

Lesson 13

Scatter Plots

Outline

Part A Correlation and Scatter Plots

- Linear Correlations
- Graphing Scatter Plots

Part B Line of Best Fit

- Calculating Line of Best Fit
- Drawing Conclusions Using the Trend Line

Targeted Review

Vocabulary

- bivariate data
- correlation
- scatter plot
- line of best fit
- interpolate
- extrapolate



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Part A: Correlation and Scatter Plots

Objectives

In this part of the lesson, you will learn about correlation and scatter plots.

By the end of this lesson, you will be able to do the following:

- ☑ Identify a correlation of a scatter plot as strong or weak, positive or negative, or no correlation.
- ☑ Explain the meanings of correlations in real-life examples.
- ☑ Create a scatter plot with accurate scale, labels, and ordered pairs.

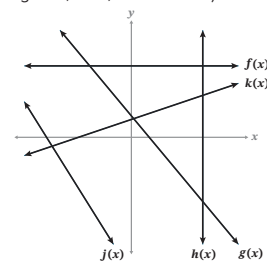
Why?

Have you ever heard the phrase “A picture is worth a thousand words”? Scatter plots are a way to see real-life data as a picture that you can use to draw conclusions, make predictions, and prepare for likely outcomes.

Warm Up

Use the graph to classify each line by its slope (positive, negative, zero, or undefined).

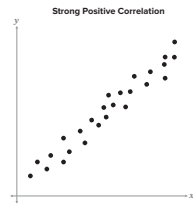
- 1) $f(x)$ **zero**
- 2) $g(x)$ **negative**
- 3) $h(x)$ **undefined**
- 4) $j(x)$ **negative**
- 5) $k(x)$ **positive**



Linear Correlations

- **Bivariate data** is a set of data that pairs independent and dependent variables to form a correlation.
- **Correlation** is a relationship between two variables or two data sets.
- A **scatter plot** is a graph that visually represents bivariate data.
- A **correlation coefficient** (r) is the number that shows how correlated, or related, the data set is.
 - The r -value will be given to you, or will be calculated by using technology.

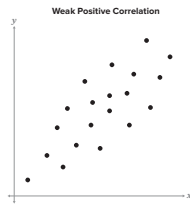
Positive Correlation



$r = 0.96$

As x increases,

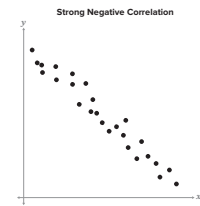
y increases.



$r = 0.42$

As x increases, y increases, but the ordered pairs are more spread apart.

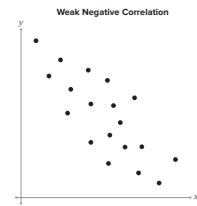
Negative Correlation



$r = -0.9$

As x increases,

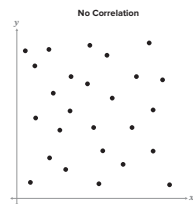
y decreases.



$r = -0.42$

As x increases, y decreases, but the ordered pairs are more spread apart.

No Correlation



$r = 0$

The r -value equals zero.

As x increases, y has no relationship to x .

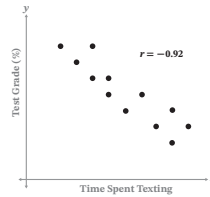
To describe correlation from a graph:

- List independent and dependent variables as ordered pairs.
- Complete sentence: As x increases, y [increases / decreases / has no relation to x]

For this course, a weak correlation will have an r -value between $|0.5|$ and 0.

Example 1

Determine the correlation for the graph. If it has a positive or negative correlation, describe the meaning of the correlation in context.



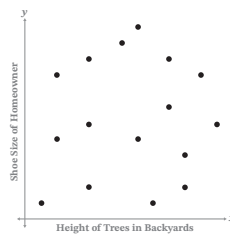
Correlation coefficient: $r = -0.92$

Independent and dependent variables:
(time spent texting, test grade)

As time spent texting increases the test grade percentage decreases.
If a student wants to improve their test score, they could try spending less time texting.

Example 2

Name and explain the correlation for the graph.



correlation coefficient: **unknown, not provided**

(height of trees , shoe size of homeowner)

Because the height of trees and the shoe size of the homeowner are not related to each other,
no correlation exists.

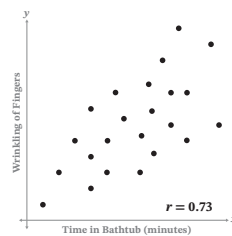
Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

- Q: What does the correlation coefficient, or r -value, tell you about the correlation?
A: It tells you how strong or weak the correlation of the data set is.

Checkpoint

Determine the correlation for the graph. If it has a positive or negative correlation, describe what the correlation means within the given context.



(minutes in bathtub, wrinkling of fingers)

The graph shows a positive correlation.

As time in the bathtub increases, the more wrinkled your fingers get.

Graphing Scatter Plots

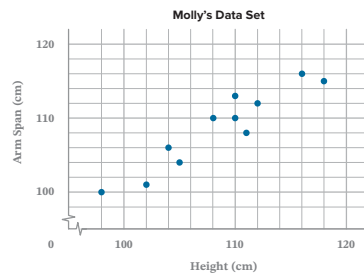
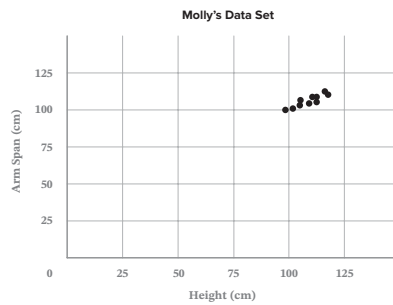
- When graphing scatter plots, it is important to graph all of the data precisely.
- Before graphing the ordered pairs, determine the scale of the graph and label the x and y-axis.
- The ordered pairs for a scatter plot will be given as a table or a relation.

Example 3

Graph the given data set to determine the relationship between the two values.

Molly collected data from children in her neighborhood. She measured their height and their arm span in centimeters and recorded the data in a table. She wants to know the relationship, if any, between the two values. She decided to make a scatter plot.

| Height (cm) | Arm Span (cm) |
|-------------|---------------|
| 102 | 101 |
| 104 | 106 |
| 111 | 108 |
| 98 | 100 |
| 108 | 110 |
| 118 | 115 |
| 116 | 116 |
| 110 | 113 |
| 112 | 112 |
| 110 | 110 |
| 105 | 104 |



- You can use a squiggle on one or both axes to denote a break at the start of a graph.
- As height increases, arm span also increases.
- This looks like a strong positive correlation between height and arm span.

Example 3

Since the r -value is not calculated, we can only estimate if there is a strong or weak correlation.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

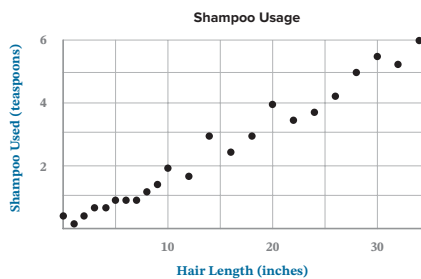
Q: What would you estimate the correlation coefficient to be for your graph?

A: Sample:

It is close to 0.95 because the graph is very close to forming a line.

 Checkpoint

Correctly label the scatter plot describing the relationship between hair length in inches and the amount of shampoo used in teaspoons. Name and explain the correlation.



(hair length, shampoo used)

As hair length increases, the amount of shampoo used increases. This is a positive correlation because the independent and dependent variables are both increasing.

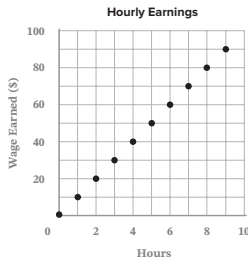
Practice 1

Complete the problems on a separate sheet of paper.

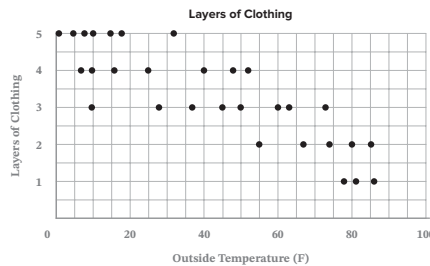
Given the type of correlation, complete the sentence and sketch an example.

- 1) Strong negative correlation, $r = -0.89$ As x increases, y decreases.
- 2) No correlation, $r = 0$ As x increases, y is not related.
- 3) Weak positive correlation, $r = 0.46$ As x increases, y increases.

- 4) Determine the correlation for the graph. If it is a positive or negative correlation, describe the meaning in context.



- 5) Determine the correlation for the graph. If it is a positive or negative correlation, describe the meaning in context.



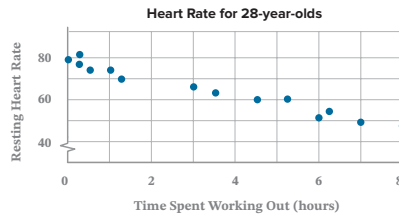
- 6) Emeril collected data from his class. He asked each person in his class their height and the grade they earned on their last test. Name the correlation and describe the relationship. Explain why your statement makes sense.

Use the data set to compare time spent working out in hours, x , to the resting heart rate, y , of 28-year-olds.

| x | 0 | 0.25 | 0.25 | 0.5 | 1 | 1.25 | 3 | 3.5 | 4.5 | 5.25 | 6 | 6.25 | 7 | 8 |
|-----|----|------|------|-----|----|------|----|-----|-----|------|----|------|----|----|
| y | 80 | 82 | 78 | 75 | 75 | 70 | 67 | 64 | 60 | 61 | 52 | 55 | 50 | 48 |

- 7) Graph.
- 8) Explain the scale of the graph.
- 9) Name the type of correlation and explain the relationship between the data sets.

The correlation is negative. As time spent working out increases, the resting heart rate decreases.



- 6) There is no correlation between the height of a person and a test score. Your height does not determine your test grade, nor does a test grade determine your height.

Q: What is the correlation coefficient for this scenario? Explain.

A: $r = 0$ because there is no correlation between the data points.

- 8) Sample:

The scale for the x -axis counts by ones because the domain is from 0 through 8. The graph shows a break on the y -axis because the data has a range from 48 through 82. Therefore, showing 0–40 on the y -axis is not necessary.

The scale refers to the number(s) you count by on each axis.

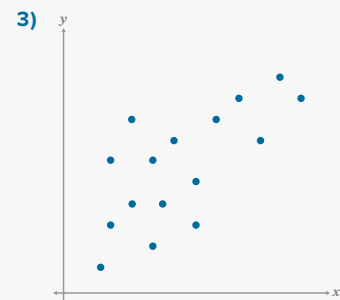
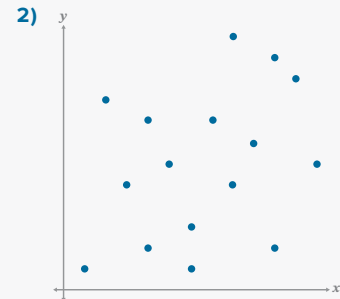
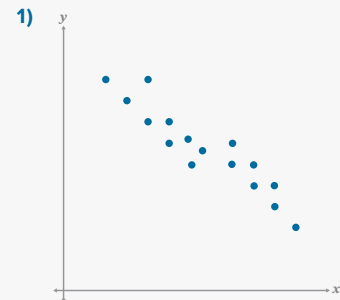
- 9) In this case, a negative correlation is good because a lower resting heart rate can show a healthier heart.

Q: If you were told that the correlation coefficient was -0.98 , what would that tell you about the data?

A: There is a very strong negative correlation.

Practice 1

Worked solutions for these problems are located in the Digital Pack.



Your student can use the r -value, or correlation coefficient, to help determine if the graph has a strong or weak correlation. The closer to 1 or -1 , the stronger the correlation.

- 4) This is a strong positive correlation. As the number of hours increases, the wages earned increase. When you get paid hourly, the more hours you work, the more you get paid.

The correlation coefficient is $r = 1$, or perfect correlation. This is because it shows a slope of \$10/1 hour worked.

Q: What is the slope for this graph?

A: $\frac{10}{1} = 10$

Q: What does this represent for the given problem?

A: Getting paid \$10 an hour worked.

- 5) This is a weak negative correlation. As the temperature increases, the number of layers of clothing decreases. The hotter it is, the less likely it will be for someone to wear a sweater or jacket.

Q: What is the slope for this graph?

A: Your student should have a negative value somewhere between -0.5 and 0.

- 11) The scale on the x -axis is counting by 25s from 550 to 850.

The scale on the y -axis is counting by 1,000s from 6,000 to 10,000. Both axes needed a break because the data is very far away from 0 on either axis.

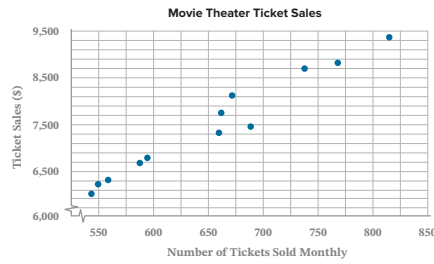
- 12) Q: If you were told that the correlation coefficient was 0.98, what would you know about the data?

A: *This means there is a very strong positive correlation between ticket sales and the number of tickets sold.*

13A PRACTICE 1

This data set compares the number of tickets sold monthly, x , to the ticket sales, y , for a local theater.

- 10) Graph.



- 11) Explain the scale of the graph.
 12) Name the type of correlation and explain the relationship between the data sets.

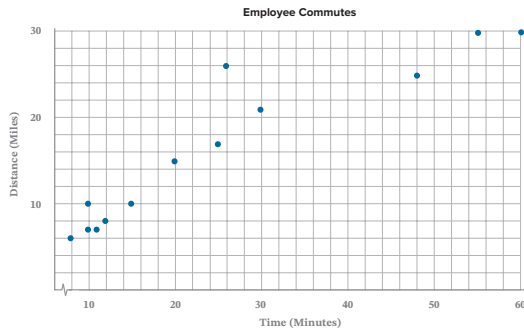
The correlation is positive. As the number of tickets sold increases, the amount of money earned increases. A movie theater will make more money if more people are buying tickets.

| Numbers of tickets sold monthly | Ticket sales (\$) |
|---------------------------------|-------------------|
| 814 | 9,390 |
| 767 | 8,838 |
| 659 | 7,340 |
| 737 | 8,713 |
| 688 | 7,475 |
| 671 | 8,138 |
| 661 | 7,763 |
| 558 | 6,330 |
| 587 | 6,693 |
| 594 | 6,805 |
| 549 | 6,240 |
| 543 | 6,038 |

Mastery Check

Show What You Know

A) Create a scatter plot for the data set.



| Time spent traveling to work | Distance traveled |
|------------------------------|-------------------|
| 20 | 15 |
| 30 | 21 |
| 26 | 26 |
| 48 | 25 |
| 10 | 7 |
| 8 | 6 |
| 11 | 7 |
| 15 | 10 |
| 25 | 17 |
| 10 | 10 |
| 60 | 30 |
| 55 | 30 |
| 12 | 8 |

B) Name the type of correlation and explain the relationship between the values.

The correlation is positive. As time increases, the distance people travel to work increases. The farther you are from a location, the longer it will take to get there.

C) What does the ordered pair (25, 17) mean in the context of this problem? Explain your thinking.

The ordered pairs represent (time in minutes, distance traveled to work in miles). The independent variable is time, and the dependent variable is miles traveled to work. The ordered pair (25, 17) means that it took a person 25 minutes to travel 17 miles to work.

D) One employee reported that it took them 25 minutes to travel 2 miles to work. This person walks to work while the other employees surveyed drive. Explain why this data point was not included.

The data collected from all the employees except this one was about driving to work. Including the point (25, 2) adds an outlier to the data set since no other person walks to work.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

A) Q: Why is it important to think about the scale of your graph before you start graphing?

A: *To ensure the data is accurately represented and that all the data points will fit on the graph.*

B) Q: What does a correlation coefficient of 0.93 tell you about this data set?

A: *There is a strong positive relationship between distance traveled to work and the time.*

D) Outliers were discussed in single-variable data in Lesson 6. They also occur in bivariate data. Using technology to graph with and without outliers provides a wonderful visual for the effect outlier(s) can have on the line of best fit. The line of best fit is covered in Part B of this lesson.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- ☑ Identify a correlation of a scatter plot as strong or weak, positive or negative, or no correlation.
- ☑ Explain the meanings of correlations in real-life examples.
- ☑ Create a scatter plot with accurate scale, labels, and ordered pairs.

Practice 2

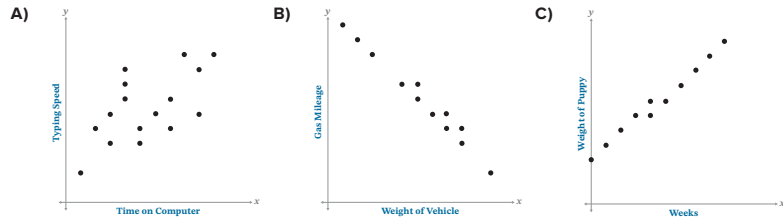
Worked solutions for these problems are located in the Digital Pack.

- 1) Be aware that your student may be tempted to switch the answers for graphs A and C. This is a good time to have a conversation with your student about subjectivity within strong and weak correlations and why the r -values are included. Try having your student explain their reasoning for the graphs they chose, and then discuss how it might be seen the other way.
- 4) As distance driven in miles increases, the gas remaining in the tank decreases. This shows a negative correlation.
- 5) As height in inches increases, the number of pairs of socks has no relation to height. This data set has no correlation.
- 6) As the age of a dishwasher increases, the number of repairs needed increases. This shows a positive correlation.
- 7) $r = 0.99$
Remember that the y -axis is in billions of people, so 0.1 billion is 100 million people.
- 8) As years continue, the population increases. This is a strong positive correlation.
- 9) The trend will continue since the data shows that each year the world population gets larger.

Practice 2

Complete the problems on a separate sheet of paper.

Choose the graph that best matches the description. Label each axis.



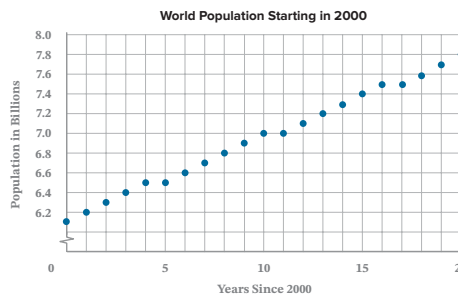
- B** 1) As the weight of the vehicle increases, the gas mileage decreases. (weight of vehicle, gas mileage) $r = -0.97$
- C** 2) As time in weeks increased, the weight of the puppy increased. (weeks, weight of puppy) $r = 0.98$
- A** 3) As time on a computer increased, typing speed increased. (time on computer, typing speed) $r = 0.48$

Explain the relationship, if any, between the given variables.

- 4) (distance driven in miles, gas remaining in tank)
- 5) (height in inches, number of pairs of socks)
- 6) (age of a dishwasher, number of repairs needed)

Use the data set to compare the year and approximate world population in billions.

- 7) Graph the data set.

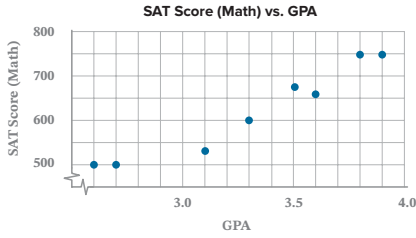


| Year | Population in billions |
|------|------------------------|
| 2020 | 7.8 |
| 2019 | 7.7 |
| 2018 | 7.6 |
| 2017 | 7.5 |
| 2016 | 7.5 |
| 2015 | 7.4 |
| 2014 | 7.3 |
| 2013 | 7.2 |
| 2012 | 7.1 |
| 2011 | 7 |
| 2010 | 7 |
| 2009 | 6.9 |
| 2008 | 6.8 |
| 2007 | 6.7 |
| 2006 | 6.6 |
| 2005 | 6.5 |
| 2004 | 6.5 |
| 2003 | 6.4 |
| 2002 | 6.3 |
| 2001 | 6.2 |
| 2000 | 6.1 |

- 8) Describe the correlation, if any exists.
- 9) Would you expect the trend you described in Problem 8 to continue? Explain.

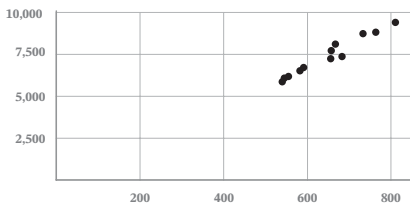
Use the data set to compare students' GPAs, x , to their SAT scores, y .

- 10) Graph the data set.



- 11) Determine the type of correlation and describe its meaning in the situation.
- 12) Explain how to improve the graph of the data set.

Ticket Sales (\$) vs. Number of Tickets Sold Monthly



| Student | GPA | SAT Score (Math) |
|---------|-----|------------------|
| 1 | 3.3 | 600 |
| 2 | 3.1 | 530 |
| 3 | 2.7 | 500 |
| 4 | 3.8 | 750 |
| 5 | 2.6 | 500 |
| 6 | 3.6 | 660 |
| 7 | 3.9 | 750 |
| 8 | 3.5 | 680 |

- 11) The graph has a positive correlation. (Strong/weak is difficult to determine because the data set is small.) As a student's GPA increases, their SAT math score increases.

$r = 0.97$

The correlation coefficient is provided if you would like to discuss with your student.

- 12) Sample:

The scatter plot does not have labels on the x -axis or y -axis. The x -axis should be the "number of tickets sold monthly." The y -axis should be "ticket sales" in dollars. The graph is not scaled correctly. There should be a break on each axis. The x -axis should start at 500 and count by 50s through 800. The y -axis should start around \$5,000 and count by 1,000s through \$10,000.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

This part of the lesson will use decimal values for the slope and y -intercept with the assistance of technology to calculate the line of best fit. The line of best fit and the correlation coefficient can be calculated using technology (e.g., graphing calculator or app). Then analysis can be completed about the meaning of the values and how strong the correlation is. However, your student will still need to understand how to hand-calculate an estimated line of best fit regardless of whether technology is available to them. If technology is available, have them practice both methods.

Part B: Line of Best Fit

Objectives

In this part of the lesson, you will learn about the line of best fit.

By the end of this lesson, you will be able to do the following:

- ☑ Estimate and draw the line of best fit for a set of data.
- ☑ Write the equation for the line of best fit in the requested form.
- ☑ Use the line of best fit to interpolate, extrapolate, and explain the context of the data.

Why?

Being able to create a line of best fit for a scatter plot can help you make better use of the data. The line of best fit brings order and meaning to the scatter of ordered pairs.

Warm Up

Write the equation of the line in slope-intercept form. Round values to the nearest hundredth where necessary.

1) $(2,001, 6.2), (2,011, 7)$

$$m = \frac{7 - 6.2}{2,011 - 2,001} = \frac{0.8}{10} = 0.08$$

$$y - 7 = 0.08(x - 2,011)$$

$$y - 7 = 0.08x - 160.88$$

$$y = 0.08x - 153.88$$

2) $(0.3, 5), (0.6, 12)$

$$m = \frac{12 - 5}{0.6 - 0.3} = \frac{7}{0.3} = \frac{70}{3}$$

$$y - 5 = \frac{70}{3}(x - 0.3)$$

$$y - 5 = \frac{70}{3}x - 7$$

$$y = \frac{70}{3}x - 2$$

$$y = 23.33x - 2$$

Calculating the Line of Best Fit

- A line of best fit can be drawn on a scatter plot that approximates the bivariate data set.
- The line of best fit is a line that passes through or as close as possible to the most number of data points in the set.
- When possible, there should be the same number of points above and below the line.
- The line of best fit is sometimes referred to as a trend line because it represents the trends, or general tendencies, between the two data sets.

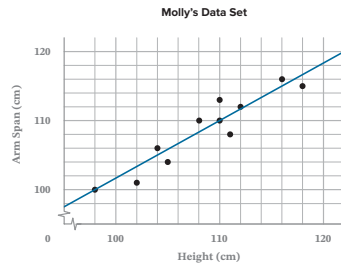
EXPLORE 13B

Example 1

Given the scatter plot, draw the line of best fit.

The line of best fit

- passes through or close to several points
- leaves a similar number of points above and below the line



Find the line of best fit by hand:

- 1) Draw a scatter plot with correct labels and scale.
- 2) Estimate the line of best fit.
- 3) Pick two points on the line.
- 4) Find the equation of the line.

Find the line of best fit and correlation coefficient using technology:

- 1) Enter all ordered pairs from the data set.
- 2) Use the regression equation $y_1 \sim mx_1 + b$.
- 3) Record slope, y -intercept, and r -value.
- 4) If you believe outliers are present, try removing them to see if this improves the r -value.

Example 2

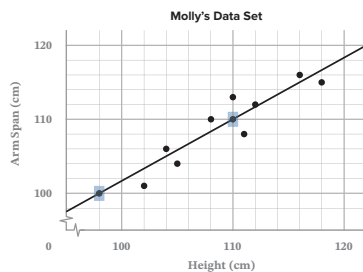
Molly chose two points on her line of best fit: (98, 100) and (110, 110).

Molly uses her chosen points to find the slope.

$$m = \frac{110 - 100}{110 - 98} = \frac{10}{12} = \frac{5}{6}$$

Next, Molly uses slope and the point (110, 110) to write the equation in point-slope form, then converts to slope-intercept form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 110 &= \frac{5}{6}(x - 110) \\ y - 110 &= \frac{5}{6}x - \frac{550}{6} \\ y &= \frac{5}{6}x - \frac{550}{6} + 110 \\ y &= \frac{5}{6}x - 91\frac{2}{3} + 110 \\ y &= \frac{5}{6}x + 18\frac{1}{3} \end{aligned}$$



Using Technology:

If using technology, the ordered pairs and the regression equation would be entered. The values will need to be rounded to the hundredth.

$$y_1 \sim mx_1 + b$$

$$m = 0.85, b = 16.17, r = 0.94$$

Keep Molly's equation, $y = 0.85x + 16.17$, in mind because you will use it later to predict other values for the children in her neighborhood.

Example 1

See the Digital Pack for more on using technology when working with scatter plots and the line of best fit.

- 3) When two points are picked from a line of best fit, they do not need to be points from the given scatter plot. They can be any two points that are on the drawn line and through grid lines on the graph.

Example 2

The r -value is only calculated when you use technology.

Example 3

A tutor decided to compare her students' GPAs to their SAT scores. She wanted a line of best fit to generalize the trend in her data.

Plan Find the equation for the line of best fit.

- Select two points on your line, through grid lines.
- Use the slope formula to find the slope of the line.
- Use point-slope form to convert the equation into slope-intercept form.

By Hand: (675, 3.5), (550, 3.0)

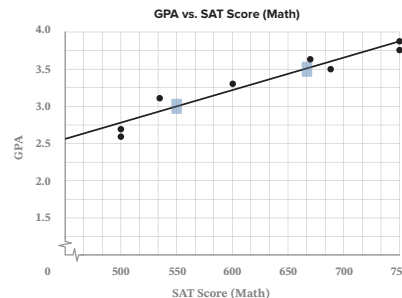
$$m = \frac{3.5 - 3.0}{675 - 550} = \frac{0.5}{125} = 0.004$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 0.004(x - 550)$$

$$y - 3 = 0.004x - 2.2$$

$$y = 0.004x + 0.8$$



Using Technology:

$$y = 0.004x + 0.537$$

$$r = 0.96$$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What does the slope mean in context?

A: The height of the tree is about 16 meters for each meter of the circumference of the trunk. Or 16 meters tall per 1 meter around the tree.

Q: What does the y-intercept mean in context? Does this make sense? Explain.

A: The y-intercept is about (0, 1), meaning with a circumference of 0 meters, the tree is 1 meter tall. This does not make sense because the tree must have some circumference to be 1 meter tall.

The data set will be in the Digital Toolbox for the student if they want to use technology to find the equation.

Checkpoint

Given the Graph: Estimate and draw the line of best fit. Then write the equation for the line in slope-intercept form using two points marked on your line of best fit.

(circumference of trunk, height of tree)

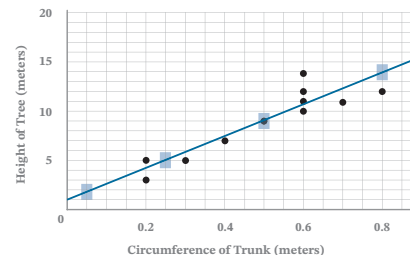
(0.5, 9) and (0.25, 5)

$$m = \frac{9 - 5}{0.5 - 0.25} = \frac{4}{0.25} = 16$$

$$y - 9 = 16(x - 0.5)$$

$$y - 9 = 16x - 8$$

$$y = 16x + 1$$



Equation using technology:

$$y = 15.98x + 1.12$$

$$r = 0.94$$

Ⓛ Drawing Conclusions Using the Trend Line

- Having an equation for the trend line allows you to make predictions about other values in the data set.
- Interpolation is the process of predicting data between the given points.
- Extrapolation is the process of predicting data outside the given points, before or after the data set.

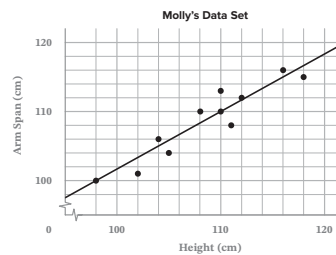
Example 4

Use Molly's graph and equation, calculated with technology, to interpolate and find the missing data.

Molly knows the height of a child that she did not collect data from is 100 centimeters. Molly uses her equation to find the arm span of the child.

Molly's equation: $y = 0.85x + 16.17$

New data point: **(100, y)**
(height, arm span)



Enter the known data into the equation:

$$y = 0.85(100) + 16.17$$

$$y = 101.17$$

Explain

When the child's height is 100 cm, the arm span is about 101 cm.

Example 5

Molly needed to determine heights for two children. She measured their arm spans but forgot to list their heights. The arm spans were 122 centimeters and 96 centimeters. This data cannot be found on the graph, so the equation must be used to extrapolate the information.

Molly's equation: $y = 0.85x + 16.17$

A) New data point: $(x, 122)$

Enter the known data into the equation:

$$122 = 0.85x + 16.17$$

$$105.83 = 0.85x$$

$$x = 124.51$$

B) New data point: $(x, 96)$

Enter the known data into the equation:

$$96 = 0.85x + 16.17$$

$$79.83 = 0.85x$$

$$x = 93.92 \approx 94 \text{ cm}$$

Explain

The height is about 125 cm when the arm span is 122 cm.

The height is about 94 cm when the arm span is 96 cm.

 Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What does the slope mean for this question?

A: For every $\frac{1}{2}$ mile traveled, 1 minute is added to the time traveling to work.

Q: What does the y-intercept mean? Does this make sense for the problem?

A: The y-intercept would be $(0, 4.4)$ or 4.4 miles traveled in 0 minutes. This does not make sense but could mean that everyone travels a minimum of 4.4 miles.

Q: What is the independent variable?

A: time traveling to work

 Checkpoint

Use the provided equation for the line of best fit to make the indicated predictions. Round to the nearest whole number.

$$y = 0.5x + 4.4 \quad (\text{time traveling to work, distance traveled})$$

A) How long will it take a person to travel 40 miles?

$$(x, 40)$$

It will take 71 minutes to travel 40 miles.

$$40 = 0.5x + 4.4$$

$$35.6 = 0.5x$$

$$x = 71.2 \approx 71$$

B) What distance was traveled when it took 45 minutes to get to work?

$$(45, y)$$

In 45 minutes, 27 miles were driven.

$$y = 0.5x(45) + 4.4$$

$$y = 22.5 + 4.4$$

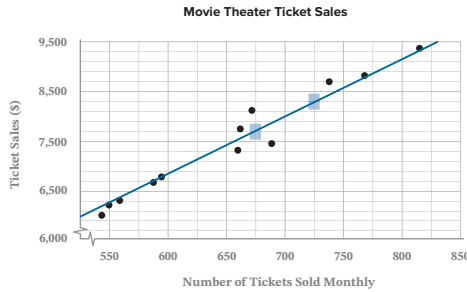
$$y = 26.9 \approx 27$$

Practice 1

Complete the problems on a separate sheet of paper.

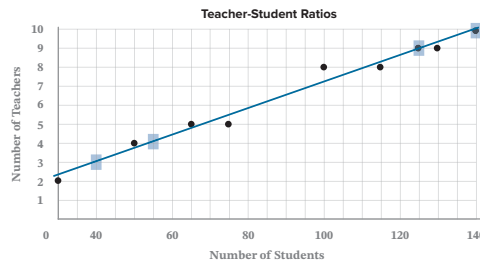
Write equations in slope-intercept form.

The scatter plot represents the data a movie theater collected comparing ticket sales and the number of tickets sold monthly.



- 1) Draw the line of best fit and mark two ordered pairs on the line.
- 2) Write the equation of your line using the ordered pairs.

The scatter plot represents the number of teachers hired compared to the number of students in the school.

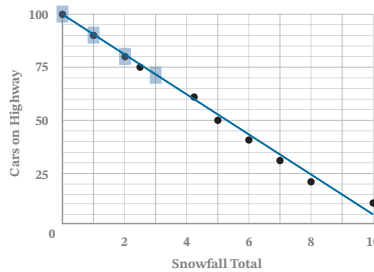


- 3) Draw the line of best fit and mark two ordered pairs on the line.
- 4) Write the equation of your line using the ordered pairs.

Sample:
 (40, 3) and (125, 9)
 $y = \frac{6}{85}x + \frac{3}{17}$

Technology:
 $y = 0.07x + 0.23; r = 0.99$

The scatter plot shows the comparison between the number of cars on the highway and the total snowfall in inches.



- 5) Draw the line of best fit and mark two ordered pairs on the line.
- 6) Write the equation of your line using the ordered pairs.

Sample:
 (0, 100), (1, 90)
 $y = -10x + 100$

Technology: $y = -9.43x + 98.65; r = -0.99$

Practice 1

Worked solutions for these problems are located in the Digital Pack.

If your student uses technology, the equations should exactly match the given technology equation. If calculating by hand from two points, the equation should be close to the given answers (sample and technology).

- 2) Sample:
 (675, 7,700) and (725, 8,300)
 $y = 12x - 400$
 Technology: $y = 11.81x - 240.27; r = 0.98$
 Q: What does the slope tell you about ticket sales?
 A: The cost was approximately \$11.81 for one ticket.

Remind your student to choose points that are both on their line and that are also going through grid lines on the graph. The points do not need to be part of the scatter plot in all cases.

- 4) Q: What does the slope mean for the problem? Try saying the number using proper place-value language.
 A: Seven-hundredths or $\frac{7}{100}$. This means there are 7 teachers for every 100 students.

Q: If there were only 50 students, how many teachers would there be? Explain.

A: Half of 7 is 3.5. This rounds to 4, so there would be 4 teachers.

- 6) Q: Why might it make sense to round the slope and y-intercept when describing their meaning in this problem?

A: Because this relates snowfall to the number of cars on the road, and you cannot drive a fraction of a car.

Q: How many cars are on the highway when there is no snow? Explain.

A: Since the y-intercept is (0, 100), or (0, 99) if you round, there are about 100 cars on the road when there is no snow.

7) Sample:

(1, 15), (8, 8)

$$y = -1x + 16$$

Technology: $y = -1x + 16$; $r = -1$

8) The slope is -1 . This means Isaiah's hours of free time decrease by 1 as the hours of work increase by 1. (The more Isaiah works, the less free time he has.)

The y -intercept (0, 16) means that if Isaiah works 0 hours, he will have 16 hours of free time.

9) $y = 6.5$

If Isaiah works for 9.5 hours, he will have 6.5 hours of free time.

10) $x = 4$

If Isaiah works for 4 hours, he will have 12 hours of free time.

11) $y = 0$

If Isaiah works for 16 hours, he will have 0 hours of free time. This is the x -intercept (16, 0).

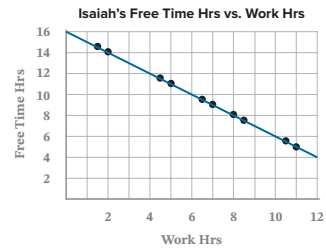
12) $x = -2$

This equation cannot be used when Isaiah spends more than 16 hours of free time because the line moves past the y -axis, and the x -values become negative. This would mean that Isaiah worked a negative number of hours which is not possible. Instead, it is more likely that Isaiah did not work at all and only slept for 6 hours.

13) $y = 146.50$

Joel's family should expect to pay \$146.50 in 2021. This is extrapolation because the number is outside of the set of data given.

Isaiah tracked his free time versus his work time on the graph below. He noticed there was a strong correlation between the two data sets.



- 7) Draw the line of best fit and write the equation. Show your work.
- 8) Using problem 7, explain the meaning of the slope and y -intercept.

Using Isaiah's equation, predict the following:

- 9) The number of hours of work if Isaiah has 9.5 hours of free time.
- 10) The number of hours of work if Isaiah has 12 hours of free time.
- 11) The hours of free time if Isaiah works 16 hours. What is this ordered pair called?
- 12) Can this equation be used to predict the number of work hours when Isaiah takes 18 hours of free time? Explain your result using numerical data.

Joel recorded his family's average monthly electric bill over the last eight years in the table below and calculated the equation $y = 2x + 130.50$.

| year | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| bill | \$131 | \$133 | \$134 | \$137 | \$138 | \$140 | \$143 | \$146 |

- 13) Approximately how much will Joel's family pay in 2021 for electricity? Explain if this is interpolation or extrapolation. (Hint: Find the number of years past 2013.)
- 14) Explain the meaning of the slope and y -intercept for Joel's equation.

(year since 2013, electric cost)

The slope is 2. This means the electric bill will increase by \$2 per year after 2013.

The y -intercept (0, 130.50) means that at year 0, or 2013, the average cost of electricity was \$130.50. This is very close to the actual amount of \$131 that Joel paid.

Mastery Check

Show What You Know

Given the scatter plot, compare the weight of a pencil to the length of a pencil in centimeters.

- A) Draw the line of best fit. Calculate the line of best fit in slope-intercept form.

Sample:
 (15, 8.5), (20, 10)

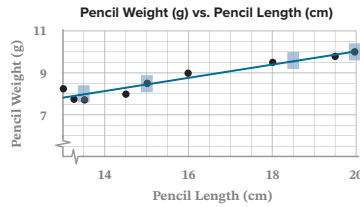
$$m = \frac{10 - 8.5}{20 - 15} = \frac{1.5}{5} = \frac{3}{10}$$

$$y - 10 = \frac{3}{10}(x - 20)$$

$$y - 10 = \frac{3}{10}x - 6$$

$$y = \frac{3}{10}x + 4$$

Technology: $y = 0.31x + 3.79$; $r = 0.96$



- B) Explain the meaning of the slope and y-intercept.

The slope of 0.31 means that the pencil weight increases 0.31 grams for every cm in length. The y-intercept is (0, 3.79). This represents a pencil with a length of 0 but a weight of 3.79 grams. This could be the weight of the eraser.

Use your equation to make predictions about the data.

- C) Predict the length of a pencil when the weight is 9 grams. Round to the nearest whole number.

$$(x, 9)$$

$$y = \frac{3}{10}x + 4$$

$$9 = \frac{3}{10}x + 4$$

$$\left(\frac{10}{3}\right)(5) = \left(\frac{10}{3}\right)\frac{3}{10}x$$

$$x = \frac{50}{3} = 16\frac{2}{3} = 16.6 \approx 17$$

Technology: $y = 0.31x + 3.79$

$$9 = 0.31x + 3.79$$

$$5.21 = 0.31x$$

$$x = 16.81 \approx 17$$

The pencil will be about 17 centimeters long when it weighs 9 grams.

- D) Predict the weight of a pencil with a length of 10 centimeters. Round to the nearest whole number.

$$(10, y)$$

$$y = \frac{3}{10}x + 4$$

$$y = \frac{3}{10}(10) + 4$$

$$y = 7$$

Technology: $y = 0.31x + 3.79$

$$y = 0.31(10) + 3.79$$

$$y = 6.89 \approx 7$$

The pencil will be about 7 grams when it is 10 centimeters long.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

- A) Even if your student uses technology, they will still need to record the line on the graph.

Q: The r -value is 0.96. What does this tell you about the scatter plot?

A: The data has a very strong positive correlation.

When calculating the line of best fit from two points, the points should be on the drawn line, not necessarily the given scatter plot points.

- B) Your student may have other reasons why/how a pencil can have weight but no length. Encourage them to make sense of the problem and explain their reasoning.

- D) The results of the equation using technology or by hand will be very close. Be sure to use your student's equation to check solutions for parts C and D.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Estimate and draw the line of best fit for a set of data.
- Write the equation for the line of best fit in the requested form.
- Use the line of best fit to interpolate, extrapolate, and explain the context of the data.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer “yes” to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer “yes” to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

4) Technology calculated regression line:
 $y = -1x + 325$

Sample:
 (50, 275), (100, 225)
 $y = -1x + 325$

Technology: $y = -x + 325; r = -1$

5) $y = -1(0) + 325$
 $y = 325$

There are 325 pages in Grant's book.

8) (75, 4), (80, 1)

$y = -\frac{3}{5}x + 49$ OR $y = -0.60x + 49$

Technology:
 $y = -0.58x + 47.78; r = -0.89$

9) $m = -\frac{3}{5}$

The Goldbergs will catch 3 fewer fish per 5° temperature increase.

10) Sample:
 (75, 4), (80, 1)

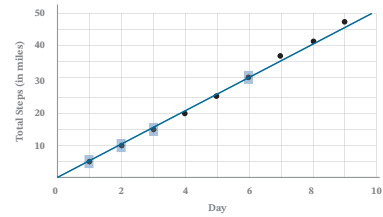
$y = 15x$

Technology: $y = 15x; r = 1$

Practice 2

Complete the problems on a separate sheet of paper.

1) The scatter plot below compares total steps in miles to the day of the month. Draw the line of best fit and mark two points on the line.



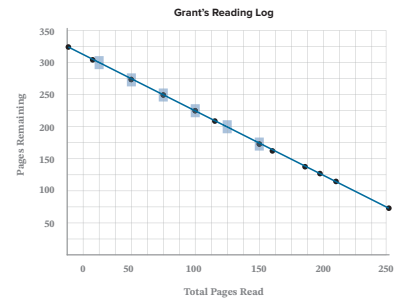
2) Write the equation for the line of best fit using your ordered pairs.

Sample:
 (1, 5), (3, 15)
 $y = 5x$

Technology:
 $y = 5.12x - 0.23;$
 $r = 0.99$

The scatter plot shows a record of Grant's reading. He compared the pages remaining in the book to the total number of pages he had read.

3) Draw the line of best fit and mark two points on the line.



4) Write the line of best fit using your points.

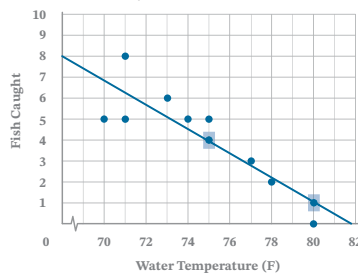
5) How many pages are in the novel Grant is reading? Explain.

6) Explain what the equation of the line of best fit can be used for.

The equation of the line of best fit can be used for predicting values through interpolation (between points) and extrapolation (outside of given points).

The Goldberg family went fishing once a week in the spring. They recorded the water temperature and the number of fish the entire family caught each trip. For each fishing trip, they stayed for three hours.

7) Draw the scatter plot and line of best fit.



| Water temperature (F) | Fish caught |
|-----------------------|-------------|
| 70 | 5 |
| 71 | 8 |
| 71 | 5 |
| 73 | 6 |
| 74 | 5 |
| 75 | 5 |
| 75 | 4 |
| 77 | 3 |
| 78 | 2 |
| 80 | 1 |
| 80 | 0 |

8) Write the equation for the line of best fit using the ordered pairs.

9) Explain the meaning of the slope for the Goldberg family.

Fred has a dog walking business. He charged the same amount for each dog walked and wanted an equation to calculate prices quickly.

- 10) Draw and write an equation to find the average charge for a dog walk.
- 11) What would the charge be for Fred to walk 11 dogs? Is this interpolation or extrapolation?
- 12) Explain the meaning of the slope and the y -intercept for Fred's dog walking business.

The slope is 15. This means that Fred charges \$15 per dog. The y -intercept is $(0, 0)$. This means that Fred will earn \$0 when no dogs have been walked.



11) $y = 165$

Fred will charge \$165 for 11 dogs. This is extrapolation because this information cannot be seen on the graph.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

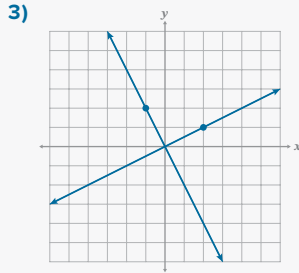
Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

2) $g(x) = \frac{1}{2}x$

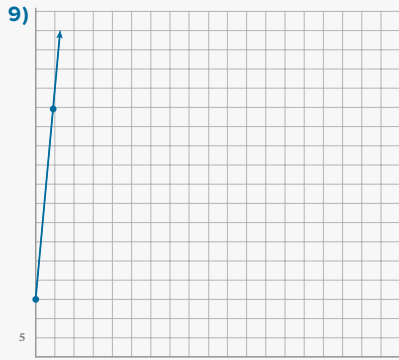


4) The slope would be negative because the trees are losing leaves.

5) The oak tree loses 50 leaves per day. The elm tree loses 40 leaves per day. The graph for the oak tree would have a steeper slope.

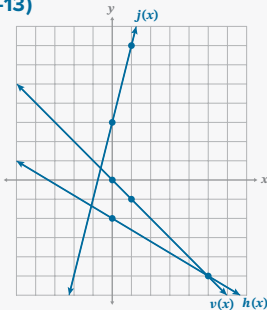
6) $c(g) = 4.25g$
 $c(10.8) = 45.9$
 The cost of 10.8 gallons of gasoline is \$45.90.

7) $x - y = 0$



No. Jessica cannot work negative hours. Therefore, she will not earn a negative amount of money.

11–13)



Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

- Write the equation of a line parallel to $y = \frac{2}{3}x - 8$, through the point $(5, -2)$. $y = \frac{2}{3}x - \frac{16}{3}$
- Write the equation of the line perpendicular to $f(x) = -2x$ through the origin. Name this new line $g(x)$.
- Graph $f(x)$ and $g(x)$ from problem 2.

During the fall, an oak tree will lose around 150 leaves every 3 days. An elm tree will lose around 80 leaves every 2 days. Assume that the trees lose their leaves at a constant rate.

- Would the slopes for these functions be positive or negative? Why?
- Which graph would have a steeper slope?

- The price of gasoline is \$4.25 for g gallons of gas. Write a function rule for the cost of the gasoline $c(g)$. Find the price for 10.8 gallons of gas.
- Find the equation of a line through $(-5, -5)$ and $(-1, -1)$. Write the equation in standard form.

Jessica charges a flat fee of \$15 for travel and \$50 an hour for a photography session.

- Write the equation of the line in slope-intercept form. $y = 50x + 15$
- Graph the line using an x scale of 1 and a y scale of 5. Will the x and y values be negative?
- How much does Jessica charge for 2.5 hours of work? $y = \$140$

Graph each of the linear functions on the same coordinate plane.

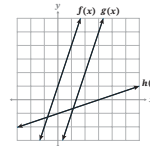
- 11) $h(x) = -\frac{3}{5}x - 2$ 12) $j(x) = 4x + 3$ 13) $v(x) = -x$

- Use the equation $x - 2y = 4$ to find the x and y -intercepts from the equation and the slope of the equation. $(4, 0), (0, -2)$ $m = \frac{1}{2}$
- Graph the equation $x - 2y = 4$.

Multiple Choice

A 16) Determine the functions that are parallel to one another from the graph.

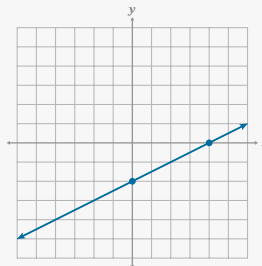
- $f(x) \parallel g(x)$
- $f(x) \parallel h(x)$
- $g(x) \parallel h(x)$
- cannot be determined



B 17) Name the slope perpendicular to the given equation: $3x - 6y = 7$

- $-\frac{1}{2}$
- -2
- $\frac{1}{2}$
- 2

15)



16) Distractor Rationale:

B, C, and D are given to make sure your student carefully names the parallel functions.

17) Distractor Rationale:

- This is the opposite of the given slope but not the reciprocal.
- This is the slope of the given line.
- This is the reciprocal of the given line but not the opposite.

| | | | | | | | | | | | | |
|----------------------|----|-------|---|-----|-------|----|------|-------|----|----|-------|--------|
| Problem | 1 | 2 | 3 | 4–5 | 6 | 7 | 8–10 | 11–13 | 14 | 15 | 16 | 17 |
| Lesson Origin | 12 | 12, 7 | 9 | 9 | 10, 7 | 11 | 10 | 9 | 11 | 11 | 12, 8 | 12, 11 |