\{-2, 0, 2, 4, 6\}$

8) $g(x) = x - 2$ with a domain of $(-\infty, \infty)$.

9) The speed of a car traveling on a local highway.

10) The total number of questions on a worksheet.

Write the equation for the line. Choose the best form of the line to write for the given scenarios.

11) A caterer charges \$25 per person plus a deposit of \$500.

12) What would the cost be for 40 people?

13) Cardi and Billie were on a road trip and reached an elevation of 14,360 feet. Seventy-five minutes later, their elevation was 3,110 feet above sea level.

14) Explain what the rate of change means in this context.

Brooke could download songs for \$1 and movies for \$5. She has a total of \$24 to spend on media this month.

15) What is the maximum number of movies Brooke is able to purchase? Explain.

16) Would Brooke be able to purchase 3 movies and 9 songs? Show or explain your work.

Siggy and her family rented a car while on vacation. Speedy Rentals charged \$17 per day. When Siggy returned the car, she paid \$283 for her 9-day rental.

17) Write an equation to model the situation. Is this a discrete or continuous function?

18) What fee did Speedy Rentals charge Siggy? Show or explain your work.

Planet Exercise charges a \$10 joiner's fee and then \$5 per month for a gym membership.

19) Determine if it is discrete or continuous. Explain your reasoning.

20) Write the equation for the line. Choose the best form of the line to write for the given scenario.

21) Explain why it does not make sense to graph the equation in the second quadrant.

22) If Nicole has a yearly budget of \$65, would she be able to afford a membership to Planet Exercise? Explain.

7) Discrete: The domain is a set of points. The graph will also be a set of points.

8) Continuous: The domain is a continous interval.

9) Continuous: Partial speeds (47.62 mph) are possible.

10) Discrete: The number of questions can be counted. You would not have a partial question.

11) Slope-intercept,
$y = 25x + 500$

12) $y = 1{,}500$
The cost for 40 people will be \$1,500.

13) Slope-intercept,
$y = -150x + 14{,}360$

14) The rate of change means that they descend 150 feet every minute.
-150 feet per minute

15) $y = 4.8$
Brooke can purchase a maximum of 4 movies if she purchases 0 songs. This needs to be rounded down because 5 movies would be \$25, which is over her budget.

16) Brooke would be able to purchase this combination of songs and movies because it is exactly \$24.

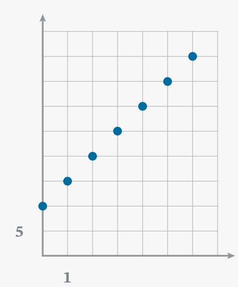
17) Point-slope,
$y - 283 = 17(x - 9)$
This is a discrete function because you have to rent a car for a number of days.

18) The fee would be the y-intercept because this is the starting value for the equation.

$y = 130$
The rental fee is \$130.

19) This is a discrete function because fees are only charged once a month.

20) Slope-intercept form,
$y = 5x + 10$



21) The second quadrant has negative x-coordinates. The variable x represents time in months, and it is not possible to go backward in time to join a gym.

22) $x = 11$
Nicole would not be able to have a gym membership for a full year because there are 12 months in a year and her budget only allows for 11 months.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Your student should look for patterns in the running times. This will help them connect the rate of change to the new topic, arithmetic sequences.

14B EXPLORE

Part B: Arithmetic Sequences

Objectives

In this part of the lesson, you will learn about arithmetic sequences.

By the end of this lesson, you will be able to do the following:

- Describe the arithmetic sequence of a given set.
- Use a sequence to find additional terms.

Why?

Linear equations all contain patterns, these can also be represented as a sequence of terms. When an arithmetic sequence occurs, it also represents a linear relationship.

Warm Up

1) Assuming a steady pace for each runner, fill in the missing race times.

	1.5 miles	2 miles	2.5 miles	3 miles
Mina	**10.5 min**	14 min	17.5 min	21 min
Albert	12 min	**16 min**	20 min	24 min
Desiree	9.75 min	13 min	16.25 min	**19.5 min**
Toby	18 min	24 min	**30 min**	36 min

2) Assuming that each of the runners maintains the same running rate for all 3 miles, write a sentence describing their rate of change.

Mina runs one mile in 7 minutes (21 min/3 miles).
Albert runs one mile in 8 minutes (24 min/3 miles).
Desiree runs one mile in 6.5 minutes (19.5 min/3 miles).
Toby runs one mile in 12 minutes (36 min/3 miles).

EXPLORE 14B

Arithmetic Sequences

- A **sequence** is a discrete function which has a domain of only natural numbers.
- Each term is labeled with a **subscript** that indicates its position in the sequence.
- **Arithmetic sequence**: a particular type of sequence used in mathematics that contains a particular pattern.
  - Begins with the first term in the sequence, a_1.
  - Adds the same value, d, to each term to get to the next term in the sequence.
    - This value, d, is referred to as the **common difference** because it is the value that results from subtracting any two consecutive terms.
  - Uses ellipsis (...) to indicate that the sequence has more terms that are not listed and continues on for n-terms.
- The explicit formula for an arithmetic sequence where n is the n^{th}-term being found is:

  $$a_n = a_1 + d(n - 1)$$

Example 1

Determine if the following sequences are arithmetic.

A) $\{20,\ 30,\ 40,\ 50,\ ...\}$

+10 +10 +10

$a_1 = 20$
$d = +10$

This is arithmetic because the numbers increase by a *constant rate* of 10. Another way of saying a constant rate is the common difference of 10.

$a_n = 20 + 10(n - 1)$

B) $\{10, 100, 1000, 10000, 100000, ...\}$

·10 ·10 ·10 ·10

This is *not* arithmetic because the numbers increase by a *multiple* of 10.

C) $\{10,\ 5,\ 0,\ -5,\ -10,\ ...\}$

−5 −5 −5 −5

This is arithmetic because the numbers decrease by a constant rate of 5.

$a_1 = 10$
$d = -5$

$a_n = 10 - 5(n - 1)$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What does the ellipsis (...) mean?

A: *This means that the arithmetic sequence continues past the given numbers.*

Q: How do you find the common difference?

A: *Subtract any two consecutive values in the sequence.*

Q: What number replaces n?

A: *The subscript number.*

Example 2

Use sequence G to answer the following questions.

$G = \{3, 7, 11, 15, 19, \ldots\}$

Plan Identify the rule for this sequence.

What term is a_1? 1 (first)

What is the value of a_1? $a_1 = 3$

What is the value of d? $d = 15 - 11$ (or any two consecutive terms) = 4

Rule in words:

Beginning with 3, continue adding 4 to each term to solve for the next term.

What term is a_3? **3 (third)**

What is the value of a_3? $a_3 = 11$

What is the value of a_5? $a_5 = 19$

What is the value of a_6? $a_6 = 19 + 4 = 23$

Using the explicit formula, $a_n = a_1 + d(n - 1)$, find a_8 and a_{21}.

$a_n = a_1 + d(n - 1)$

$a_n = 3 + 4(n - 1)$ Rewrite the formula with a_1 and d.

$n = 8$	$n = 21$
$a_n = 3 + 4(n - 1)$	$a_n = 3 + 4(n - 1)$
$a_8 = 3 + 4(8 - 1)$	$a_{21} = 3 + 4(21 - 1)$
$a_8 = 3 + 4(7)$	$a_{21} = 3 + 4(20)$
$a_8 = 31$	$a_{21} = 83$

Checkpoint

Given the following sequence, find a_1 and d.
Then, use the formula $a_n = a_1 + d(n - 1)$ to solve for a_3 and a_{32}.

$A = \{4, 7, 10, 13, \ldots\}$

$a_1 = 4$	$a_3 = 4 + 3(3 - 1) = 4 + 3(2)$	$a_{32} = 4 + 3(32 - 1) = 4 + 3(31)$
$d = 7 - 4 = 3$	$a_3 = 10$	$a_{32} = 97$
$a_n = 4 + 3(n - 1)$		

Practice 1

Complete the problems on a separate sheet of paper.

Determine if the following sequences are arithmetic. If so, find the common difference and the next three terms in the sequence (a_5, a_6, a_7).

1) $\{2, 4, 8, 16, \ldots\}$

2) $\{2, 4, 6, 8, \ldots\}$

3) $\{6, 5.5, 5, 4.5, \ldots\}$

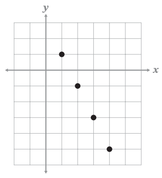
4) $\{20, 10, 5, 2.5, \ldots\}$

Use the arithmetic sequence formula, $a_n = a_1 + d(n - 1)$.

5) $\{2, -1, -4, -7, -10, \ldots\}$
What is the value of a_1?
What is the value of d?
Use the formula to find a_5, a_{10}, and a_{12}.

6) $\left\{-5, -4\frac{2}{3}, -4\frac{1}{3}, -4, \ldots\right\}$
What is the value of a_1?
What is the value of d?
Use the formula to find a_5, a_{10}, and a_{12}.

When a sequence is graphed, the domain values represent the terms of the sequence $\{1, 2, 3, \ldots, n\}$. The range is represented by the value of a_n. Another way to think about this is (n, a_n). The graph below represents an arithmetic sequence.



7) List the first 4 values of the range.

8) List the first 4 values of the domain.

9) Write the formula for the sequence for the n^{th} term.

10) Write the arithmetic sequence equation found in problem 9, in slope-intercept form and function notation.

11) The common difference of an arithmetic sequence is $\frac{1}{3}$. If $a_7 = 5$, what is a_1? Show your work.

12) The common difference in an arithmetic sequence is -4. If $a_{11} = -2$, what is a_1?

Aria and Reza were trying to go on as many rides as possible at the amusement park. After 1 hour, they had been on 2 rides, in 1.5 hours they had been on 5 rides, and in 2 hours they had been on 8 rides.

13) Write the arithmetic sequence supposing this continues until the park closes.

14) Write the formula for the sequence.

15) Suppose the park was open for 12 hours, how many rides did Aria and Reza go on?

Eliza's job requires her to be able to lift boxes that weigh up to 40 pounds. She is moving boxes in the warehouse and wants to track the total amount she lifted in one day. Below is the table Eliza used to track her totals for the day.

Box #	1	2	3	4	5
Total weight	32	64	96	128	160

16) Write the formula for the arithmetic sequence.

17) How many pounds will Eliza have moved for a_{15}? Show your work.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) not arithmetic

Q: How can you determine if a sequence is arithmetic?

A: *Make sure there is a common difference between each pair of consecutive numbers.*

2) arithmetic,
$d = 2, a_5 = 10, a_6 = 12, a_7 = 14$

3) arithmetic,
$d = -0.5, a_5 = 4, a_6 = 3.5, a_7 = 3$

4) not arithmetic

5) $a_1 = 2$
$d = -3$
$a_5 = -10$
$a_{10} = -25$
$a_{12} = -31$

Q: What value do you substitute for n?

A: *The subscript number.*

6) $a_1 = -5$
$d = \frac{1}{3}$
$a_5 = -\frac{11}{3}$
$a_{10} = -2$
$a_{12} = -\frac{4}{3}$

7) $\{1, -1, -3, -5\}$

Q: What would the difference be if this was a linear equation?

A: *The points would be connected with a line.*

8) $\{1, 2, 3, 4\}$

Q: What variable will replace a_n?

A: y

9) $a_1 = 1, d = -2$
$a_n = 1 - 2(n - 1)$

10) $y = -2n + 3$
$f(x) = -2n + 3$

11) $a_1 = 3$

Be sure your student substitutes all values into the formula carefully.

Q: What value will replace n in the formula?

A: *7*

Q: What value will replace a_n?

A: *5*

12) Another way that may help your student is to remind them that the subscript is the input, and the term is the output.

$a_1 = 38$

Q: What number will replace n?

A: 11 (the subscript number)

Q: What number will replace a_n?

A: *−2*

13) $\{2, 5, 8, \ldots\}$

Q: What are the independent and dependent variables in this question?

A: *(time, rides)*

14) $a_n = 2 + 3(n - 1)$

15) $a_{12} = 35$

If the park is open for 12 hours, Aria and Reza can go on 35 rides.

Q: How would you change your sequence knowing that the park is open for twelve hours?

A: *The sequence would have an end number {2, 5, 8, ..., 35}*

16) $a_n = 32 + 32(n - 1)$

17) $a_{15} = 480$

Q: Do you think there is a maximum number of pounds that Eliza can lift in one day? Explain.

A: *Yes, because at some point, Eliza will get tired and not be able to lift anything else, or she will stop lifting boxes when she leaves work for the day.*

Eliza will have moved 480 pounds total.

Mastery Check

Show What You Know

Have your student list the arithmetic sequence in set brackets if they are unsure of how to start. $\{18, 23, 28, \ldots, 58\}$

Remind your student to find the common difference.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words.
If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Describe the arithmetic sequence of a given set.
- Use the sequence to find additional terms.

14B MASTERY CHECK

Mastery Check

Show What You Know

Kalen was stacking chairs in the school gym after a music concert. Starting with one chair, the stack was 18 inches off the ground. With two chairs, the stack was 23 inches off the ground, and with three chairs, it was 28 inches.

A) Write a formula to find the n^{th} number of chairs in a stack.

$a_1 = 18, d = 5$
$a_n = 18 + 5(n - 1)$

B) If the maximum height allowed for stacking chairs is 58 inches, how many chairs can be put in a stack?

$58 = 18 + 5(n - 1)$
$40 = 5n - 5$
$45 = 5n$
$n = 9$
There can be a maximum of 9 chairs in a stack.

C) Write the domain and range for this problem. Why does this sequence have an ending point?

domain: $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
range: $\{18, 23, 28, 33, 38, 43, 48, 53, 58\}$
This sequence ends because the chairs can only be stacked to 58 inches high.

D) Write the equation in slope-intercept form. What do the variables x and y represent?

$a_n = 18 + 5(n - 1)$
$a_n = 18 + 5n - 5$
$y = 5x + 13$
x and y represent: (number of chairs, height of chairs) $(0, 13)$

E) Why does the y-intercept *not* make sense for the context of this problem?

The y-intercept does not make sense because it would mean that 0 chairs have a height of 13 inches.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Lesson and Unit Tests

As students reach the end of of this unit, they can complete the Lesson Test and the Unit Test.

Students should complete the Lesson Test in the same manner as they have in previous lessons.

The next day, they may need to review Lessons 7–14, using their guided notes, Mastery Checks, and Lesson Tests before beginning the Unit Test.

PRACTICE 2 14B

Practice 2

Complete the problems on a separate sheet of paper.

Determine if the following sequences are arithmetic. If so, find the common difference and the next three terms in the sequence (a_5, a_6, a_7).

1) $\{1, 4, 9, 16, ...\}$

2) $\{3, 9, 27, 81, ...\}$

3) $\{-1, -2, -3, -4, ...\}$

4) $\{20, 15, 10, 5, ...\}$

Use the formula for an arithmetic sequence, $a_n = a_1 + d(n-1)$, to find a_5, a_{10}, and a_{12}

5) $d = -\frac{1}{2}, a_1 = 26$ **$a_5 = 24$, $a_{10} = \frac{43}{2}$, $a_{12} = \frac{41}{2}$**

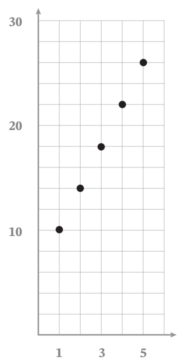
6) $d = 4, a_1 = -5$ **$a_5 = 11$, $a_{10} = 31$, $a_{12} = 39$**

7) The first four houses on Green Street are numbered 201, 208, 215, and 222. What house (term) is numbered 264?

8) Rolanda drives 22 miles each day. Today her odometer read 13,426 miles. Write a formula for the n^{th} term, then solve for the odometer reading on day 45.

9) The common difference for an arithmetic sequence is -0.25. If $a_{21} = 6$, what is a_1?

Mina Rees wanted to determine a formula for the following graphed arithmetic sequence.



10) Find the formula for the arithmetic sequence.

11) Write an equation in slope-intercept form to represent the graph.

12) What do the common difference, d, and the rate of change, m have in common?

Create your own arithmetic sequence with a common difference of $\frac{1}{5}$.

13) List the first 5 terms of your sequence.

14) Write the formula for the n^{th} term.

15) Find a_9 using your formula.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

1) Not arithmetic

2) Not arithmetic

3) arithmetic,
$d = -1, a_5 = -5, a_6 = -6, a_7 = -7$

4) arithmetic,
$d = -5, a_5 = 0, a_6 = -5, a_7 = -10$

7) $a_{10} = 264$
The 10^{th} house is 264 OR 264 Green Street.

8) $a_{45} = 13{,}426 + 22(n-1)$
$a_{45} = 14{,}394$
Rolanda's odometer reading on day 45 would be 14,394.

9) $a_n = a_1 - 0.25(n-1)$
$a_1 = 11$

10) $a_n = 10 + 4(n-1)$

11) $y = 4n + 6$

12) Both d and m show the change in y over the change in x, or the slope.

13) See student work, terms should increase by $\frac{1}{5}$ each time.

14) $a_n = a_1 + \frac{1}{5}(n-1)$
a_1 will be determined by the sequence your student wrote in problem 13.

15) Your student should substitute 9 for n and solve for a_n.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note regarding Lesson and Unit Tests.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

3) Tripp painted faster. He painted pictures three times faster than Rook.

4) $x = 4$ (vertical), $y = 5$ (horizontal)

7) Jimena spends $25 per week on lunch or $m = -\frac{25}{1}$.

8) Point-slope form, $y - 520 = 20(x - 6)$

9) Distractor Rationale:

A) and B) represent a negative correlation when the graph shows a positive correlation.

B) and C) represent a weak correlation when this is a strong correlation.

10) Distractor Rationale:

B) common error: taking the constants to form an ordered pair

C) common error: thinking the coefficient of x will be the correct solution

D) common error: not checking the signs of the values

11) Distractor Rationale:

B) and D) have slopes too large for the scale of the graph.

C) and D) represent positive slopes when the scatter plot has negative correlation.

12) Distractor Rationale:

A) and C) incorrectly calculate slope as run over rise.

C) and D) represent a negatively sloped function.

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Write the equation in slope-intercept form given the points $(-4, -2)$ and $(-3, 1)$. $y = 3x + 10$

2) Convert the equation for the line from standard form to slope-intercept form: $2x - 3y = -6$ $y = \frac{2}{3}x + 2$

3) Rook and Tripp painted pictures. The equations below represent how many pictures they had ready for sale after a given number of hours. Determine who painted faster and how much faster he painted than the other.

Rook: $p = 4h + 5$ Tripp: $p = 12h - 2$

4) Write the equation for the horizontal line and vertical line that passes through the point $(4, 5)$.

5) Is relation M a function? Explain your answer.
M: $\{(1, 3), (4, -2), (9, 1), (3, 4), (5, 3)\}$ **Yes, for every input (x) there is only one output (y).**

6) Write the equation perpendicular to the given line, $y = -\frac{1}{4}x + 11$, through $(-3, 8)$. $y = 4x + 20$

7) Jimena used the equation $r(w) = -25w + 500$ to represent her lunch budget for her first semester at college, where w represents weeks and $r(w)$ represents the remaining money after w weeks. Explain the meaning of the slope and y-intercept related to the problem.

8) Write a linear equation. Hector saved $20 each week from his paycheck. After working for 6 weeks, he had $520 in his savings account. Write a linear equation to model this situation. Name the form of the equation used.

Multiple Choice

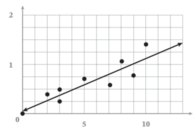
__D__ **9)** Determine the correlation coefficient that best matches the scatter plot and trend line.

A) $r = -0.91$

B) $r = -0.42$

C) $r = 0.43$

D) $r = 0.91$



__A__ **10)** Which point lies on the line $2x + y = 1$?

A) $(2, -3)$

B) $(2, 1)$

C) $(2, 0)$

D) $(2, 3)$

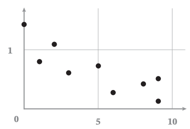
__A__ **11)** Choose the equation that best matches the given scatter plot.

A) $y = -0.1x + 1.15$

B) $y = -10x + 1.15$

C) $y = 0.1x + 1.15$

D) $y = 10x + 15$



__B__ **12)** Given the table, write the equation in slope-intercept form.

A) $y = \frac{4}{3}x$

B) $y = \frac{3}{4}x$

C) $y = -\frac{4}{3}x$

D) $y = -\frac{3}{4}x$

x	$z(x)$
−4	−3
0	0
4	3
8	6

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	10	11	10	11	7	12	10	10	13	7	13	10