Lesson 12
Parallel and Perpendicular Lines

Outline

Part A:
Parallel Lines

- Parallel Lines
- Parallel Lines from Equations
- Parallel Line through a Point

Part B
Perpendicular Lines

- Perpendicular Lines
- Perpendicular Lines from Equations
- Perpendicular Line through a Point
- Special Perpendicular Lines

Targeted Review

Vocabulary

- parallel lines
- perpendicular lines

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Q: If two fractions are written in different forms, can they be equal?

A: *Yes, as long as they simplify to the same number.*

Part A: Parallel Lines

Objectives

In this part of the lesson, you will learn about parallel lines.

By the end of this lesson, you will be able to do the following:

- Identify parallel lines.
- Write the equation of a line that is parallel to another known line and passes through a given point.

Why?

Parallel lines do not show up often in algebra. However, it is critical to understand their uniqueness when you encounter problems that include them.

Warm Up

Simplify the following ratios.

1) $\frac{5}{15}$ $\frac{1}{3}$

2) $\frac{25}{60}$ $\frac{5}{12}$

3) $\frac{16}{24}$ $\frac{2}{3}$

Solve.

4) $\frac{3}{v} = \frac{9}{15}$

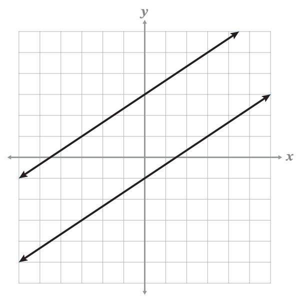
$(3)(15) = 9v$
$45 = 9v$
$v = 5$

5) $\frac{6}{4} = \frac{r}{20}$

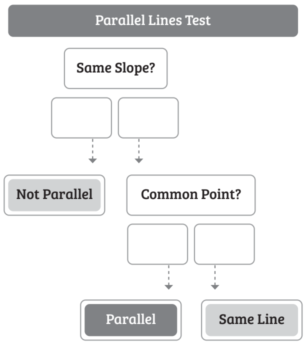
$(6)(20) = 4r$
$120 = 4r$
$r = 30$

Parallel Lines

- Parallel lines (∥) have **equal** slopes, but **different** y-intercepts.
- Parallel lines have **zero (no)** points in common.
- Parallel lines can also be thought of as **translations** of the same line.



EXPLORE 12A



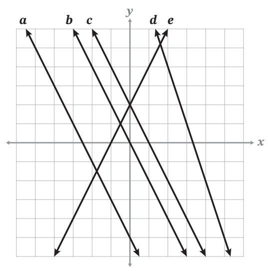
Example 1

Determine which lines represented on the graph are parallel.

Plan Determine the slope of each line. Determine the y-intercepts for lines with the same slopes.

Implement

Line	m	b
Line a	-2	-5
Line b	-2	0
Line c	-2	2
Line d	-3	—
Line e	2	—



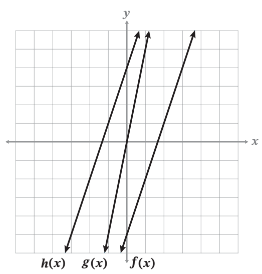
$a \parallel b$
$a \parallel c$ **OR** $a \parallel b \parallel c$
$b \parallel c$

Lines a, b, and c all have the same slope and different y-intercepts. These three lines are parallel.

Checkpoint

Name the slope of each line. Then determine the set(s) of parallel lines.

$h(x)$: $m = 3$
$g(x)$: $m = 5$ $h(x) \parallel f(x)$
$f(x)$: $m = 3$



Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Do any of the lines share an intercept?

A: *no*

Q: How can you determine if the lines are parallel?

A: *They have equal slopes and different y-intercepts.*

12A EXPLORE

Parallel Lines from Equations

- $y = mx + b$ is particularly useful for determining if lines are parallel because you are given the slope and the y-intercept.
- Efficient ways to compare equations:
  - Write all equations in the same form.
  - Find m and b.
- All horizontal lines are parallel to one another because $m = 0$ for every horizontal line and they have different y-intercepts.
- All vertical lines are parallel to one another because they have undefined slopes and different x-intercepts.

Example 2

Determine which lines are parallel. Write your answer in a sentence, then write your answer using math shorthand.

Line m: $y = 3$

Line n: $y = 5$

Line p: $x = 3$

Line q: $x = -2$

Lines m and n are horizontal lines, so $m \parallel n$.

Lines p and q are vertical lines, so $p \parallel q$

.

EXPLORE 12A

Example 3

Determine which of the given lines are parallel.

Line a: $y = -\frac{2}{3}x - 1$

Line b: $2x + 3y = 6$

Line c: $y - 3 = -2(x - 2)$

Line d: $y = -\frac{2}{3}$

Line e: $x = -\frac{2}{3}$

Plan List the slopes and y-intercepts.
Use a method to compare the equations.

Implement

Line	m	b	$y = mx + b$
a: $y = -\frac{2}{3}x - 1$	$-\frac{2}{3}$	-1	$y = -\frac{2}{3}x - 1$
b: $2x + 3y = 6$	$-\frac{2}{3}$	2	$y = -\frac{2}{3}x + 2$
c: $y - 3 = -2(x - 2)$	-2	7	$y = -2x + 7$
d: $y = -\frac{2}{3}$	0	$-\frac{2}{3}$	$y = 0x - \frac{2}{3}$
e: $x = -\frac{2}{3}$	undefined	none	$x = -\frac{2}{3}$

Lines a and b have the same slope and different y-intercepts, so $a \parallel b$.

Checkpoint

Are the given lines parallel? Explain.

Line a: $2x - 3y = -15$ Line b: $y = \frac{2}{3}x + 4.5$ Line c: $y = \frac{3}{2}$

Line a: $m = \frac{2}{3}$, $b = 5$ **Line b: $m = \frac{2}{3}$, $b = 4.5$** **Line c: $m = 0$, $b = \frac{2}{3}$**

$a \parallel b$ (or Line a is parallel to line b.)

Example 3

Encourage your student to complete the table before watching the video to see if they can use what they have learned in Lessons 7–11.

At this point, you would choose the most efficient way to compare the equations. However, in this example, both methods are demonstrated.

Method 1: Rewrite each line in slope-intercept form.

a: $y = -\frac{2}{3}x - 1$

b: $2x + 3y = 6$

$m = -\left(\frac{A}{B}\right), b = \frac{C}{B}$

$y = -\left(\frac{2}{3}\right)x + \left(\frac{6}{3}\right)$

$y = -\frac{2}{3}x + 2$

c: $y - 3 = -2(x - 2)$

$y = -2x + 4 + 3$

$y = -2x + 7$

d: $y = -\frac{2}{3}$; this is a horizontal line.

e: $x = -\frac{2}{3}$; this is a vertical line.

Method 2: Compare slopes and y-intercepts

a: $m = -\frac{2}{3}$ and $b = -1$

b: $m = -\left(\frac{A}{B}\right) = -\left(\frac{2}{3}\right) = -\frac{2}{3}$ and $b = \frac{C}{B} = \frac{6}{3} = 2$

c: $m = -2$ (since m is different, we do not need to check b.)

d: $m = 0, b = -\frac{2}{3}$

e: $m =$ undefined, $b =$ does not exist

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

$2x - 3y = -15$ written in slope-intercept form is $y = \frac{2}{3}x + 5$.

If your student needs a visual, you can have them graph each equation and then compare. However, you also want your student to determine parallel lines algebraically.

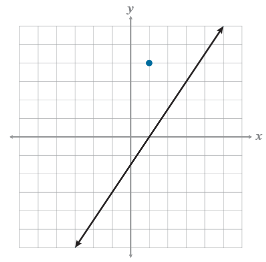
12A EXPLORE

Parallel Lines through a Point

- Sometimes lines need to be **parallel** and one of the lines must travel **through** a specific point.
- In this course, if the **form** for the solution of a linear equation is not specified, you should give the final solution in the **same form** in which it was originally presented.

Example 4

Find the equation of the line in slope-intercept form that travels through the point (1, 4) and is parallel to the given line on the coordinate plane.



Plan Find the slope of the provided line.
Write down the point for the new line.
Write the new line in point-slope form.
Write the equation of the line in slope-intercept form.

Implement

Slope of provided line: $m = \frac{3}{2}$

Point on the new line: $(1, 4)$

$m = \frac{3}{2} \parallel m = \frac{3}{2}$

$(1, 4)$

$$y - 4 = \frac{3}{2}(x - 1)$$
$$y - 4 = \frac{3}{2}x - \frac{3}{2}$$
$$+4 \qquad +4$$
$$y = \frac{3}{2}x - \frac{3}{2} + \frac{8}{2}$$
$$y = \frac{3}{2}x + \frac{5}{2}$$

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Example 5

Find the equation of the line parallel to $x - 4y = -3$ that travels through the point $(-2, 7)$.

Plan Determine the slope from the given equation.
Substitute the new point and the slope into point-slope form.
Rewrite in standard form.

Implement

$m = -\left(\frac{A}{B}\right) = -\left(-\frac{1}{4}\right) = \frac{1}{4}$

$m = \frac{1}{4} \parallel m = \frac{1}{4}$

$(-2, 7)$

$$y - 7 = \frac{1}{4}(x + 2)$$
$$y - 7 = \frac{1}{4}x + \frac{1}{2}$$
$$y = \frac{1}{4}x + \frac{15}{2}$$
$$-\frac{1}{4}x + y = \frac{15}{2}$$
$$-4\left(-\frac{1}{4}x + y = \frac{15}{2}\right)$$
$$x - 4y = -30$$

Checkpoint

Determine the line parallel to $y = 2x - 4$ and traveling through the point $(3, -2)$.

$m = 2 \parallel m = 2$

$(3, -2)$

$$y + 2 = 2(x - 3)$$
$$y + 2 = 2x - 6$$
$$y = 2x - 8$$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the slope from the given equation?

A: *2*

Q: What will the slope of the new line be? Explain.

A: *The slope of this line will also be 2 because parallel lines have equal slopes.*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) Sample:
First, determine if the lines have equal slopes. If the slopes are equal, then determine if the lines have different y-intercepts. If the y-intercepts are different, the lines are parallel.

2–5)
Your student should be able to identify the slope and y-intercept of each set of lines using what they have mastered in previous lessons.

7) Your student is free to use one method of their choosing. Finding the slope using $m = -\left(\frac{A}{B}\right)$ is the most efficient way to solve the problem.

Practice 1

Complete the practice problems on a separate sheet of paper.

1) How do you determine if two lines are parallel?

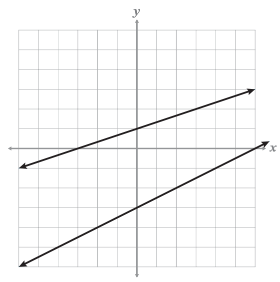
Determine if the lines are parallel, the same, or neither.

2) $y = 2x + 1$ **$m = 2, b = 1$**
$y = 2x + 4$ **$m = 2, b = 4$; parallel**

3) $y - 3 = 3(x + 2)$ **$m = 3, b = 9$**
$y + 3 = 3(x + 4)$ **$m = 3, b = 9$; parallel**

4) $4x + 3y = 5$ **$m = -\frac{4}{3}, b = \frac{5}{3}$**
$4x + 3y = 7$ **$m = -\frac{4}{3}, b = \frac{7}{3}$; parallel**

5)

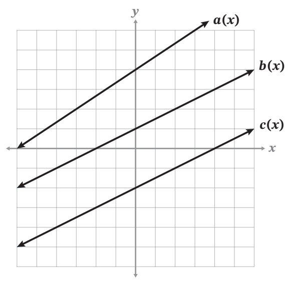


$m = \frac{1}{3}$, $b = 1$

$m = \frac{1}{2}$, $b = -3$;

neither

6) Given the graph of $a(x)$, $b(x)$, and $c(x)$, which, if any, of these lines are parallel? Explain.



$b(x) \parallel c(x)$

These lines have equal slopes and different y-intercepts. Therefore, they are parallel.

7) Determine which lines below are parallel to $2x - y = 5$.

Line a: $4x - 2y = 7$ **The given line is parallel to line a and line c. given line $\parallel a \parallel c$**

Line b: $6x + 3y = 15$

Line c: $8x - 4y = 9$

PRACTICE 1 12A

8) Determine if the following lines are parallel to $y - 3 = \frac{2}{3}(x + 3)$.

Line a: $y - 5 = \frac{2}{3}(x - 3)$

Line b: $y + 2 = \frac{2}{3}(x - 3)$

Line c: $y - 1 = \frac{2}{3}(x - 6)$

Lines a and b are parallel to $y - 3 = \frac{2}{3}(x + 3)$.
Line c and the given line are the same line.
given line $\parallel a \parallel b$

9) Determine if the following lines are parallel to $y = \frac{1}{4}x$.

Line a: $y - 2 = 4(x + 1)$

Line b: $2x - 8y = 1$

Line c: $x - 4y = 3$

Lines b and c are parallel to $y = \frac{1}{4}x$. given line $\parallel b \parallel c$

Find the equation of the line that is parallel to the given line through the given point.

10) $y - 3 = 5(x + 2)$ through the point $(-4, 6)$ in point-slope form. $\mathbf{y - 6 = 5(x + 4)}$

11) $y = -\frac{2}{5}x - 1$ through the point $(1, 1)$ in slope-intercept form. $\mathbf{y = -\frac{2}{5}x + \frac{7}{5}}$

12) $8x - 3y = 24$ through $(1, 1)$ in standard form. $\mathbf{8x - 3y = 5}$

13) $x = 4$ through the point $(-2, 5)$. $\mathbf{x = -2}$

14) A city planner is determining the needed roads in a new neighborhood. He decides that all streets named after trees, Oak and Elm, will run parallel to one another. The center of the neighborhood is at $(5, -5)$ and Oak Street runs along the line $y = 4x + 1$. Find the equation for Elm Street that is parallel to Oak Street and goes through $(5, -5)$. $\mathbf{y = 4x - 25}$

15) While laying tile for swim lanes at the bottom of a pool, the contractor noticed that the edge of the pool fit the equation of $y = -2$. The pool required four parallel lanes, each translated over four positive units. What are the equations for the next three lanes? $\mathbf{y = 2, y = 6, y = 10}$

8) Your student may wish to graph this to compare the lines. This would be another method to demonstrate this visually.

Though there are two methods, only one will be shown (the one that is most efficient).

10) Q: What form should you write the equation in if it is not stated?

A: *In the same form as the given line.*

13) Q: What type of line does the equation $x = 4$ represent?

A: *A vertical line.*

15) Your student may need to draw the lanes to visualize what is happening.

Q: How far apart are the lanes?

A: *Four units.*

Q: What should you do to find the other equations?

A: *Add 4.*

Mastery Check

Show What You Know

B) Prove algebraically means that your student needs to show work. In this case, they should use the slope formula.

$\overline{EH}$ y-intercept $= 2$

$\overline{FG}$ y-intercept $= -4$

C) Your student should be able to write this equation without finding the slope because it is a vertical line.

D) Q: What is the y-intercept of this line?

A: *2*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Identify parallel lines.
- Write the equation of a line that is parallel to another known line and passes through a given point.

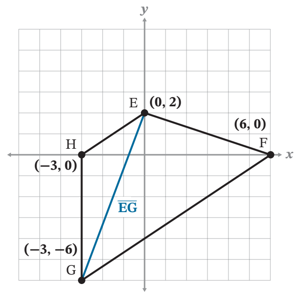
Mastery Check

Show What You Know

A trapezoid is a quadrilateral with one set of parallel sides.

A) State the parallel sides.

$\overline{EH} \parallel \overline{FG}$



B) Prove algebraically that the lines in part A are parallel.

The slope of $\overline{EH} = \frac{2-0}{0+3} = \frac{2}{3}$

The slope of $\overline{FG} = \frac{0+6}{6+3} = \frac{2}{3}$

Since the slopes are equal, and the y-intercepts are different, the lines are parallel.

C) Find the equation of side $\overline{GH}$. Explain your thinking.

G $(-3, 6)$ H $(-3, 0)$

$x = -3$

Because the x-coordinates are equal, side $\overline{GH}$ is a vertical line.

D) Connect points E and G with a segment. What is the equation of $\overline{EG}$ written in standard form?

E $(0, 2)$ G $(-3, -6)$

$m = \frac{-6-2}{-3-0} = \frac{8}{3}$

$m = \frac{8}{3}$

$(0, 2)$

$$y = \frac{8}{3}x + 2$$
$$-\frac{8}{3}x + y = 2$$
$$-3\left(-\frac{8}{3}x + y = 2\right)$$
$$8x - 3y = -6$$

Say What You Know

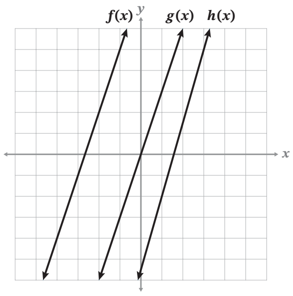
In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

PRACTICE 2 12A

Practice 2

Complete the practice problems on a separate sheet of paper.

1) Given the graphs of $f(x)$, $g(x)$, and $h(x)$, which, if any, are parallel? Explain.



$f(x) \parallel g(x)$

The slopes of lines $f(x)$ and $g(x)$ are parallel because they are equal, but their y-intercepts are different.

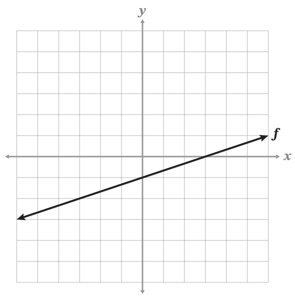
2) Determine if the following lines are parallel to the line in the graph.

Line a: $3x - y = 1$

Line b: $y + 1 = \frac{1}{3}(x - 9)$

Line c: $y - 1 = \frac{1}{3}(x - 6)$

Line d: $x - 3y = 2$



$b \parallel f$ AND $d \parallel f$

OR

$b \parallel d \parallel f$

Line a has a different slope, so it is not parallel to line f. Line c is the same as line f.

3) Determine if the following lines are parallel to $6x + 7y = 3$.

Line a: $12x + 14y = 9$ **Line a is parallel to $6x + 7y = 3$.**

Line b: $7x + 6y = 3$

Line c: $6x - 7y = 4$

Practice 2

Worked solutions for these problems are located in the Digital Pack.

12A PRACTICE 2

4) Determine if the following lines are parallel to $y = 6$.

Line a: $y - 5 = 0$ **Line a is parallel to $y = 6$, because all horizontal lines are parallel. Lines b and c are not horizontal lines.**

Line b: $y = -4x$

Line c: $x = 2$

5) Determine if the following lines are parallel to $y = \frac{3}{4}x - 7$.

Line a: $y + 5 = -\frac{3}{4}(x + 3)$ **Lines b and c are parallel to $y = \frac{3}{4}x - 7$.**

Line b: $y + 5 = \frac{3}{4}(x + 3)$

Line c: $6x - 8y = 9$

6) Determine if the following lines are parallel to $y + 1 = -\frac{5}{3}(x + 3)$.

Line a: $5x - 3y = 9$ **Line b is parallel to line $y + 1 = -\frac{5}{3}(x + 3)$.**

Line b: $y = -\frac{5}{3}x - 5$

Line c: $y + 1 = -\frac{5}{3}(x + 3)$

Find the equation of the line that is parallel to the given line through the given point.

7) $y + 10 = \frac{2}{3}(x + 7)$ through the point $(5, 0)$ in point-slope form. $\mathbf{y - 0 = \frac{2}{3}(x - 5)}$

8) $y = \frac{3}{4}x$ through the point $(8, 3)$ in slope-intercept form. $\mathbf{y = \frac{3}{4}x - 3}$

9) $4x - 3y = 6$ through $(-12, 12)$ in standard form. $\mathbf{4x - 3y = -78}$

10) $x = 7$ through $(-3, 1)$. $\mathbf{x = -3}$

11) A gardener is making parallel rows for planting. The first row fits the equation $y = 3x - 4$. The second row must go through the point $(0, 1)$ and the third row must go through the point $(3, -2)$. Find the equations of the second and third rows in slope-intercept form. **Second Row: $y = 3x + 1$; Third Row: $y = 3x - 11$**

12) A local tech company is installing a new fiber optic cable. The equation for the old line is $y = -\frac{1}{2}x - 5$. The new cable needs to run parallel to the old line but translated up three units. What is the equation for the new fiber optic cable? $\mathbf{y = -\frac{1}{2}x - 2}$

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

EXPLORE 12B

Part B: Perpendicular Lines

Objectives

In this part of the lesson, you will learn about perpendicular lines.

By the end of this lesson, you will be able to do the following:

- Identify perpendicular lines.
- Write the equation of a line that is perpendicular to a given line and passes through a given point.

Why?

Perpendicular lines do not show up too often in algebra. However, it is critical to understand their uniqueness when you encounter problems that include them.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Write the value that is the reciprocal of the value shown.

1) $\frac{3}{2}$ reciprocal: $\frac{2}{3}$ **2)** $\frac{1}{2}$ reciprocal: **2** **3)** -3 reciprocal: $-\frac{1}{3}$

Determine the opposite of the value shown.

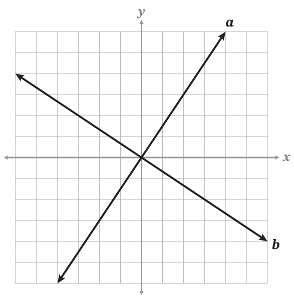
4) $\frac{2}{3}$ opposite: $-\frac{2}{3}$ **5)** -2 opposite: **2**

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

The reciprocal of a number is not equal to the given number. Make sure your student does not write an equal sign between the two numbers. This is also true of a number and its opposite.

Perpendicular Lines



- **Perpendicular lines (⊥)** intersect each other in the same plane and form right angles.
- Slopes for perpendicular lines:
  - Have **opposite** signs.
  - Are **reciprocals** of one another.
- The product of the slopes of perpendicular lines will always be **-1**.
- Perpendicular lines can have the same ***y*-intercepts**.
- Only the **slope** needs to be considered when determining if lines are perpendicular.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Your student can find the slope from the graph. It is good practice to write it down even when not stated in the directions so that your student does not have to remember all of the slopes.

12B EXPLORE

Example 1

Determine the lines that are perpendicular to each other.

Slope of line b: $-\frac{2}{3}$

Slope of line c: $-\frac{2}{3}$

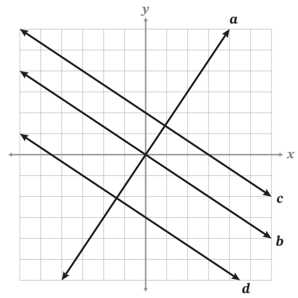
Slope of line d: $-\frac{2}{3}$

Slope of line a: $\frac{3}{2}$

$\left(\frac{3}{2}\right)\left(-\frac{2}{3}\right) = -1$

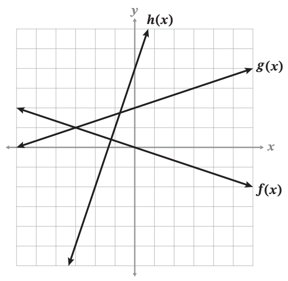
$a \perp b$ $\quad a \perp c$

$a \perp d$



Checkpoint

Determine the lines perpendicular to one another. Explain.



$f(x)$: $m = -\frac{1}{3}$

$g(x)$: $m = \frac{1}{3}$

$h(x)$: $m = 3$

$f(x) \perp h(x)$ because $\left(-\frac{1}{3}\right)(3) = -1$

Perpendicular Lines from an Equation

- The slope from **slope-intercept** form or **point-slope** form can be used to determine whether lines are perpendicular.
- If lines are given in different forms, first **identify the slope** of each line to compare.

EXPLORE 12B

Example 2

Determine which lines are perpendicular.

Line a: $y - 3 = -\frac{1}{2}(x - 2)$ Line b: $y = 3x + 4$ Line c: $x + 3y = 12$

Plan Identify slopes.
Compare.

Implement

$m_a = -\frac{1}{2}$ $m_b = 3$ $m_c = -\frac{A}{B} = \left(-\frac{1}{3}\right) = -\frac{1}{3}$

Another way to note the line you are referring to without writing out "line a" is to use m_a, which means the slope of line a.

Since $(3)\left(-\frac{1}{3}\right) = -1$, then $b \perp c$.

Example 3

Determine which lines are perpendicular.

Line a: $5y + 3x = 1$

Line b: $5x - 3y = -3$

Line c: $5x + 3y = 4$

Use the formula for slope from standard form, $m = -\left(\frac{A}{B}\right)$.

Line a: $m = -\frac{3}{5}$

Line b: $m = -\left(\frac{5}{-3}\right) = \frac{5}{3}$

Line c: $m = -\frac{5}{3}$

$m_b = \frac{5}{3}$ and $m_a = -\frac{3}{5}$

Since $\left(\frac{5}{3}\right)\left(-\frac{3}{5}\right) = -1$, then $a \perp b$.

Checkpoint

Name the slope of each line. Determine which lines are perpendicular.

Line a: $y = \frac{1}{4}x + 2$ $m = \frac{1}{4}$

Line b: $y - 3 = 4(x - 2)$ $m = 4$

Line c: $4x + y = \frac{1}{3}$ $m = -4$

$a \perp c$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How do you know that the lines you chose are perpendicular?

A: *When you multiply the slopes together, the product is negative one.*

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How did you determine the new slope?

A: *Find the opposite and reciprocal of the given slope.*

12B EXPLORE

Perpendicular Lines through a Point

- Perpendicular lines can be determined given an **equation** and a **point**.
- First, determine the **slope** of the given line.
- Then find the **opposite reciprocal** of that slope.
- Use the new slope and the point to find a **new equation**.
- Organize the information this way: **Given m ⊥ new m**
  (x, y)

Example 4

Determine the line that is perpendicular to $y = \frac{3}{5}x + 1$ and travels through the point $(-1, 2)$.

Plan: Determine the slope of the new line.
Create the equation for the new line.

Implement:

$m = \frac{3}{5} \perp m = -\frac{5}{3}$

$(-1, 2)$

$$y - 2 = -\frac{5}{3}(x + 1)$$
$$y - 2 = -\frac{5}{3}x - \frac{5}{3}$$
$$y = -\frac{5}{3}x - \frac{5}{3} + 2$$
$$y = -\frac{5}{3}x - \frac{5}{3} + \frac{6}{3}$$
$$y = -\frac{5}{3}x + \frac{1}{3}$$

$y = \frac{3}{5}x + 1 \perp y = -\frac{5}{3}x + \frac{1}{3}$ through $(-1, 2)$.

Checkpoint

Determine the line that is perpendicular to $x + 2y = 3$ and travels through the point $(4, 5)$. Write the equation in point-slope form.

Given m ⊥ new m

$m = -\frac{1}{2} \perp m = 2$

$(4, 5)$

$$y - 5 = 2(x - 4)$$

EXPLORE 12B

Special Perpendicular Lines

- The x-axis and y-axis are a set of perpendicular lines that form the **coordinate plane**.
- The slope of every **horizontal** line is $m = \frac{0}{1} = 0$.
- The slope of every **vertical** line is $m = \frac{1}{0} =$ *undefined*.
- *Every* horizontal line is **⊥** to *every* vertical line on the coordinate plane.

Example 5

Write *horizontal* or *vertical* next to each equation. Then determine which lines are perpendicular.

Line a: $x = 5$ **vertical** $a \perp b$

Line b: $y = 5$ **horizontal** $b \perp c$

Line c: $x = -2$ **vertical** $a \parallel c$

Example 6

Determine the line that is perpendicular to $y = -1$ and travels through the point $(-3, 5)$.

The line given is **horizontal**.

Any line **perpendicular** to this line will be **vertical**.

The vertical line that travels through the point $(-3, 5)$ is $x = -3$.

Checkpoint

Determine the line that is perpendicular to $x = \frac{2}{3}$ and travels through the point $(1, 2)$.

$y = 2$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

The line given is vertical. Any line perpendicular to this line will be horizontal. The horizontal line that travels through the point $(1, 2)$ is $y = 2$.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1–5)

It may help your student to write the perpendicular slope next to the given slope and then try to match to find the correct answer.

6) $m_{b(x)} = \frac{2}{3}$, $m_{f(x)} = -\frac{3}{2}$, and $m_{j(x)} = -\frac{3}{2}$

Since $\left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1$, then $b(x) \perp f(x)$ and $b(x) \perp j(x)$. Since m is equal in $f(x)$ and $j(x)$ and the intercepts are different, these lines are ||.

Be sure your student finds the slopes mathematically. Some lines will "look" perpendicular when they are not perpendicular mathematically.

7) $m_{r(x)} = 2$, $m_{m(x)} = 1$, and $m_{a(x)} = -\frac{1}{2}$

Since $(2)\left(-\frac{1}{2}\right) = -1$, then $r(x) \perp a(x)$. There are no || lines on this graph.

8–11)

Have your student use the perpendicular symbol rather than writing out each statement in words.

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12B PRACTICE 1

Practice 1

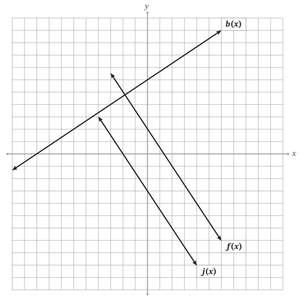
Complete the practice problems on a separate sheet of paper.

Match each of the given slopes to its corresponding perpendicular slope.

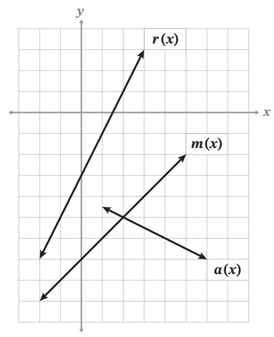
	Given		Perpendicular Slopes
C	1)	$m = 2$	A) -2
E	2)	$m = \frac{6}{5}$	B) 1
B	3)	$m = -1$	C) $-\frac{1}{2}$
A	4)	$m = \frac{1}{2}$	D) $\frac{6}{5}$
D	5)	$m = -\frac{5}{6}$	E) $-\frac{5}{6}$

Determine which lines are perpendicular and which are parallel.

6)



7)



Determine which lines are perpendicular (⊥). Name the slope of each line. Write a sentence describing the relationship between perpendicular lines. Indicate any lines that are vertical or horizontal. (Hint: there can be multiple sets of ⊥ lines.)

8) Line a: $y = -\frac{3}{4}x - 5$ $\quad m_a = -\frac{3}{4}$

Line b: $y = \frac{4}{3}x + 1$ $\quad m_b = \frac{4}{3}$

Line c: $y = -\frac{5}{4}x - 9$ $\quad m_c = -\frac{5}{4}$

Line d: $y = \frac{4}{5}x + 10$ $\quad m_d = \frac{4}{5}$

Since $\left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$, then $a \perp b$.

Since $\left(-\frac{5}{4}\right)\left(\frac{4}{5}\right) = -1$, then $c \perp d$.

9) Line a: $4x + 3y = 12$ $\quad m_a = -\frac{4}{3}$

Line b: $16x + 10y = 17$ $\quad m_b = -\frac{8}{5}$

Line c: $6x - 8y = 5$ $\quad m_c = \frac{3}{4}$

Line d: $5x - 8y = 2$ $\quad m_d = \frac{5}{8}$

Since $\left(-\frac{4}{3}\right)\left(\frac{3}{4}\right) = -1$, then $a \perp c$.

Since $\left(-\frac{8}{5}\right)\left(\frac{5}{8}\right) = -1$, then $b \perp d$.

PRACTICE 1 12B

Determine which lines are perpendicular (⊥). Name the slope of each line. Write a sentence describing the relationship between perpendicular lines. Indicate any lines that are vertical or horizontal. (Hint: there can be multiple sets of ⊥ lines.)

10) Line a: $y-1=-4(x+2)$ $\boldsymbol{m_a=-4}$

Line b: $y+9=3(x-8)$ $\boldsymbol{m_b=3}$

Line c: $y+9=-\frac{1}{3}(x-8)$ $\boldsymbol{m_c=-\frac{1}{3}}$

Line d: $y-1=\frac{1}{4}(x+2)$ $\boldsymbol{m_d=\frac{1}{4}}$

11) Line a: $14x-16y=3$ $\boldsymbol{m_a=\frac{7}{8}}$

Line b: $y=17$ $\boldsymbol{m_b=0}$ **(horizontal)**

Line c: $y-3=2(x-6)$ $\boldsymbol{m_c=2}$

Line d: $y=-\frac{8}{7}x+5$ $\boldsymbol{m_d=-\frac{8}{7}}$

Line e: $2x+4y=9$ $\boldsymbol{m_e=-\frac{1}{2}}$

Line f: $x=10$ $\boldsymbol{m_f=}$ **undefined (vertical)**

Write the equation of the line that is perpendicular to the given line and passes through the given point.

12) $y=3x-7$; $(-6,3)$ in slope-intercept form. $\boldsymbol{y=-\frac{1}{3}x+1}$

13) $y+4=\frac{1}{2}(x+1)$; $(-1,-4)$ in point-slope form. $\boldsymbol{y+4=-2(x+1)}$

14) $8x+6y=3$; $(2,-2)$ in standard form. $\boldsymbol{3x-4y=14}$

15) $y=5$; $(-10,11)$ $\boldsymbol{x=-10}$

16) Determine if the following lines are perpendicular to the line in the graph below. Equations a–e should be solved algebraically.

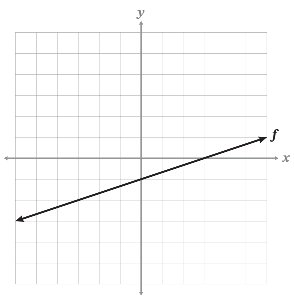
Line a: $3x-y=2$

Line b: $y+1=-\frac{1}{3}(x-1)$

Line c: $3x+y=1$

Line d: $y+1=-3(x-6)$

Line e: $x-3y=-3$



The graph: $\boldsymbol{m=\frac{1}{3}}$

Line $\boldsymbol{a}$**:** $\boldsymbol{m_a=3}$

Line $\boldsymbol{b}$**:** $\boldsymbol{m_b=-\frac{1}{3}}$

Line $\boldsymbol{c}$**:** $\boldsymbol{m_c=-3}$

Line $\boldsymbol{d}$**:** $\boldsymbol{m_d=-3}$

Line $\boldsymbol{e}$**:** $\boldsymbol{m_e=\frac{1}{3}}$

Since $\boldsymbol{\left(\frac{1}{3}\right)(-3)=-1,}$ **then** $\boldsymbol{c\perp f}$ **and** $\boldsymbol{d\perp f}$**.**

17) Renee Street and Descartes Street are perpendicular. The equation for Renee Street is $y=5x+1$. Descartes Street goes through the point $(15,-5)$. Find the equation of Descartes Street in slope-intercept form. $\boldsymbol{y=-\frac{1}{5}x-2}$

10) Since $(-4)\left(\frac{1}{4}\right)=-1$, then $a\perp d$.

Since $(3)\left(-\frac{1}{3}\right)=-1$, then $b\perp d$.

11) Since $\left(\frac{7}{8}\right)\left(-\frac{8}{7}\right)=-1$, then $a\perp d$.

Since line b is horizontal and line f is vertical, then $b\perp f$.

Since $(2)\left(-\frac{1}{2}\right)=-1$, then $c\perp e$.

15) Q: What type of line is $y=5$?

A: *Horizontal*

Q: What type of line is always perpendicular to this?

A: *A vertical line.*

16) Q: What is the slope of the graphed line?

A: $m=\frac{1}{3}$

Q: What line(s) are parallel to this line? How do you know?

A: *Line e, because the slopes are equal.*

17) Renee Descartes is a famous mathematician that helped develop the coordinate plane, sometimes called the Cartesian plane.

Mastery Check

Show What You Know

C) Q: Why should you have four perpendicular statements?

A: *Because there are four corners (or vertices) of a rectangle.*

D) Encourage your student to use their formula sheet to look up the area of a triangle.

Q: What is the area of a triangle?

A: $A = \frac{1}{2}bh$

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

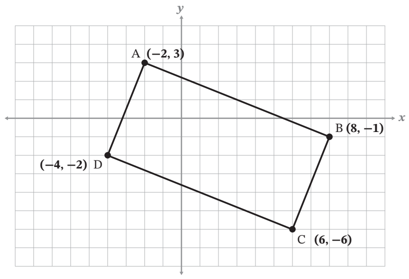
- Identify perpendicular lines.
- Write the equation of a line that is perpendicular to another known line and passes through a given point.

12B MASTERY CHECK

Mastery Check

Show What You Know

A rectangle is a quadrilateral with special properties. Opposite sides must be parallel and adjacent sides must be perpendicular.



Prove algebraically that rectangle ABCD is, in fact, a rectangle.

A) Determine the slopes of all sides of the rectangle algebraically.

The slope of $\overline{AB} = \frac{-1-3}{8-(-2)} = -\frac{2}{5}$

The slope of $\overline{CD} = \frac{-6-(-2)}{6-(-4)} = -\frac{2}{5}$

The slope of $\overline{BC} = \frac{-6-(-1)}{6-8} = \frac{5}{2}$

The slope of $\overline{AD} = \frac{-2-3}{-4-(-2)} = \frac{5}{2}$

B) Name the sides parallel to one another.

Since the slopes are equal, $\overline{AB} \parallel \overline{CD}$.

Since the slopes are equal, $\overline{BC} \parallel \overline{AD}$.

C) Name the sides perpendicular to one another.

$m = -\frac{2}{5} \perp m = \frac{5}{2}$, so

$\overline{AB} \perp \overline{AD}$ $\overline{BC} \perp \overline{CD}$

$\overline{AB} \perp \overline{BC}$ $\overline{CD} \perp \overline{AD}$

D) A segment was drawn connecting A and C to make two triangles. If the length of side $\overline{AD}$ is 5.4 cm and the length of side $\overline{CD}$ is 10.8 cm, what would the area be for one of the triangles formed by segment $\overline{AC}$?

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(5.4)(10.8)$

$A = 29.16$ square cm

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

246 Lesson 12: Parallel and Perpendicular Lines > Part B: Perpendicular Lines > Mastery Check Algebra 1 Student Worktext

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

PRACTICE 2 12B

Practice 2

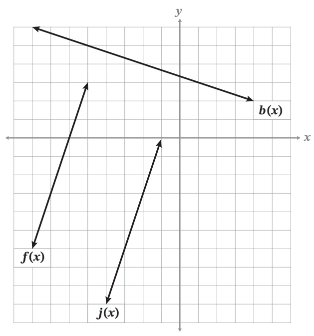
Complete the practice problems on a separate sheet of paper.

Match each of the given slopes to its corresponding perpendicular slope.

	Given		Perpendicular Slopes
D	1) $m = 4$	A)	undefined
B	2) $m = -\frac{7}{9}$	B)	$\frac{9}{7}$
A	3) $m = 0$	C)	$-\frac{7}{4}$
E	4) $m = -\frac{2}{3}$	D)	$-\frac{1}{4}$
C	5) $m = \frac{4}{7}$	E)	$\frac{3}{2}$

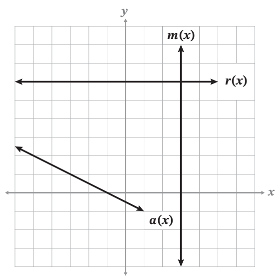
Find *all* pairs of perpendicular lines.

6)



Since $\left(-\frac{1}{3}\right)(3) = -1$, then $b(x) \perp f(x)$ and $b(x) \perp j(x)$

7)



Since $r(x)$ is horizontal and $m(x)$ is vertical, then $r(x) \perp m(x)$

Determine which lines are perpendicular. Name the slope of each line. Write a sentence describing the relationship between perpendicular lines. Indicate any lines that are vertical or horizontal. (Hint: there can be multiple sets of ⊥ lines.)

8) Determine which lines are perpendicular.

Line a: $y = -10x$ $\quad m_a = -10$

Line b: $y = x - 7$ $\quad m_b = 1$

Line c: $y = \frac{1}{10}x - 9$ $\quad m_c = \frac{1}{10}$

Line d: $y = -x + 10$ $\quad m_d = -1$

Since $(-10)\left(\frac{1}{10}\right) = -1$, then $a \perp c$.
Since $(1)(-1) = -1$, then $b \perp d$.

9) Determine which lines are perpendicular.

Line a: $2x + 7y = 5$ $\quad m_a = -\frac{2}{7}$

Line b: $14x - 4y = 9$ $\quad m_b = \frac{7}{2}$

Line c: $x + y = 5$ $\quad m_c = -1$

Line d: $x - y = 3$ $\quad m_d = 1$

Since $\left(-\frac{2}{7}\right)\left(\frac{7}{2}\right) = -1$, then $a \perp b$.
Since $(-1)(1) = -1$, then $c \perp d$.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

10) Since $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right) = -1$, then $a \perp d$.
Since $(4)\left(-\frac{1}{4}\right) = -1$, then $b \perp c$.

11) Since vertical and horizontal lines are perpendicular, $a \perp c$.
Since $\left(-\frac{3}{5}\right)\left(\frac{5}{3}\right) = -1$, there is no line perpendicular to b or d.
Since $\left(\frac{2}{7}\right)\left(-\frac{7}{2}\right) = -1$, then $e \perp f$.

16) The student found the reciprocal slope but did not change the sign.
Correction:

$$y - 2 = \frac{1}{5}(x - (-3))$$
$$y - 2 = \frac{1}{5}(x + 3)$$
$$y - 2 = \frac{1}{5}x + \frac{3}{5}$$
$$y = \frac{1}{5}x + \frac{13}{5}$$

If needed, have your student go back to the Mastery Check and reapply what they have learned to show and say what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Determine which lines are perpendicular. Name the slope of each line. Write a sentence describing the relationship between perpendicular lines. Indicate any lines that are vertical or horizontal. (Hint: there can be multiple sets of ⊥ lines.)

10) Determine which lines are perpendicular.

Line a: $y - 6 = -\frac{2}{3}(x + 4)$ $\quad \boldsymbol{m_a = -\frac{2}{3}}$

Line b: $y - 1 = 4(x - 8)$ $\quad \boldsymbol{m_b = 4}$

Line c: $y - 6 = -\frac{1}{4}(x + 4)$ $\quad \boldsymbol{m_c = -\frac{1}{4}}$

Line d: $y - 1 = \frac{3}{2}(x - 8)$ $\quad \boldsymbol{m_d = \frac{3}{2}}$

11) Determine which lines are perpendicular.

Line a: $x = 0$ $\quad$ **m_a = undefined (vertical)**

Line b: $y = -\frac{3}{5}x$ $\quad \boldsymbol{m_b = -\frac{3}{5}}$

Line c: $y = 0$ $\quad$ **$m_c = 0$ (horizontal)**

Line d: $3x + 5y = 15$ $\quad \boldsymbol{m_d = -\frac{3}{5}}$

Line e: $y + 7 = \frac{2}{7}(x - 2)$ $\quad \boldsymbol{m_e = \frac{2}{7}}$

Line f: $y = -\frac{7}{2}x + 9$ $\quad \boldsymbol{m_f = -\frac{7}{2}}$

Write the equation of the line that is perpendicular to the given line and passes through the given point.

12) $y = \frac{5}{6}x - 7$; $(-1, 3)$ in slope-intercept form. $\quad \boldsymbol{y = -\frac{6}{5}x + \frac{9}{5}}$

13) $y + 2 = 4(x + 2)$; $(8, -2)$ in point-slope form. $\quad \boldsymbol{y + 2 = -\frac{1}{4}(x - 8)}$

14) $x + 3y = 4$; $(-6, 7)$ in standard form. $\quad \boldsymbol{3x - y = -25}$

15) Find the line perpendicular to the given line, $x = -12$, that passes through $(-4, 9)$. Explain your thinking. **This line is vertical. Any line perpendicular to this line will be horizontal. $y = 9$**

16) Find the error, then correct the work.

A student was trying to find the equation of a line in slope-intercept form that is perpendicular to $y = -5x + 2$ and passes through the point $(-3, 2)$. Their work was as follows:

$y - 2 = -\frac{1}{5}(x + 3)$, or $y = -\frac{1}{5}x + \frac{7}{10}$

17) A construction worker was installing a railing for the stairs. The slope of the railing was a 7-inch rise by a 10-inch run. The worker needed to build a perpendicular baluster through the point $(-4, -6)$. Write the equation of the line in point-slope form. $\quad \boldsymbol{y + 6 = -\frac{10}{7}(x + 4)}$

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Graph the horizontal and vertical lines that pass through the point $(-3, 2)$. Label the point.

2) Write the equation of the horizontal line that passes through the point $(-3, 2)$. $y = 2$

3) Write the equation of the vertical line that passes through the point $(-3, 2)$. $x = -3$

4) Write an equation in point-slope form for a line with a slope of 2 that travels through the point $(-1, 3)$. $y - 3 = 2(x + 1)$

5) Graph your equation from problem 4.

6) Joseph sold lemonade for one week in the summer. He sold the lemonade for \$0.75 per cup and had earned \$15.25 after selling 47 cups of lemonade.

Define your variables as an ordered pair, and write a linear equation.
(cups of lemonade, money earned); $y = 0.75x - 20$

Explain what you think the y-intercept means in the context of this problem. **The y-intercept is -20. This means that Joseph had to borrow \$20 to start his business (or that he owes \$20 to someone).**

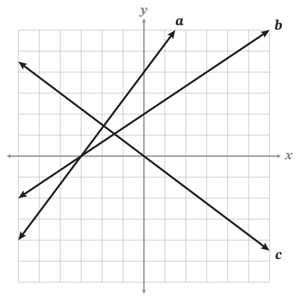
7) Write the equation in standard form.

$y = -\frac{2}{3}x + 2$ $2x + 3y = 6$

8) Determine the x and y-intercepts for the given equation.

$7x + 3y = 18$ $\left(\frac{18}{7}, 0\right)$ **and** $(0, 6)$

9) Which line does each solution below satisfy?



$f(0) = 0$ **line c**

$f(-3) = 0$ **lines a and b**

$f(0) = 4$ **line a**

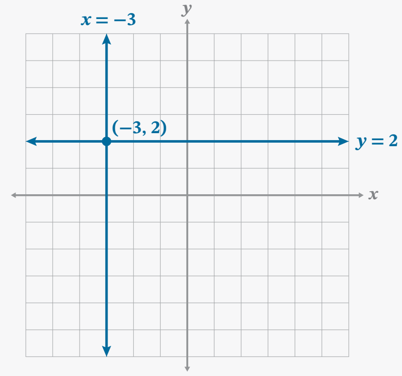
$f(3) = 4$ **line b**

Targeted Review

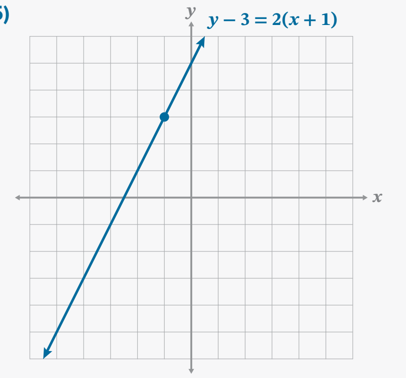
Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

1)



5)



Problem	1	2	3	4	5	6	7	8	9	10	11	12	13
Lesson Origin	11	11	9	9	10	11	11	7	8	7	11	11	11

10) Your student may also realize that the lines are parallel from what they learned in Lesson 12. However, this is a review question from Lesson 8.

12) Distractor Rationale:

A and D correctly match the graph but are *not* in standard form. B has the incorrect y-intercept.

13) Distractor Rationale:

A and B represent equations of vertical lines. C uses the x-coordinate rather than the y-coordinate for the equation.

TARGETED REVIEW 12

10) Explain how the graphs of $f(x) = -2x$ and $g(x) = -2x + 2$ are translations of one another.
The graph of $g(x)$ is translated up two units from the graph of $f(x)$.

11) Name the domain and range. Determine if the given relation is a function.

x	$f(x)$
-3	-9
0	0
1	3
2	6

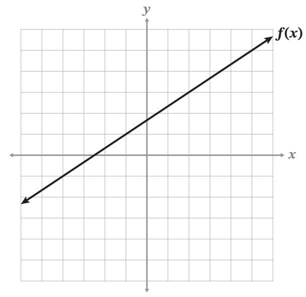
domain: $\{-3, 0, 1, 2\}$
range: $\{-9, 0, 3, 6\}$

This is a function because the domain values do not repeat.

Multiple Choice

C **12)** Determine the equation in standard form that represents the function $f(x)$ in the graph below.

A) $-\frac{2}{3}x + y = \frac{5}{3}$
B) $-2x + 3y = -5$
C) $\mathbf{2x - 3y = -5}$
D) $y = \frac{2}{3}x + \frac{5}{3}$



D **13)** What is the equation for the horizontal line through the point $(-8, 14)$?

A) $x = -8$
B) $x = 14$
C) $y = -8$
D) $\mathbf{y = 14}$

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13
Lesson Origin	11	11	9	9	10	11	11	7	8	7	11	11	11