Lesson 7
Functions

Outline

Part A
Relations and Functions

- The Coordinate Plane
- Relations
- Functions
- The Vertical Line Test

Part B
Understanding Functions

- Function Notation
- Independent and Dependent Variables
- Using a Function Rule

Targeted Review

Vocabulary

- coordinate plane
- origin
- quadrant
- coordinates
- ordered pairs
- relation
- domain
- range
- function
- mapping
- vertical line test
- independent variable
- dependent variable
- repeated substitution

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

2) Horizontal is left-to-right like the horizon (they share the same y-value.)

3) Vertical points are aligned up and down (have the same x-value)

Part A: Relations and Functions

Objectives

In this part of the lesson, you will learn about relations and functions.

By the end of this lesson, you will be able to do the following:

- Find the domain and range of a relation from a graph, table, and mapping.
- Express the definition of a function using words and diagrams.

Why?

Seeing relations and functions in different ways will help you recognize them in word problems and in your world.

Warm Up

1) Plot the ordered pairs on the graph provided and label them with the letter indicated.

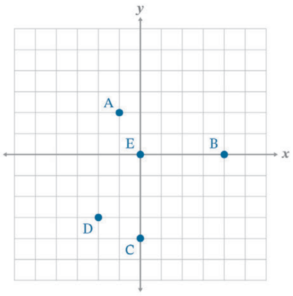
A: $(-1, 2)$

B: $(4, 0)$

C: $(0, -4)$

D: $(-2, -3)$

E: $(0, 0)$



2) Which points align horizontally? Explain.

E and B align horizontally because they share the same y-value.

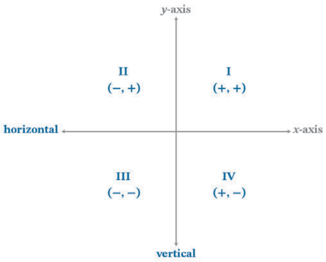
3) Which points align vertically? Explain.

E and C align vertically because they share the same x-value.

The Coordinate Plane

- The **coordinate plane** is a two-dimensional graph formed by the x-axis and y-axis.
- The point $(0, 0)$ where the x-axis and y-axis intersect is called the **origin**.
- A quarter of the coordinate plane is called a **quadrant**.
- Quadrants are labeled **counter-clockwise** starting at the upper-right quadrant.

- The signs for an ordered pair in each quadrant are:
- **Ordered pairs** are sets of numbers that represent points on the coordinate plane.
- They are written as **(x, y)**, where x represents the **horizontal** direction of the point and y represents the **vertical** direction of the point.
  - x is positive in quadrants: **I, IV**
  - x is negative in quadrants: **II, III**
  - y is positive in quadrants: **I, II**
  - y is negative in quadrants: **III, IV**



- In this course, if the graph does not have a scale listed, it will be assumed the scale is **one**.

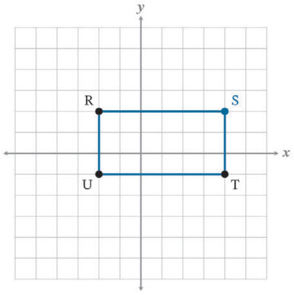
Example 1

Create a rectangle on the coordinate plane using the three vertices provided. Plot point S to finish the rectangle.

Plan Visually determine where the fourth point belongs. Then, plot the point.

OR

Take the x-value from point T and the y-value from point R to make the new ordered pair for S. Then, plot the point.



Point S: **(4, 2)**

Name the horizontal sides of the rectangle.

$\overline{RS}$ and $\overline{TU}$

Name the vertical sides of the rectangle.

$\overline{RU}$ and $\overline{ST}$

Point is another name for an ordered pair, which are always ordered (x, y).

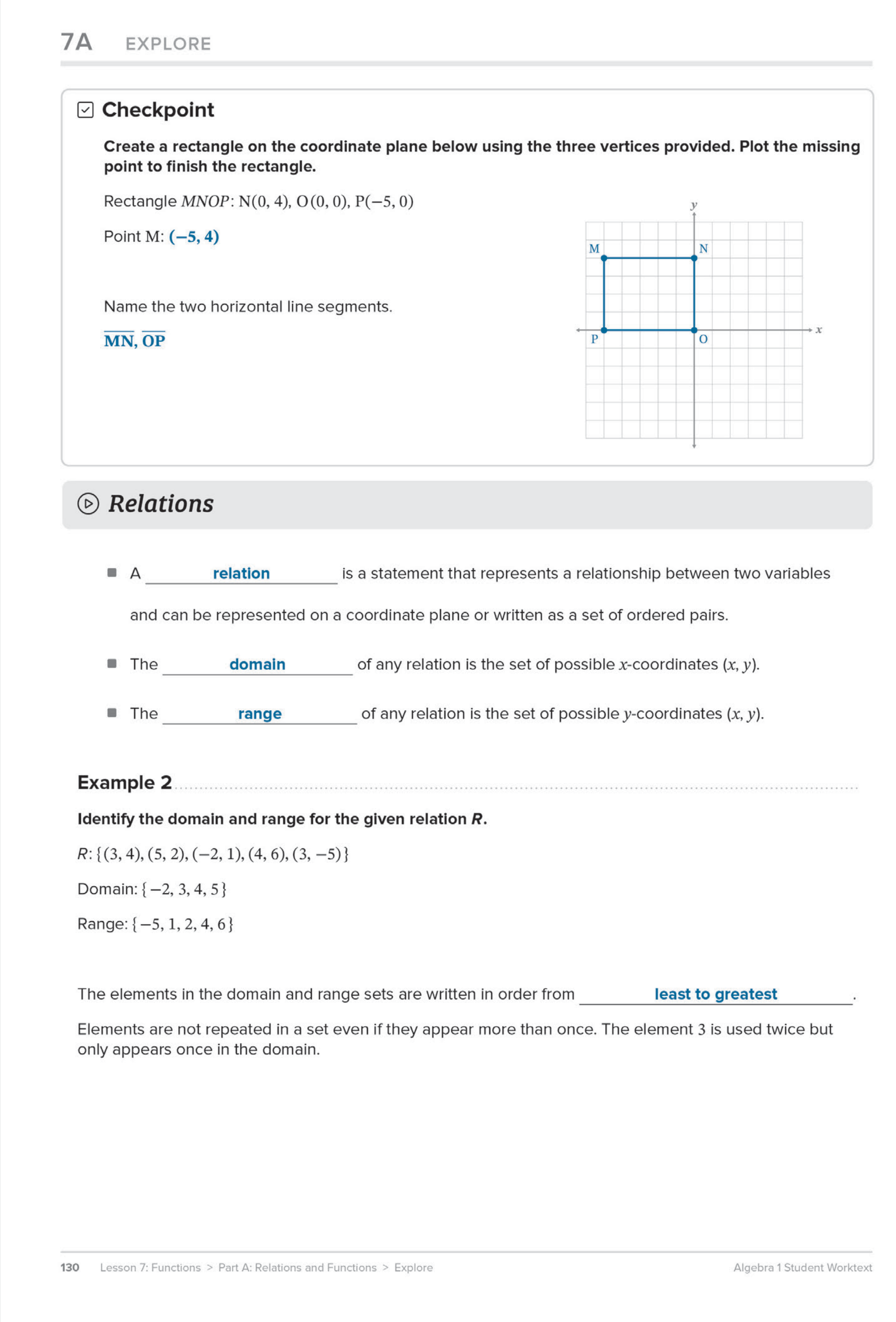
☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Is horizontal left to right or up and down?
A: *left to right*

Q: What are the two vertical line segments?
A: $\overline{MP}$, $\overline{NO}$

Q: What quadrant is Point M located in?
A: *QII, Quadrant 2*



☑ Checkpoint

Create a rectangle on the coordinate plane below using the three vertices provided. Plot the missing point to finish the rectangle.

Rectangle $MNOP$: N(0, 4), O(0, 0), P(−5, 0)

Point M: **(−5, 4)**

Name the two horizontal line segments.

$\overline{MN}$, $\overline{OP}$

▷ *Relations*

- A **relation** is a statement that represents a relationship between two variables and can be represented on a coordinate plane or written as a set of ordered pairs.
- The **domain** of any relation is the set of possible x-coordinates (x, y).
- The **range** of any relation is the set of possible y-coordinates (x, y).

Example 2

Identify the domain and range for the given relation *R*.

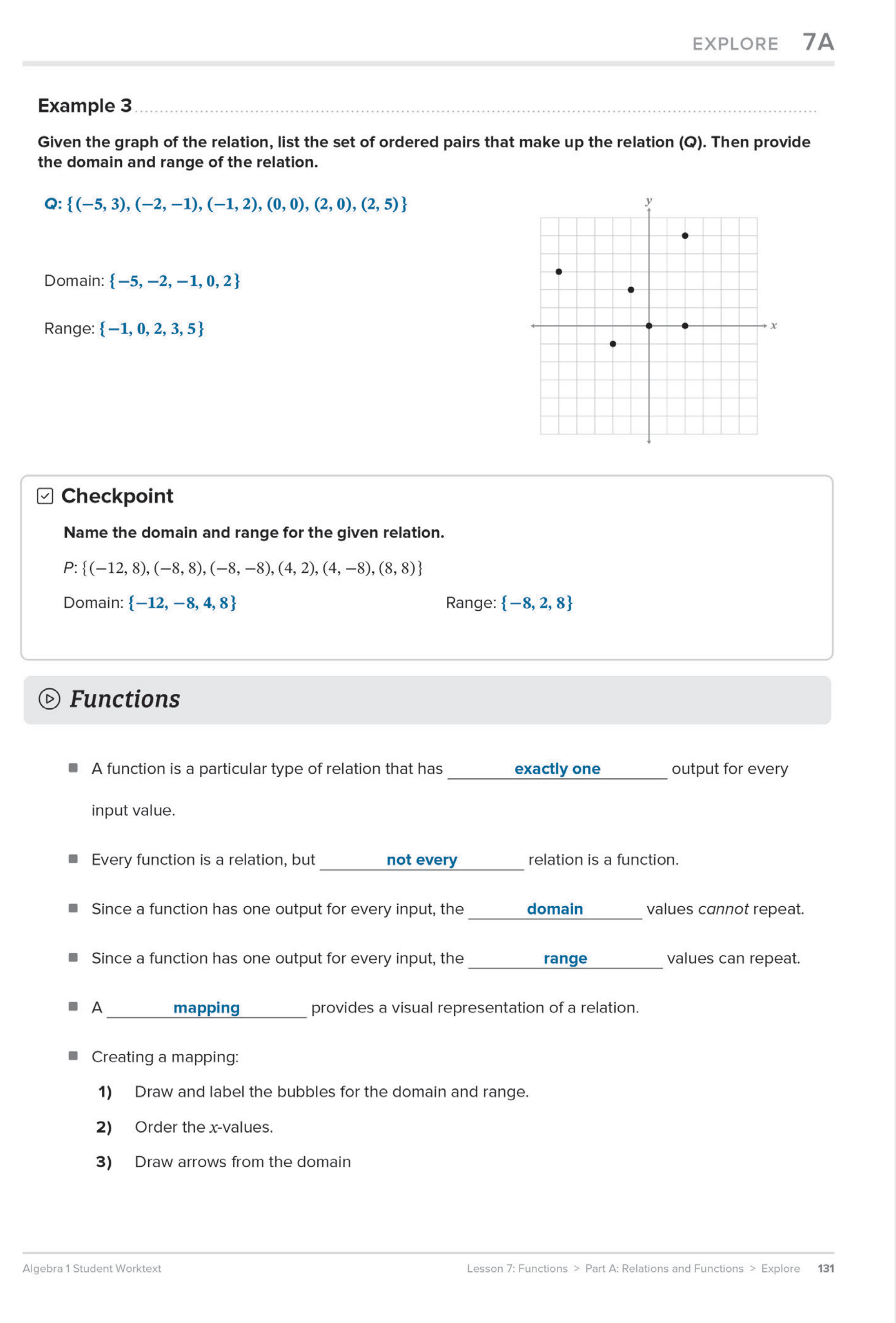
R: $\{(3, 4), (5, 2), (-2, 1), (4, 6), (3, -5)\}$

Domain: $\{-2, 3, 4, 5\}$

Range: $\{-5, 1, 2, 4, 6\}$

The elements in the domain and range sets are written in order from **least to greatest**.

Elements are not repeated in a set even if they appear more than once. The element 3 is used twice but only appears once in the domain.



Example 3

Given the graph of the relation, list the set of ordered pairs that make up the relation (*Q*). Then provide the domain and range of the relation.

Q: {(−5, 3), (−2, −1), (−1, 2), (0, 0), (2, 0), (2, 5)}

Domain: **{−5, −2, −1, 0, 2}**

Range: **{−1, 0, 2, 3, 5}**

☑ Checkpoint

Name the domain and range for the given relation.

P: $\{(-12, 8), (-8, 8), (-8, -8), (4, 2), (4, -8), (8, 8)\}$

Domain: **{−12, −8, 4, 8}**

Range: **{−8, 2, 8}**

Functions

- A function is a particular type of relation that has **exactly one** output for every input value.
- Every function is a relation, but **not every** relation is a function.
- Since a function has one output for every input, the **domain** values *cannot* repeat.
- Since a function has one output for every input, the **range** values can repeat.
- A **mapping** provides a visual representation of a relation.
- Creating a mapping:
  1. Draw and label the bubbles for the domain and range.
  2. Order the x-values.
  3. Draw arrows from the domain

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What values are represented by the domain?

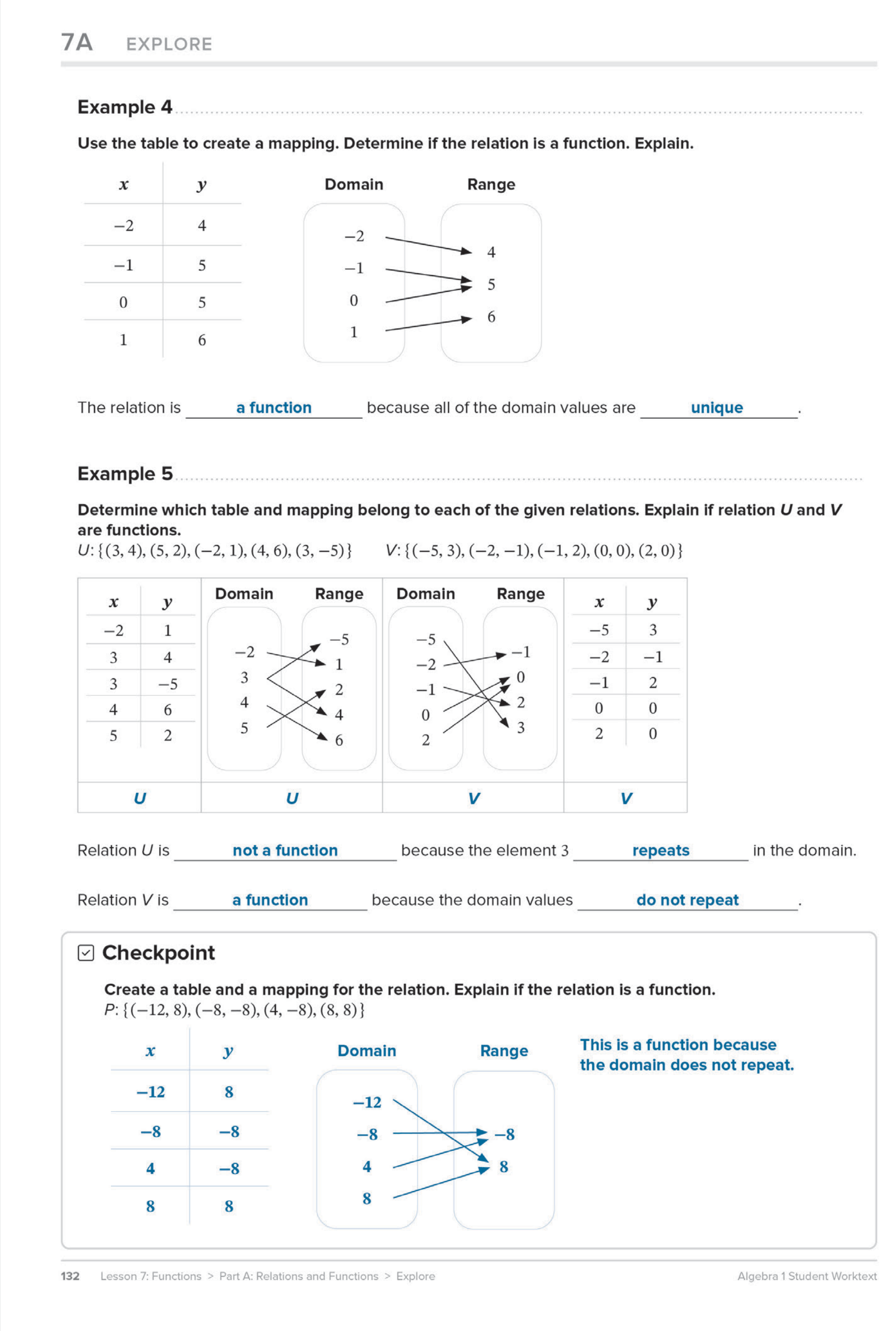
A: *x-values*

Q: What values are represented by the range?

A: *y-values*

Q: Why do values not repeat when listing the domain and range?

A: *Because this is a relation, not a data set.*



Example 4

Use the table to create a mapping. Determine if the relation is a function. Explain.

x	y
−2	4
−1	5
0	5
1	6

The relation is **a function** because all of the domain values are **unique**.

Example 5

Determine which table and mapping belong to each of the given relations. Explain if relation *U* and *V* are functions.

U: $\{(3, 4), (5, 2), (-2, 1), (4, 6), (3, -5)\}$ V: $\{(-5, 3), (-2, -1), (-1, 2), (0, 0), (2, 0)\}$

x	y
−2	1
3	4
3	−5
4	6
5	2

U

x	y
−5	3
−2	−1
−1	2
0	0
2	0

V

Relation *U* is **not a function** because the element 3 **repeats** in the domain.

Relation *V* is **a function** because the domain values **do not repeat**.

☑ Checkpoint

Create a table and a mapping for the relation. Explain if the relation is a function.

P: $\{(-12, 8), (-8, -8), (4, -8), (8, 8)\}$

x	y
−12	8
−8	−8
4	−8
8	8

This is a function because the domain does not repeat.

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Having your student make the table first will help them organize the mapping. Recall that the mapping has only one occurrence of any domain value or any range value, and it should be in ascending order.

Q: Can the range values repeat when you have a function?

A: *Yes*

Q: Can the domain values repeat in a function?

A: *No*

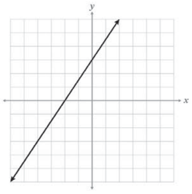
The Vertical Line Test

- The **vertical line test (VLT)** compares a vertical line to every x-value on a graph.
- If you move a vertical line across a graph and it touches **only one point at a time** ______, then the equation is a function.
- If you move a vertical line across a graph and it touches **more than one point** ______ for any x-value, then the equation is not a function.
- **Vertical** lines are not functions because every x-value has the same coordinate.
- Functions can be identified using a **relation**, **table**, **mapping**, and **graph**.

Example 6

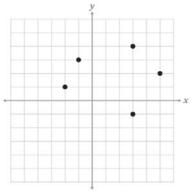
Determine if the graphs below are functions.

A)



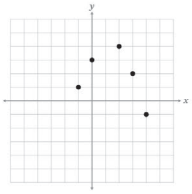
Function

B)



Not a function

C)

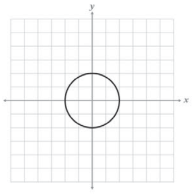


Function

☑ Checkpoint

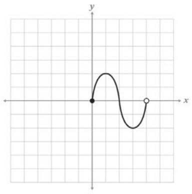
Determine whether the following relations are functions. Explain.

A)



This is not a function because it fails the vertical line test (VLT).

B)



This is a function because it passes the VLT (and all the domain values are unique).

Example 6

A) This is a function because it passes the vertical line test (VLT).

B) This fails the VLT when $x = 3$.

Have your student draw a vertical line through $x = 3$.

C) This is a function because it passes the VLT. (No repeating domain or x-values)

☑ Checkpoint

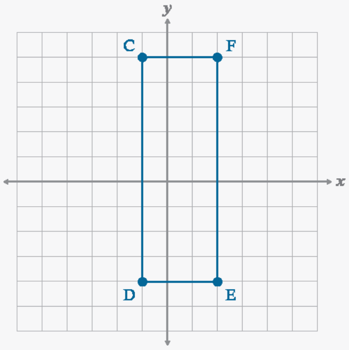
To continue past this checkpoint, students should confidently and correctly answer this problem.

A) Saying that a function fails the vertical line test is enough to explain why a graph is not a function. This method cannot be used when a list of ordered pairs is given unless the ordered pairs are graphed first.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1)



Point D: $(-1, -4)$

Q: In what quadrant is point D located?

A: *QIII, Quadrant 3*

3) Q: Are the domain values the variable x or y?

A: x

4)

x	y
-4	8
-2	6
0	4
2	2
4	0

Q: If the same number occurs in the range, can the relation be a function?

A: *Yes, the only thing that cannot repeat are domain values.*

6)

x	y
-1	-3
0	-2
1	-1
1	3
2	0

Q: What happens on the graph when you do not have a function?

A: *There are points that line up vertically.*

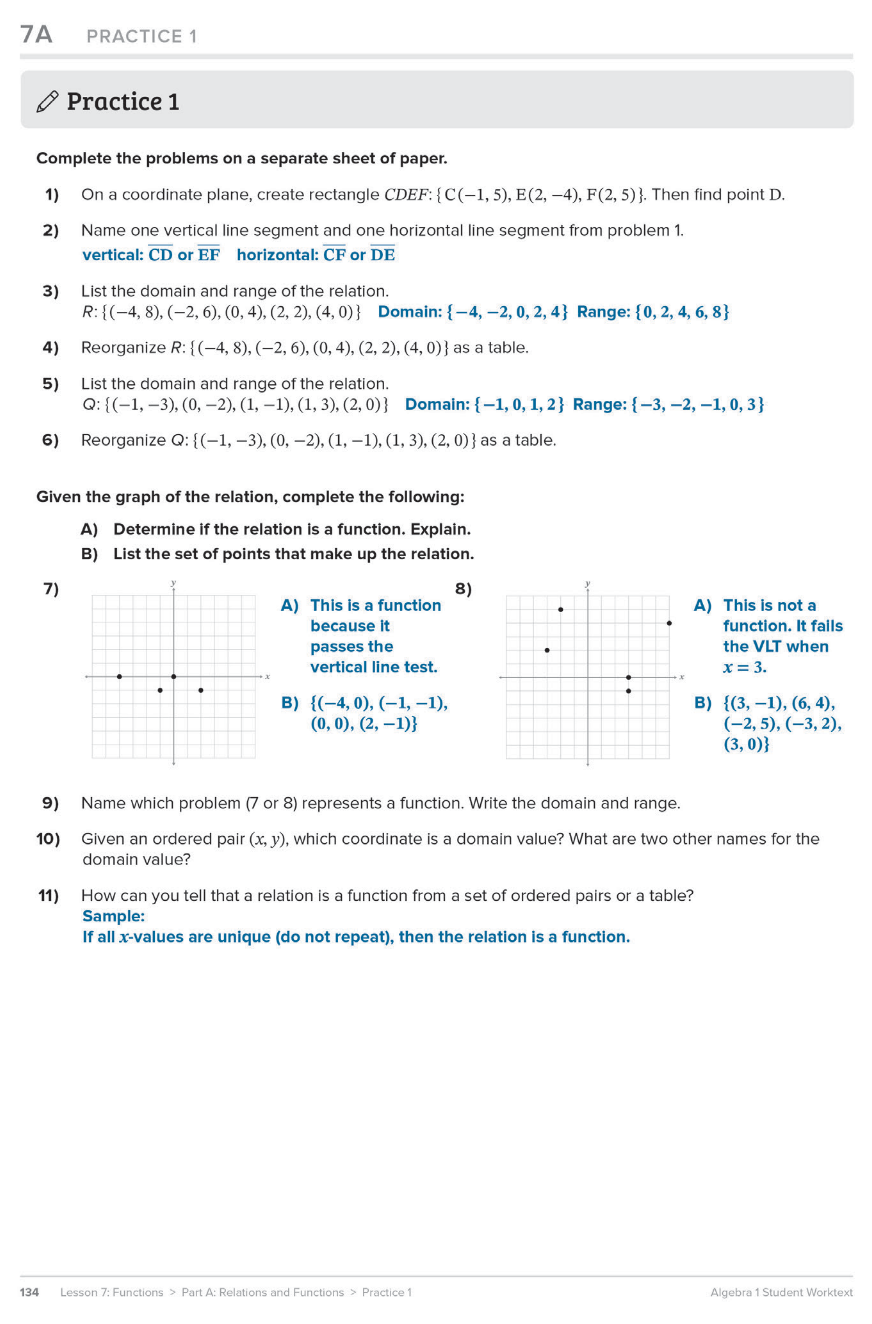
7) B) Q: Do the points have to be ordered in a certain way when writing a relation?

A: *No, you just need to make sure that all points are listed.*

9) Problem 7 represents a function. Domain: $\{-4, -1, 0, 2\}$ Range: $\{-1, 0\}$

10) The x-coordinate represents the domain value. Other names for the domain include x-value or input.

In part B of this lesson, your student will also be able to identify independent variables as x-values or the domain.



7A PRACTICE 1

Practice 1

Complete the problems on a separate sheet of paper.

1) On a coordinate plane, create rectangle $CDEF$: $\{C(-1, 5), E(2, -4), F(2, 5)\}$. Then find point D.

2) Name one vertical line segment and one horizontal line segment from problem 1.
vertical: $\overline{CD}$ or $\overline{EF}$ horizontal: $\overline{CF}$ or $\overline{DE}$

3) List the domain and range of the relation.
R: $\{(-4, 8), (-2, 6), (0, 4), (2, 2), (4, 0)\}$ **Domain: $\{-4, -2, 0, 2, 4\}$ Range: $\{0, 2, 4, 6, 8\}$**

4) Reorganize R: $\{(-4, 8), (-2, 6), (0, 4), (2, 2), (4, 0)\}$ as a table.

5) List the domain and range of the relation.
Q: $\{(-1, -3), (0, -2), (1, -1), (1, 3), (2, 0)\}$ **Domain: $\{-1, 0, 1, 2\}$ Range: $\{-3, -2, -1, 0, 3\}$**

6) Reorganize Q: $\{(-1, -3), (0, -2), (1, -1), (1, 3), (2, 0)\}$ as a table.

Given the graph of the relation, complete the following:

A) Determine if the relation is a function. Explain.
B) List the set of points that make up the relation.

7)

A) This is a function because it passes the vertical line test.

B) $\{(-4, 0), (-1, -1), (0, 0), (2, -1)\}$

8)

A) This is not a function. It fails the VLT when $x = 3$.

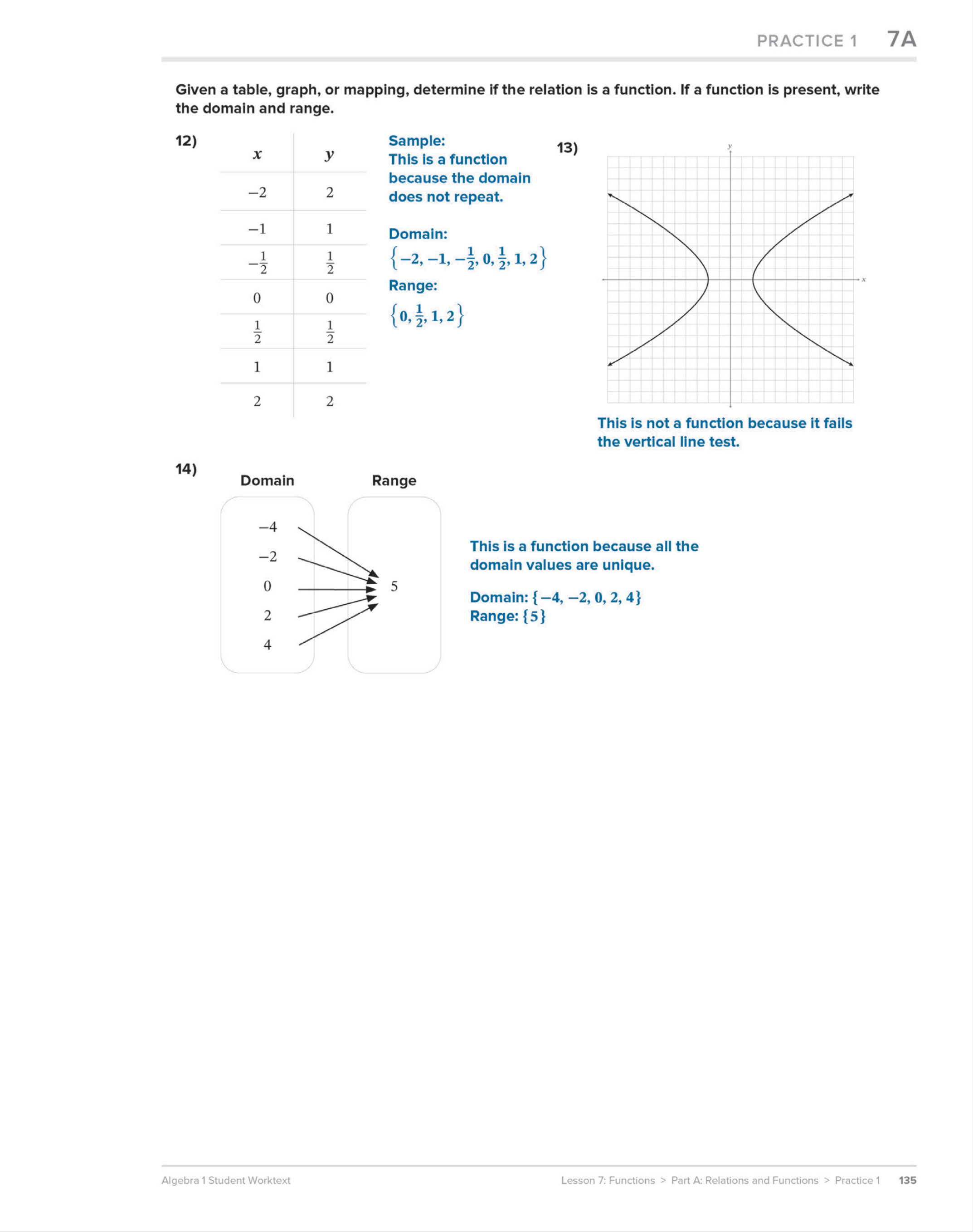
B) $\{(3, -1), (6, 4), (-2, 5), (-3, 2), (3, 0)\}$

9) Name which problem (7 or 8) represents a function. Write the domain and range.

10) Given an ordered pair (x, y), which coordinate is a domain value? What are two other names for the domain value?

11) How can you tell that a relation is a function from a set of ordered pairs or a table?
Sample:
If all x-values are unique (do not repeat), then the relation is a function.

134 Lesson 7: Functions > Part A: Relations and Functions > Practice 1 Algebra 1 Student Worktext



Given a table, graph, or mapping, determine if the relation is a function. If a function is present, write the domain and range.

12)

x	y
-2	2
-1	1
$-\frac{1}{2}$	$\frac{1}{2}$
0	0
$\frac{1}{2}$	$\frac{1}{2}$
1	1
2	2

Sample:
This is a function because the domain does not repeat.

Domain:
$\left\{-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2\right\}$

Range:
$\left\{0, \frac{1}{2}, 1, 2\right\}$

13)

This is not a function because it fails the vertical line test.

14)

This is a function because all the domain values are unique.

Domain: $\{-4, -2, 0, 2, 4\}$
Range: $\{5\}$

12) Q: What should you look at to determine if you have a function?

A: *The x-values*

14) If your student is not convinced that this can be a function, have them graph the ordered pairs $(-4, 5)$, $(-2, 5)$, $(0, 5)$, $(2, 5)$, $(4, 5)$. The result will be a horizontal line that will pass the vertical line test.

Mastery Check

Show What You Know

B) Your student should have a table with unique x-values. The y-values can also be unique but are allowed to repeat.

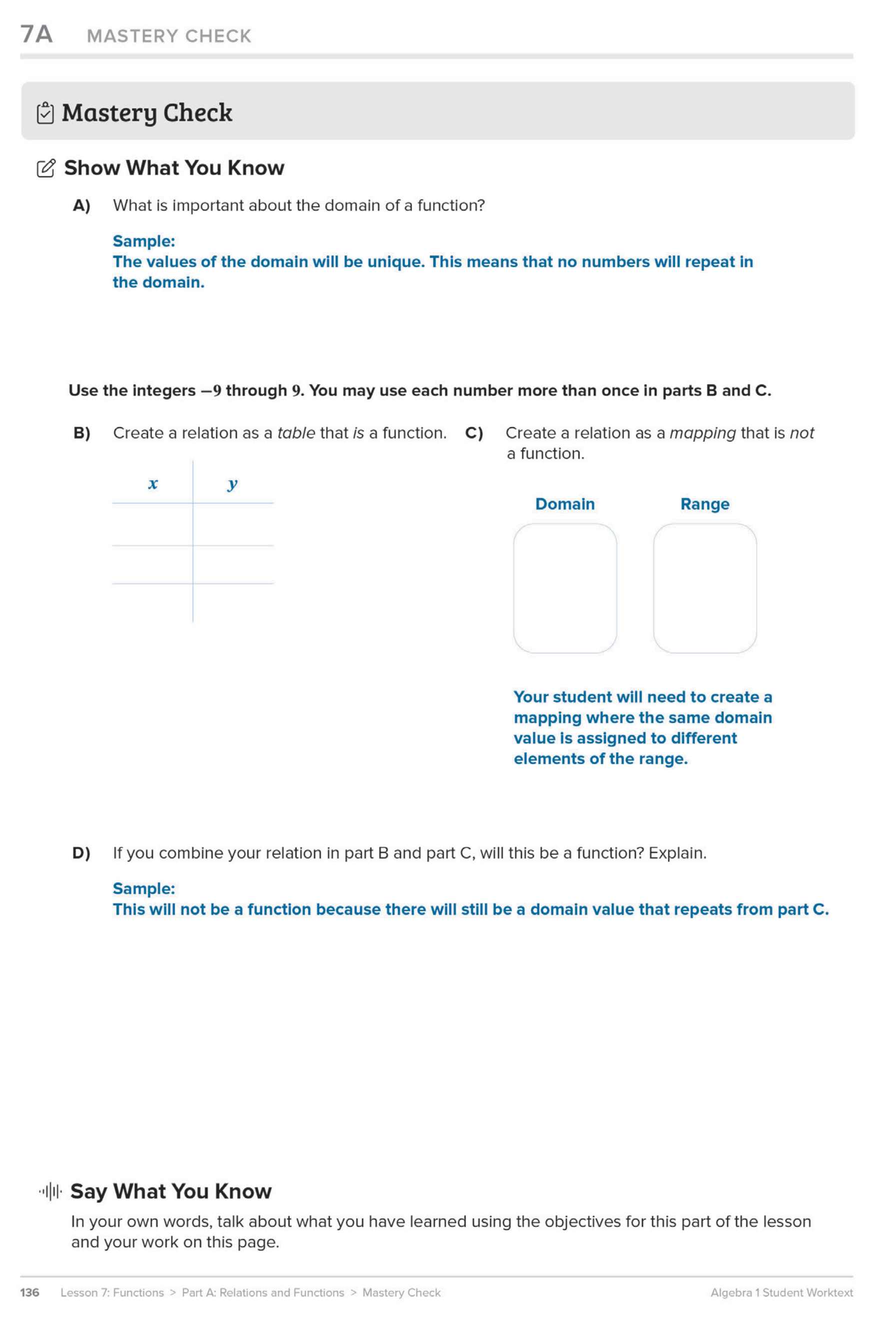
C) Example 5 in the Guided Notes can be used as a guide.

Because your student may use the numbers −9 to 9 more than once, combining the two relations will likely result in more repeated elements of the domain. Since part C is not a function, combining it with part B will still result in a relation that is not a function.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- ✓ Find the domain and range of a relation from a graph, table, and mapping.
- ✓ Express the definition of a function using words and diagrams.



Mastery Check

Show What You Know

A) What is important about the domain of a function?

Sample:
The values of the domain will be unique. This means that no numbers will repeat in the domain.

Use the integers −9 through 9. You may use each number more than once in parts B and C.

B) Create a relation as a *table* that *is* a function.

x	y

C) Create a relation as a *mapping* that is *not* a function.

Domain **Range**

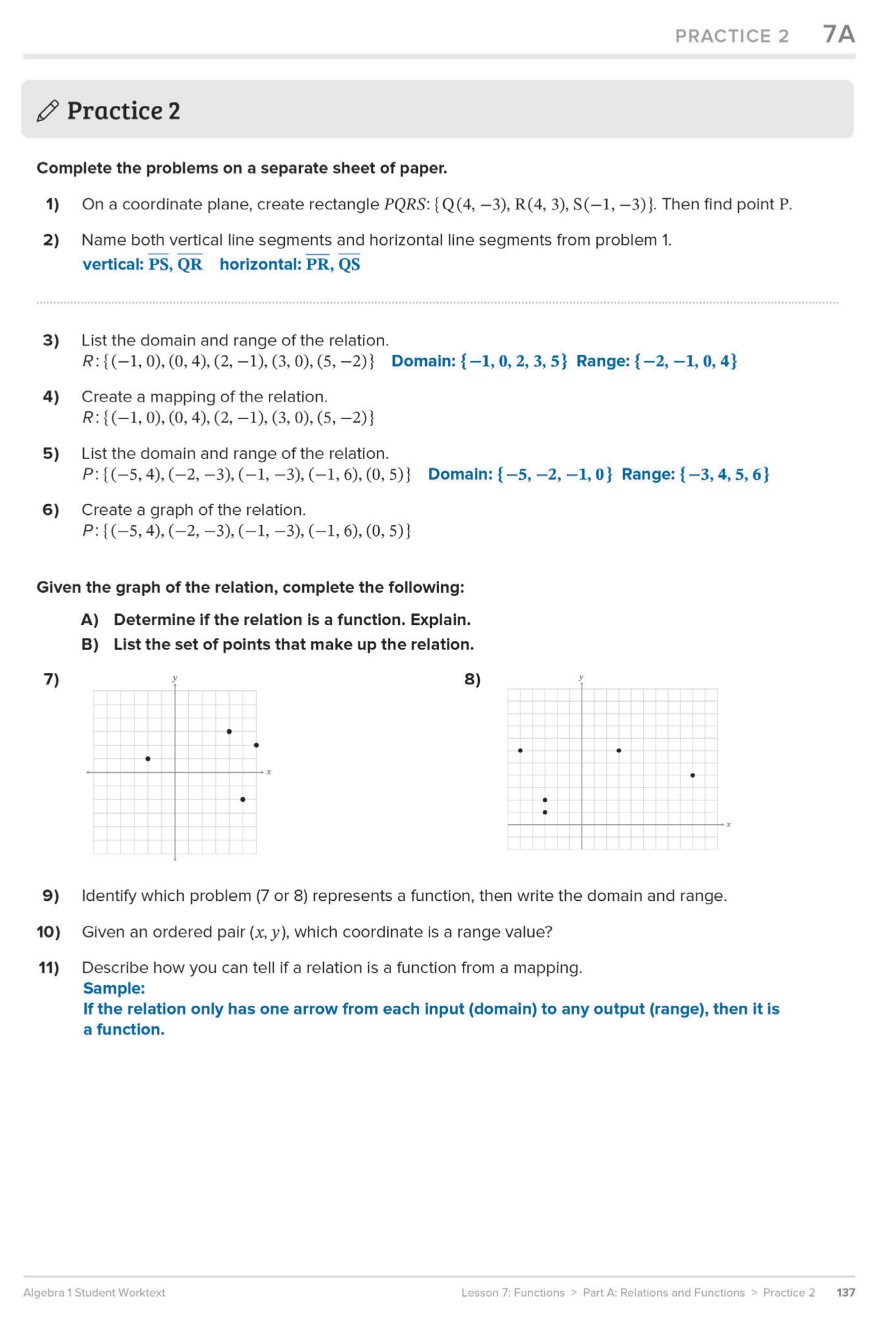
Your student will need to create a mapping where the same domain value is assigned to different elements of the range.

D) If you combine your relation in part B and part C, will this be a function? Explain.

Sample:
This will not be a function because there will still be a domain value that repeats from part C.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.



Practice 2

Complete the problems on a separate sheet of paper.

1) On a coordinate plane, create rectangle $PQRS$: $\{Q(4, -3), R(4, 3), S(-1, -3)\}$. Then find point P.

2) Name both vertical line segments and horizontal line segments from problem 1.
vertical: $\overline{PS}$, $\overline{QR}$ horizontal: $\overline{PR}$, $\overline{QS}$

3) List the domain and range of the relation.
$R: \{(-1, 0), (0, 4), (2, -1), (3, 0), (5, -2)\}$ **Domain: $\{-1, 0, 2, 3, 5\}$ Range: $\{-2, -1, 0, 4\}$**

4) Create a mapping of the relation.
$R: \{(-1, 0), (0, 4), (2, -1), (3, 0), (5, -2)\}$

5) List the domain and range of the relation.
$P: \{(-5, 4), (-2, -3), (-1, -3), (-1, 6), (0, 5)\}$ **Domain: $\{-5, -2, -1, 0\}$ Range: $\{-3, 4, 5, 6\}$**

6) Create a graph of the relation.
$P: \{(-5, 4), (-2, -3), (-1, -3), (-1, 6), (0, 5)\}$

Given the graph of the relation, complete the following:

A) **Determine if the relation is a function. Explain.**
B) **List the set of points that make up the relation.**

7)

8)

9) Identify which problem (7 or 8) represents a function, then write the domain and range.

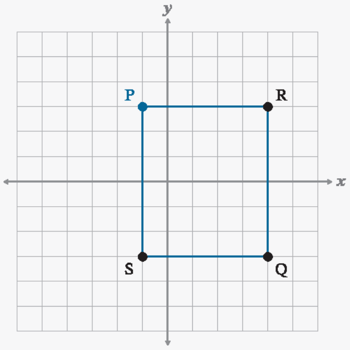
10) Given an ordered pair (x, y), which coordinate is a range value?

11) Describe how you can tell if a relation is a function from a mapping.
Sample:
If the relation only has one arrow from each input (domain) to any output (range), then it is a function.

Practice 2

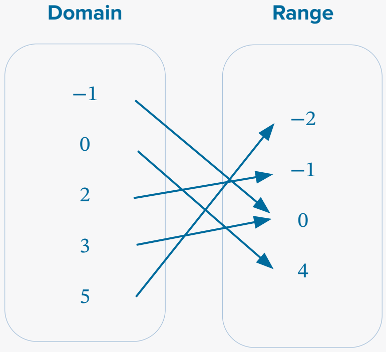
Worked solutions for these problems are located in the Digital Pack.

1)

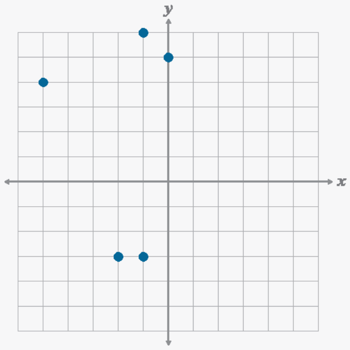


Point P: $(-1, 3)$

4)



6)



7) A) This is a function because it passes the VLT (or domain values are unique/do not repeat).

B) $R: \{(-2, 1), (4, 3), (5, -2), (7, 2)\}$

8) A) This graph fails the vertical line test when $x = -3$. Therefore, it is not a function.

B) $R: \{(-5, 6), (-3, 1), (-3, 2), (3, 6), (9, 4)\}$

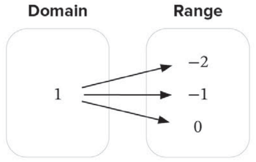
9) Problem 7 represents a function.
Domain: $\{-2, 4, 5, 6\}$ Range: $\{-2, 1, 2, 3\}$

10) The y-coordinate represents the range value.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Given a table, graph, or mapping, determine if the relation is a function. If a function is present, write the domain and range.

12)



Sample:
This is not a function because the domain repeats for each range value.

13)

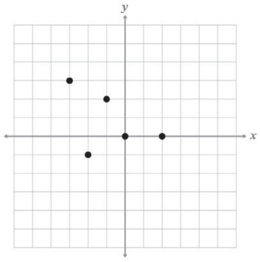
x	y
−4	−2
−2	0
0	2
$\frac{1}{2}$	$\frac{5}{2}$
2	4

Sample:
This is a function because the domain values are unique.

domain: $\left\{-4, -2, 0, \frac{1}{2}, 2\right\}$

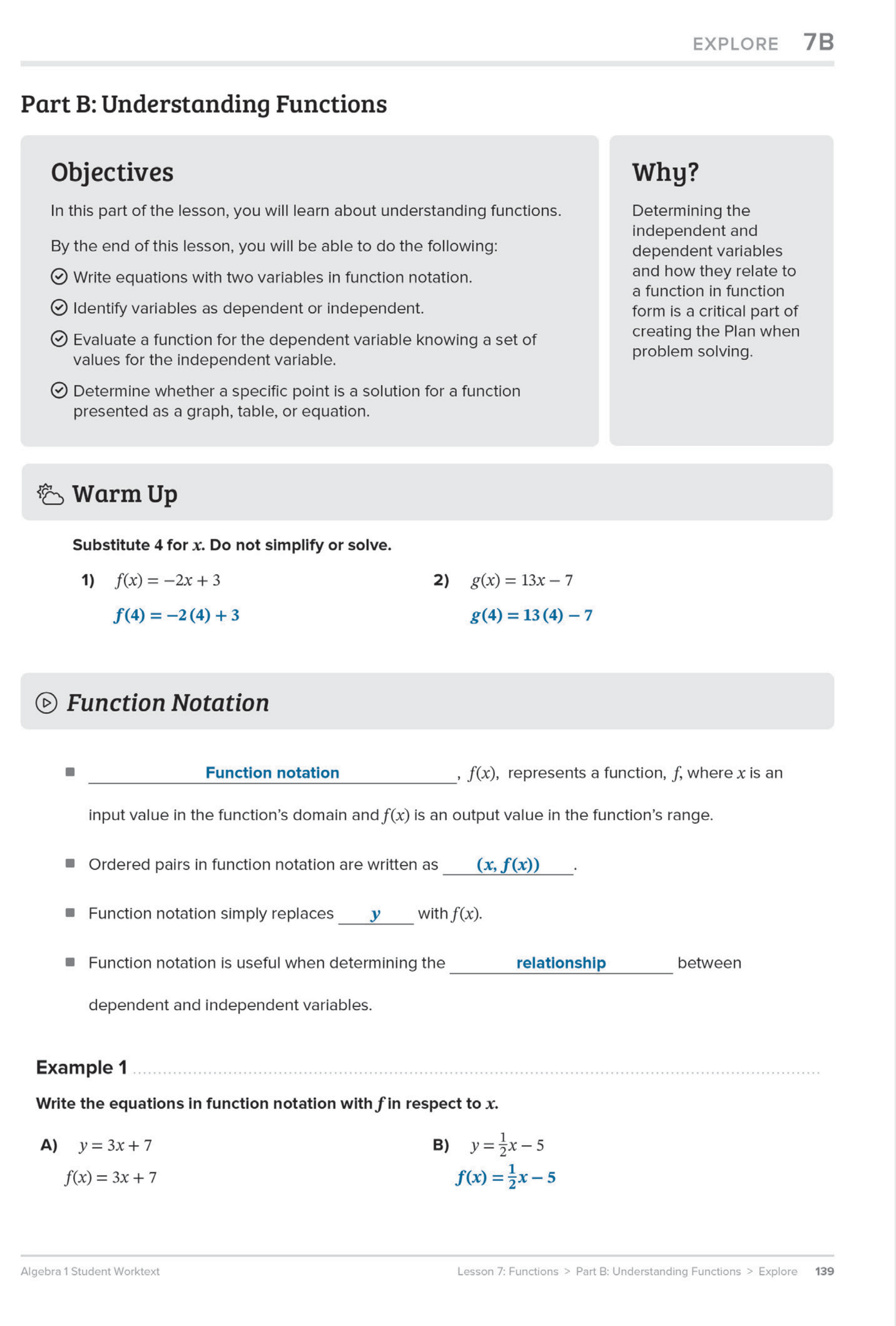
range: $\left\{-2, 0, 2, \frac{5}{2}, 4\right\}$

14)



Sample:
This is a function because the domain does not repeat.

domain: $\{-3, -2, -1, 0, 2\}$
range: $\{-1, 0, 2, 3\}$



Part B: Understanding Functions

Objectives

In this part of the lesson, you will learn about understanding functions.

By the end of this lesson, you will be able to do the following:

- ✓ Write equations with two variables in function notation.
- ✓ Identify variables as dependent or independent.
- ✓ Evaluate a function for the dependent variable knowing a set of values for the independent variable.
- ✓ Determine whether a specific point is a solution for a function presented as a graph, table, or equation.

Why?

Determining the independent and dependent variables and how they relate to a function in function form is a critical part of creating the Plan when problem solving.

Warm Up

Substitute 4 for x. Do not simplify or solve.

1) $f(x) = -2x + 3$

$f(4) = -2(4) + 3$

2) $g(x) = 13x - 7$

$g(4) = 13(4) - 7$

Function Notation

- **Function notation**, $f(x)$, represents a function, f, where x is an input value in the function's domain and $f(x)$ is an output value in the function's range.
- Ordered pairs in function notation are written as **$(x, f(x))$**.
- Function notation simply replaces **y** with $f(x)$.
- Function notation is useful when determining the **relationship** between dependent and independent variables.

Example 1

Write the equations in function notation with f in respect to x.

A) $y = 3x + 7$

$f(x) = 3x + 7$

B) $y = \frac{1}{2}x - 5$

$f(x) = \frac{1}{2}x - 5$

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

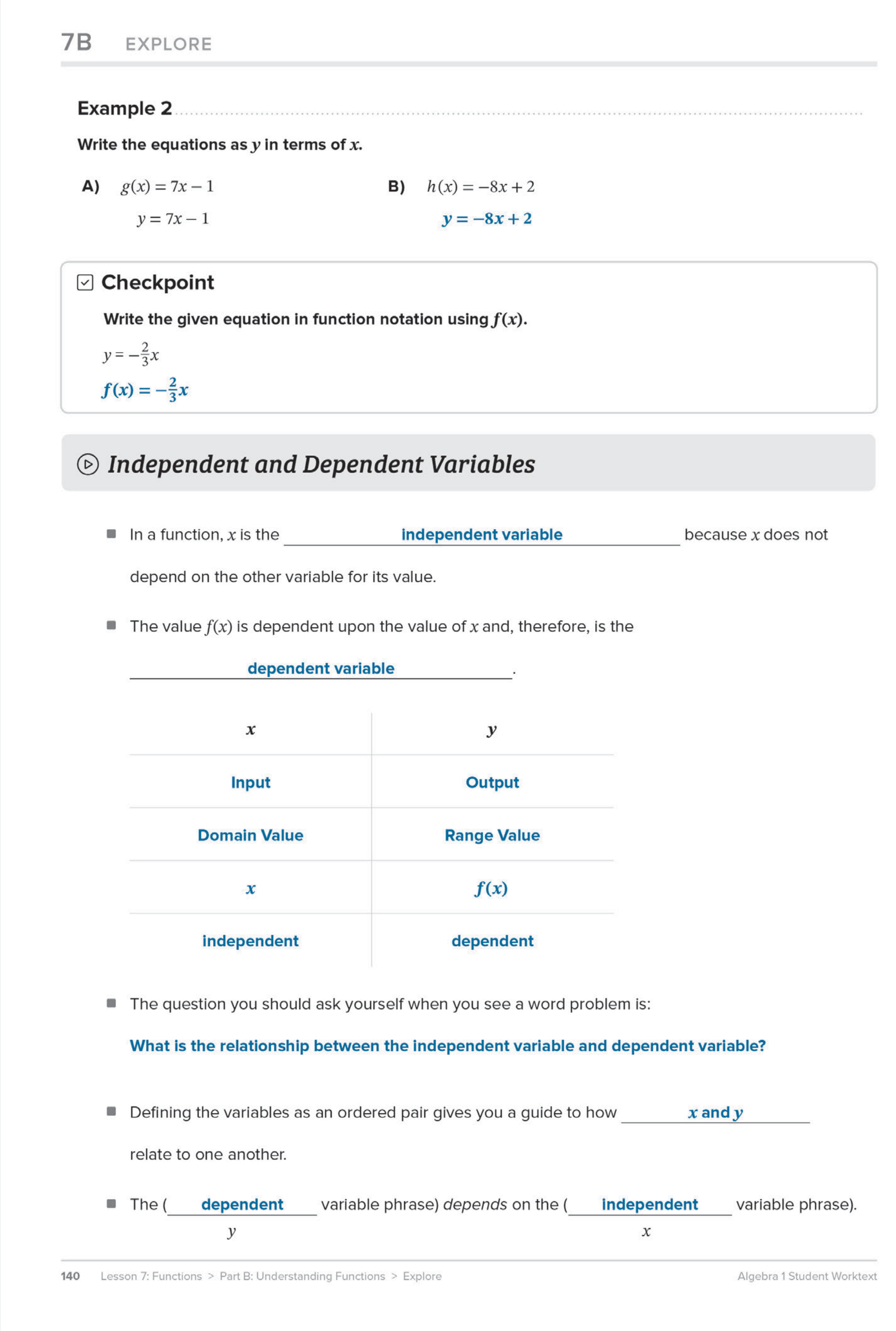
This part of the lesson is about writing equations in different forms. All your student needs to do is replace all of the x's with the number 4.

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: If an equation is already in function notation, what is true of the domain?

A: *All domain values are unique, or the domain does not repeat.*



Example 2

Write the equations as y in terms of x.

A) $g(x) = 7x - 1$

$y = 7x - 1$

B) $h(x) = -8x + 2$

$\boldsymbol{y = -8x + 2}$

☑ Checkpoint

Write the given equation in function notation using $f(x)$.

$y = -\frac{2}{3}x$

$\boldsymbol{f(x) = -\frac{2}{3}x}$

Independent and Dependent Variables

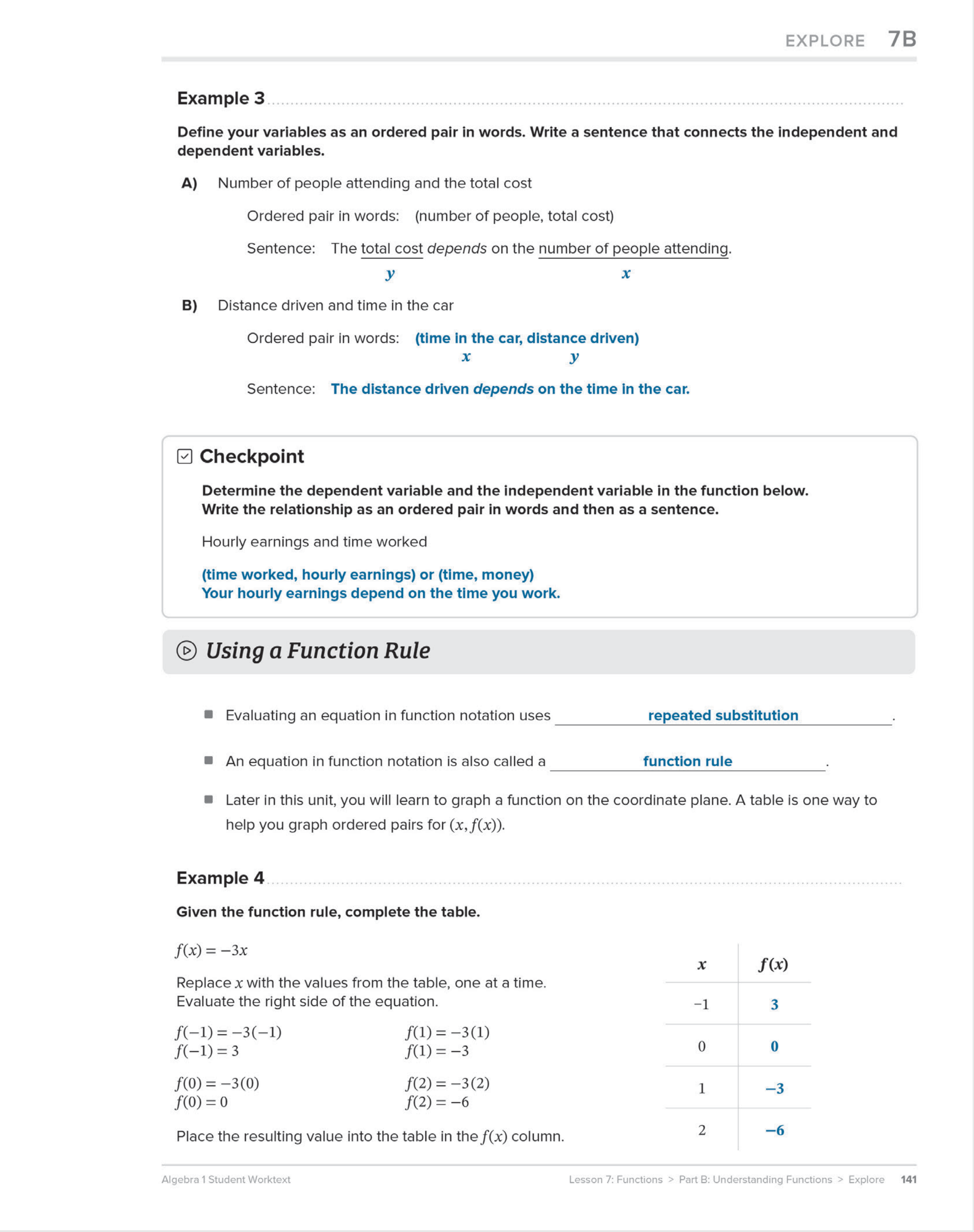
- In a function, x is the **independent variable** because x does not depend on the other variable for its value.
- The value $f(x)$ is dependent upon the value of x and, therefore, is the **dependent variable**.

x	y
Input	**Output**
Domain Value	**Range Value**
$\boldsymbol{x}$	$\boldsymbol{f(x)}$
independent	**dependent**

- The question you should ask yourself when you see a word problem is:

  What is the relationship between the independent variable and dependent variable?
- Defining the variables as an ordered pair gives you a guide to how **x and y** relate to one another.
- The (**dependent** variable phrase) *depends* on the (**independent** variable phrase).

  y x



Example 3

Define your variables as an ordered pair in words. Write a sentence that connects the independent and dependent variables.

A) Number of people attending and the total cost

Ordered pair in words: (number of people, total cost)

Sentence: The $\underbrace{\text{total cost}}_{y}$ *depends* on the $\underbrace{\text{number of people attending}}_{x}$.

B) Distance driven and time in the car

Ordered pair in words: **(time in the car, distance driven)** x y

Sentence: **The distance driven *depends* on the time in the car.**

☑ Checkpoint

Determine the dependent variable and the independent variable in the function below. Write the relationship as an ordered pair in words and then as a sentence.

Hourly earnings and time worked

(time worked, hourly earnings) or (time, money)
Your hourly earnings depend on the time you work.

Using a Function Rule

- Evaluating an equation in function notation uses **repeated substitution**.
- An equation in function notation is also called a **function rule**.
- Later in this unit, you will learn to graph a function on the coordinate plane. A table is one way to help you graph ordered pairs for $(x, f(x))$.

Example 4

Given the function rule, complete the table.

$f(x) = -3x$

Replace x with the values from the table, one at a time.
Evaluate the right side of the equation.

$f(-1) = -3(-1)$
$f(-1) = 3$

$f(1) = -3(1)$
$f(1) = -3$

$f(0) = -3(0)$
$f(0) = 0$

$f(2) = -3(2)$
$f(2) = -6$

Place the resulting value into the table in the $f(x)$ column.

x	$f(x)$
-1	**3**
0	**0**
1	**-3**
2	**-6**

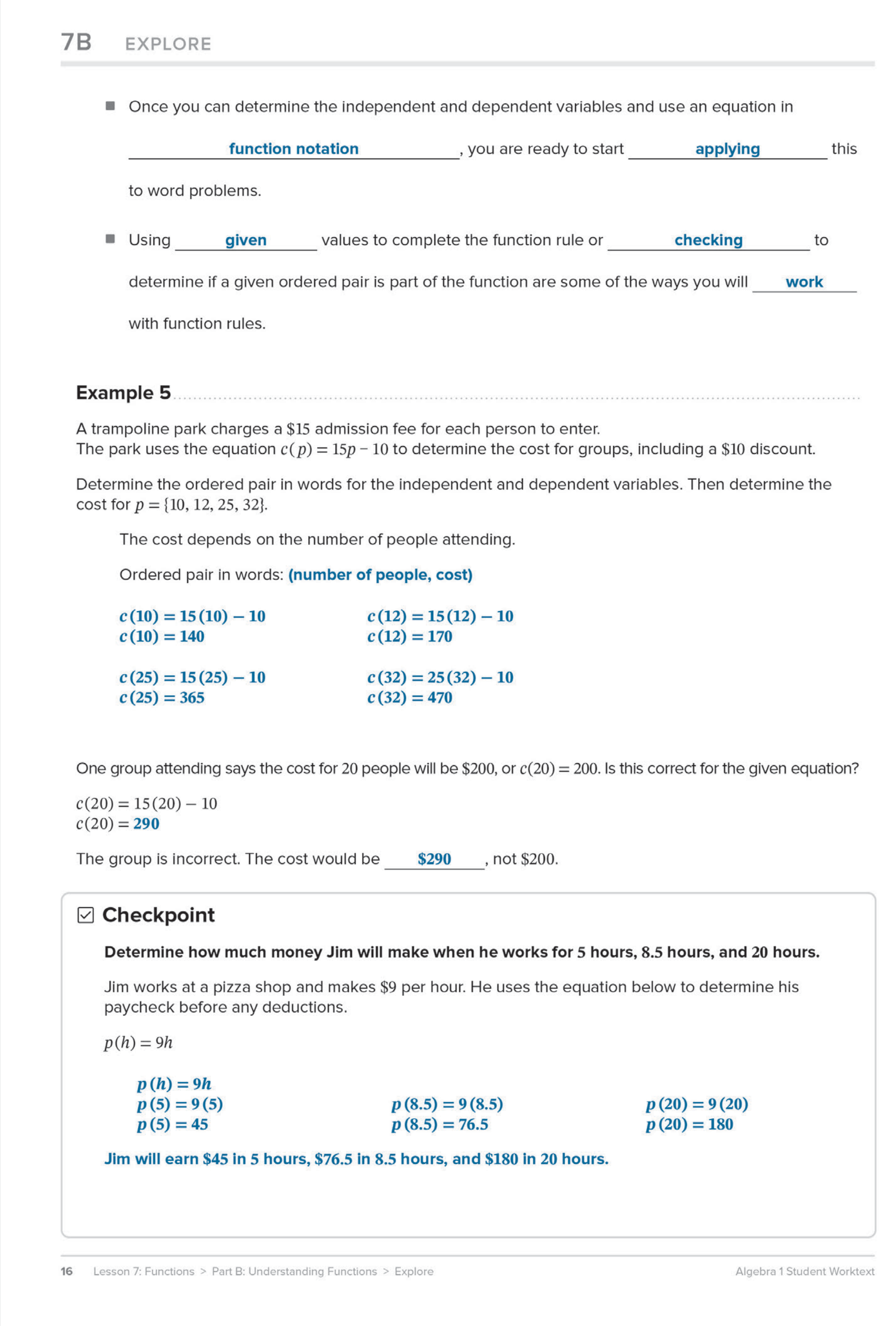
Example 3

As your student becomes more comfortable writing the ordered pair in words, they may move to (time, distance) rather than a phrase for each variable.

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

If your student needs help, ask them, "If you make $10 an hour and work for 5 hours, how much money will you make?" What happens if you work fewer than 5 hours? ($50, fewer hours = less money)



- Once you can determine the independent and dependent variables and use an equation in **function notation**, you are ready to start **applying** this to word problems.
- Using **given** values to complete the function rule or **checking** to determine if a given ordered pair is part of the function are some of the ways you will **work** with function rules.

Example 5

A trampoline park charges a $15 admission fee for each person to enter.
The park uses the equation $c(p) = 15p - 10$ to determine the cost for groups, including a $10 discount.

Determine the ordered pair in words for the independent and dependent variables. Then determine the cost for $p = \{10, 12, 25, 32\}$.

The cost depends on the number of people attending.

Ordered pair in words: **(number of people, cost)**

$c(10) = 15(10) - 10$
$c(10) = 140$

$c(12) = 15(12) - 10$
$c(12) = 170$

$c(25) = 15(25) - 10$
$c(25) = 365$

$c(32) = 25(32) - 10$
$c(32) = 470$

One group attending says the cost for 20 people will be $200, or $c(20) = 200$. Is this correct for the given equation?

$c(20) = 15(20) - 10$
$c(20) = 290$

The group is incorrect. The cost would be **$290**, not $200.

☑ Checkpoint

Determine how much money Jim will make when he works for 5 hours, 8.5 hours, and 20 hours.

Jim works at a pizza shop and makes $9 per hour. He uses the equation below to determine his paycheck before any deductions.

$p(h) = 9h$

$p(h) = 9h$
$p(5) = 9(5)$
$p(5) = 45$

$p(8.5) = 9(8.5)$
$p(8.5) = 76.5$

$p(20) = 9(20)$
$p(20) = 180$

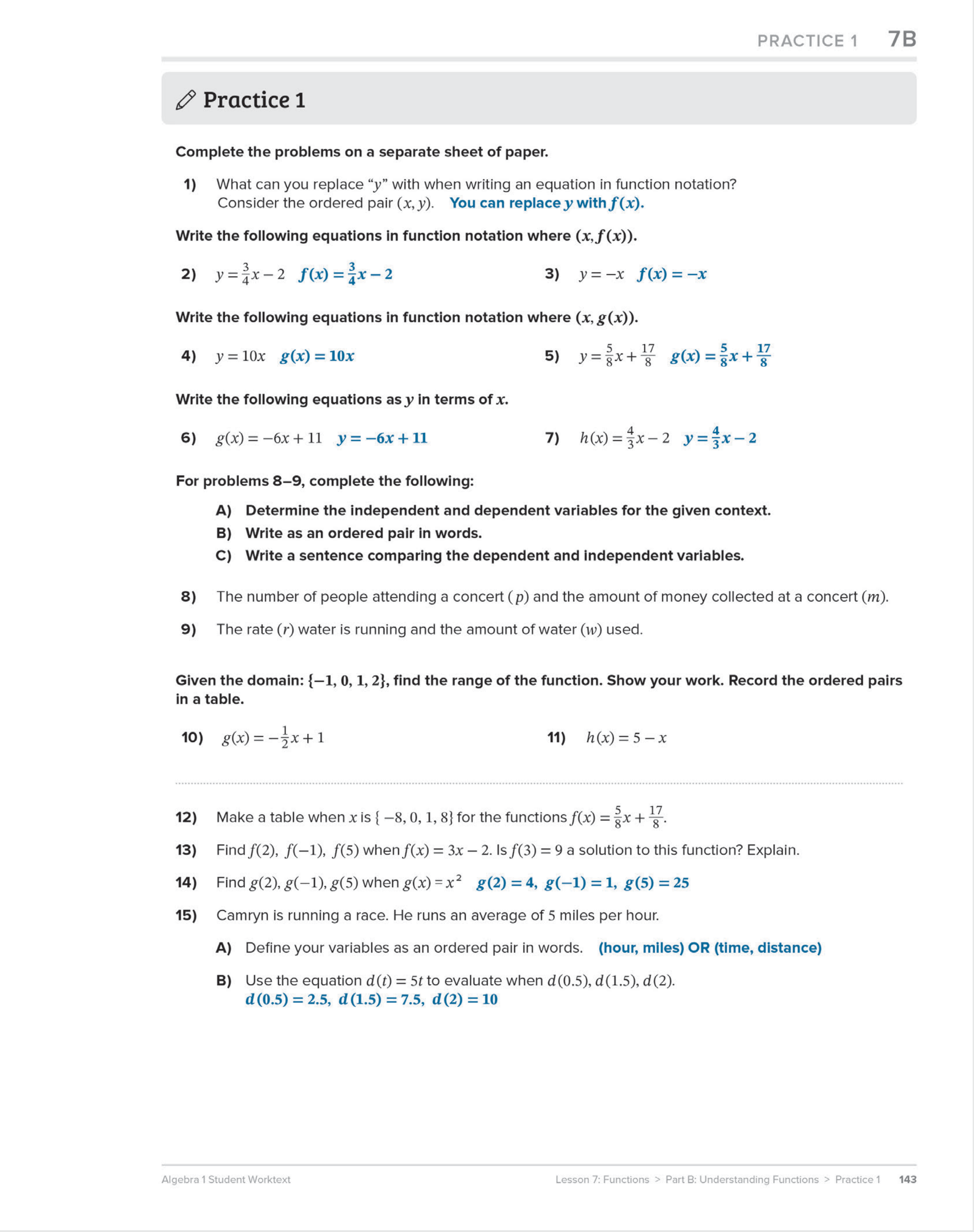
Jim will earn $45 in 5 hours, $76.5 in 8.5 hours, and $180 in 20 hours.

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the ordered pair in words for this function?

A: *(time worked, money earned)*



PRACTICE 1 7B

Practice 1

Complete the problems on a separate sheet of paper.

1) What can you replace "y" with when writing an equation in function notation? Consider the ordered pair (x, y). **You can replace y with $f(x)$.**

Write the following equations in function notation where $(x, f(x))$.

2) $y = \frac{3}{4}x - 2$ **$f(x) = \frac{3}{4}x - 2$**

3) $y = -x$ **$f(x) = -x$**

Write the following equations in function notation where $(x, g(x))$.

4) $y = 10x$ **$g(x) = 10x$**

5) $y = \frac{5}{8}x + \frac{17}{8}$ **$g(x) = \frac{5}{8}x + \frac{17}{8}$**

Write the following equations as y in terms of x.

6) $g(x) = -6x + 11$ **$y = -6x + 11$**

7) $h(x) = \frac{4}{3}x - 2$ **$y = \frac{4}{3}x - 2$**

For problems 8–9, complete the following:

- **A)** **Determine the independent and dependent variables for the given context.**
- **B)** **Write as an ordered pair in words.**
- **C)** **Write a sentence comparing the dependent and independent variables.**

8) The number of people attending a concert (p) and the amount of money collected at a concert (m).

9) The rate (r) water is running and the amount of water (w) used.

Given the domain: $\{-1, 0, 1, 2\}$, find the range of the function. Show your work. Record the ordered pairs in a table.

10) $g(x) = -\frac{1}{2}x + 1$

11) $h(x) = 5 - x$

12) Make a table when x is $\{-8, 0, 1, 8\}$ for the functions $f(x) = \frac{5}{8}x + \frac{17}{8}$.

13) Find $f(2)$, $f(-1)$, $f(5)$ when $f(x) = 3x - 2$. Is $f(3) = 9$ a solution to this function? Explain.

14) Find $g(2), g(-1), g(5)$ when $g(x) = x^2$ **$g(2) = 4,\ g(-1) = 1,\ g(5) = 25$**

15) Camryn is running a race. He runs an average of 5 miles per hour.

- **A)** Define your variables as an ordered pair in words. **(hour, miles) OR (time, distance)**
- **B)** Use the equation $d(t) = 5t$ to evaluate when $d(0.5), d(1.5), d(2)$.
  $d(0.5) = 2.5,\ d(1.5) = 7.5,\ d(2) = 10$

Algebra 1 Student Worktext Lesson 7: Functions > Part B: Understanding Functions > Practice 1 143

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) Your student can use $f(x)$, $g(x)$, or $h(x)$. However, $f(x)$ is most common in function notation in Algebra.

4–5)

While using $f(x)$ will still result in an equation in function notation, make sure your student carefully follows the directions and uses the correct variable.

6–7)

Changing between forms will become more important as your student moves through high school mathematics. Understanding this is the same equation in different forms now will make other topics flow more smoothly later on.

8) A) Dependent variable: m
Independent variable: p

B) (people attending, money collected)

C) The money collected depends on the number of people attending.

9) A) Dependent variable: w
Independent variable: r

B) (rate water running, water used)

C) The water used *depends* on the rate the water is running.

10) $g(2) = -\frac{1}{2}(2) + 1 = 0$

$g(x) = \left\{\frac{3}{2}, 1, \frac{1}{2}, 0\right\}$

Your student should evaluate the function four times since they are given four elements in the domain.

Q: How many times will you need to evaluate this function? Explain.

A: *Four, because there are four given domain values.*

Writing the $g(x)$ values using decimal values rather than improper fractions is also correct. If your student prefers this, know that a fraction provides more directions than a decimal when graphing slope.

Q: What is the decimal equivalent of $\frac{3}{2}$?

A: *1.5*

11) $h(2) = 5 - (2) = 3$

$h(x) = \{6, 5, 4, 3\}$

12)

x	$f(x)$
-8	$-\frac{23}{8}$
0	$\frac{17}{8}$
1	$\frac{11}{4}$
8	$\frac{57}{8}$

It is preferred that your student leave the answers in fraction form since the equation already has a common denominator.

13) $f(2) = 4, f(-1) = -5, f(5) = 13$
$f(3) = 9$ is not a solution to this function because it does not follow the function rule.

Q: What are the domain values?

A: *domain: {2, −1, 5}*

You could also use the terms x-values or input values.

14) When your student squares -1, the result is 1 because a negative number multiplied by a negative number is a positive number.

15) Miles per hour is another way of providing the independent and dependent variables. Have your student look at this phrase or the equation in part B if they are uncertain how to write the ordered pair in words.

Mastery Check

Show What You Know

A) If your student is not sure which is the dependent variable, have them say the sentence both ways. Usually the one that sounds better is the correct order and names the dependent and independent variables.

B) Your student does not have to show their work for Part B, but it will help if they have incorrect values.

C) Q: What are you solving for in this question?

A: *number of days on vacation*

Q: What will the 500 replace in the equation?

A: $c(d)$

D) This is a multi-step problem. Have your student think about what number they need to start with, and the operations needed to find the total number of excursions.

Other parts of the Mastery Check may be needed to answer part D.

Q: What is the maximum amount the Ross family can spend?

A: *$3,000*

Q: If they round the final number to 5, ask “What is 5 times $400?”

A: *$2,000*

Q: Does the Ross family have this to spend after food and gas?

A: *No*

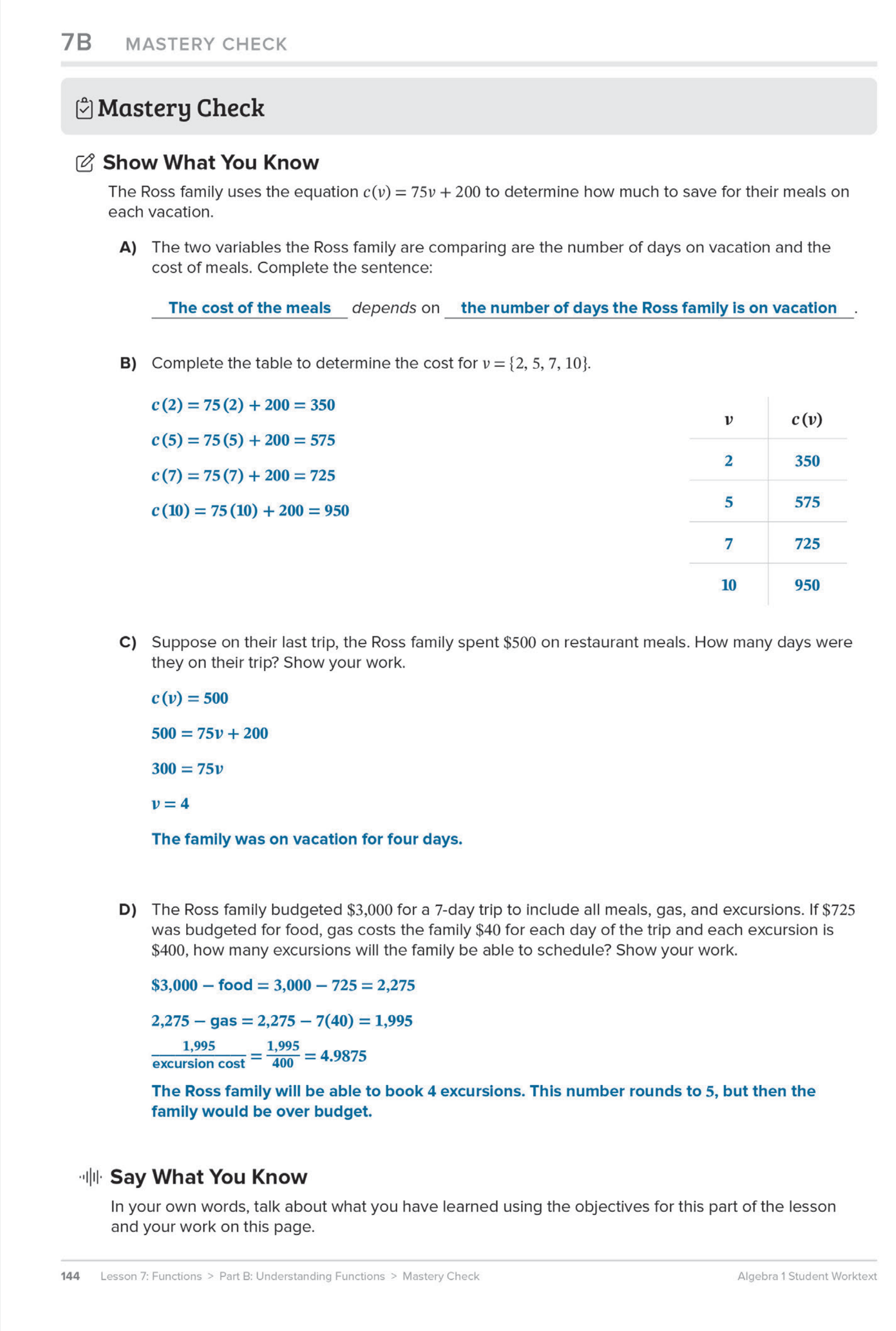
Q: Should the number round up or down in this case?

A: *Down*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Write equations with two variables in function notation.
- Identify variables as dependent or independent.
- Evaluate a function for the dependent variable knowing a set of values for the independent variable.
- Determine whether a specific point is a solution for a function presented as a graph, table, or equation.



7B MASTERY CHECK

Mastery Check

Show What You Know

The Ross family uses the equation $c(v) = 75v + 200$ to determine how much to save for their meals on each vacation.

A) The two variables the Ross family are comparing are the number of days on vacation and the cost of meals. Complete the sentence:

The cost of the meals *depends* on **the number of days the Ross family is on vacation**.

B) Complete the table to determine the cost for $v = \{2, 5, 7, 10\}$.

$c(2) = 75(2) + 200 = 350$

$c(5) = 75(5) + 200 = 575$

$c(7) = 75(7) + 200 = 725$

$c(10) = 75(10) + 200 = 950$

v	$c(v)$
2	350
5	575
7	725
10	950

C) Suppose on their last trip, the Ross family spent $500 on restaurant meals. How many days were they on their trip? Show your work.

$c(v) = 500$

$500 = 75v + 200$

$300 = 75v$

$v = 4$

The family was on vacation for four days.

D) The Ross family budgeted $3,000 for a 7-day trip to include all meals, gas, and excursions. If $725 was budgeted for food, gas costs the family $40 for each day of the trip and each excursion is $400, how many excursions will the family be able to schedule? Show your work.

$\$3{,}000 - \text{food} = 3{,}000 - 725 = 2{,}275$

$2{,}275 - \text{gas} = 2{,}275 - 7(40) = 1{,}995$

$\frac{1{,}995}{\text{excursion cost}} = \frac{1{,}995}{400} = 4.9875$

The Ross family will be able to book 4 excursions. This number rounds to 5, but then the family would be over budget.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

144 Lesson 7: Functions > Part B: Understanding Functions > Mastery Check Algebra 1 Student Worktext

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

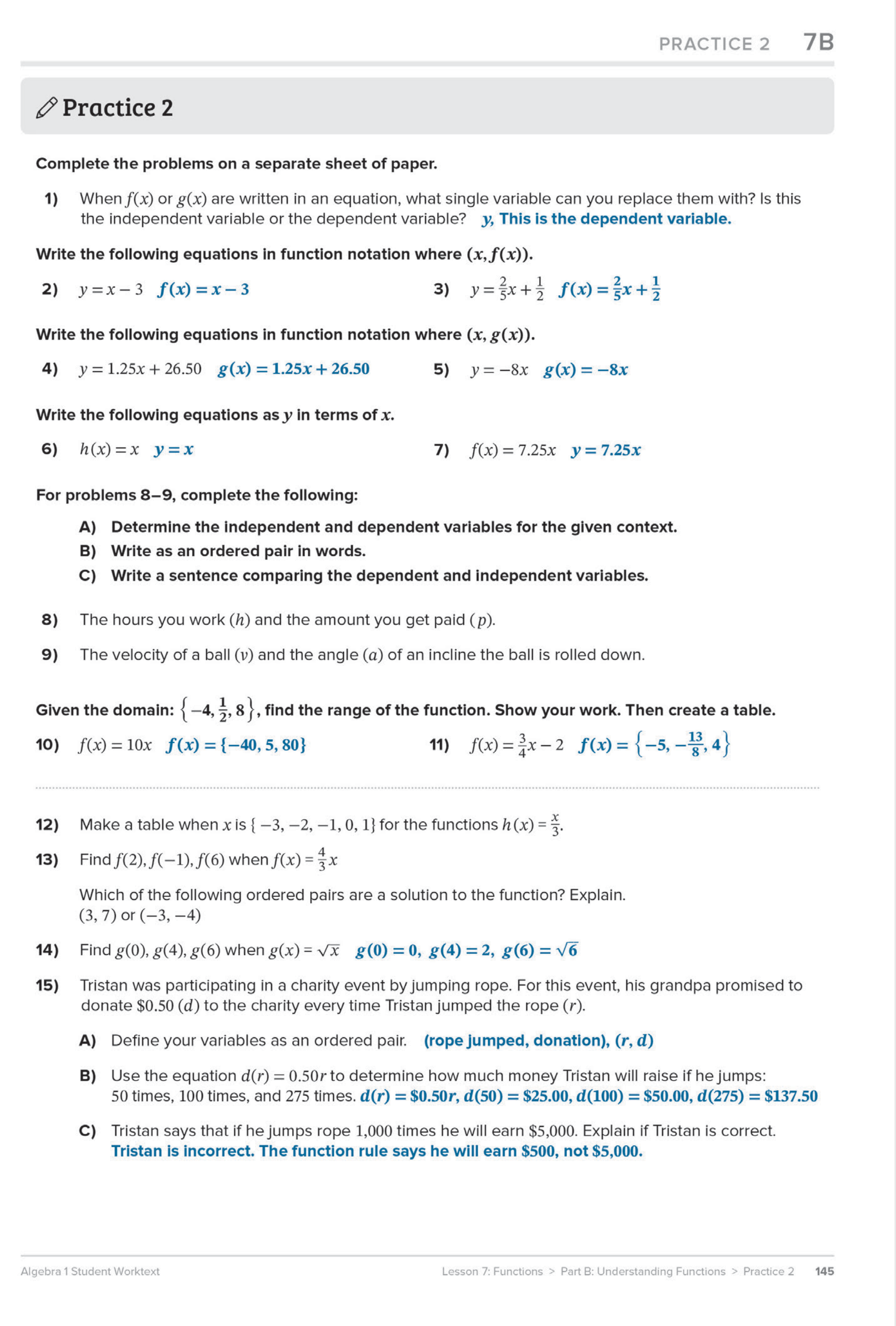
YES

If your student can answer “yes” to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer “yes” to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.



Practice 2

Complete the problems on a separate sheet of paper.

1) When $f(x)$ or $g(x)$ are written in an equation, what single variable can you replace them with? Is this the independent variable or the dependent variable? **y, This is the dependent variable.**

Write the following equations in function notation where $(x, f(x))$.

2) $y = x - 3$ $\quad f(x) = x - 3$

3) $y = \frac{2}{5}x + \frac{1}{2}$ $\quad f(x) = \frac{2}{5}x + \frac{1}{2}$

Write the following equations in function notation where $(x, g(x))$.

4) $y = 1.25x + 26.50$ $\quad g(x) = 1.25x + 26.50$

5) $y = -8x$ $\quad g(x) = -8x$

Write the following equations as y in terms of x.

6) $h(x) = x$ $\quad y = x$

7) $f(x) = 7.25x$ $\quad y = 7.25x$

For problems 8–9, complete the following:

- **A)** **Determine the independent and dependent variables for the given context.**
- **B)** **Write as an ordered pair in words.**
- **C)** **Write a sentence comparing the dependent and independent variables.**

8) The hours you work (h) and the amount you get paid (p).

9) The velocity of a ball (v) and the angle (a) of an incline the ball is rolled down.

Given the domain: $\left\{-4, \frac{1}{2}, 8\right\}$, find the range of the function. Show your work. Then create a table.

10) $f(x) = 10x$ $\quad f(x) = \{-40, 5, 80\}$

11) $f(x) = \frac{3}{4}x - 2$ $\quad f(x) = \left\{-5, -\frac{13}{8}, 4\right\}$

12) Make a table when x is $\{-3, -2, -1, 0, 1\}$ for the functions $h(x) = \frac{x}{3}$.

13) Find $f(2), f(-1), f(6)$ when $f(x) = \frac{4}{3}x$

Which of the following ordered pairs are a solution to the function? Explain.
$(3, 7)$ or $(-3, -4)$

14) Find $g(0), g(4), g(6)$ when $g(x) = \sqrt{x}$ $\quad g(0) = 0,\ g(4) = 2,\ g(6) = \sqrt{6}$

15) Tristan was participating in a charity event by jumping rope. For this event, his grandpa promised to donate \$0.50 (d) to the charity every time Tristan jumped the rope (r).

- **A)** Define your variables as an ordered pair. **(rope jumped, donation), (r, d)**
- **B)** Use the equation $d(r) = 0.50r$ to determine how much money Tristan will raise if he jumps: 50 times, 100 times, and 275 times. $d(r) = \$0.50r,\ d(50) = \$25.00,\ d(100) = \$50.00,\ d(275) = \$137.50$
- **C)** Tristan says that if he jumps rope 1,000 times he will earn \$5,000. Explain if Tristan is correct. **Tristan is incorrect. The function rule says he will earn \$500, not \$5,000.**

Practice 2

Worked solutions for these problems are located in the Digital Pack.

8) A) Dependent variable: p
Independent variable: h

B) (hours worked, amount paid), (h, p)

C) The amount you get paid depends on the hours you work.

9) A) Dependent variable: v
Independent variable: a

B) (angle of incline, velocity of ball), (a, v)

C) The velocity of the ball depends on the angle of incline.

12)

x	$f(x)$
-3	-1
-2	$-\frac{2}{3}$
-1	$-\frac{1}{3}$
0	0
1	$\frac{1}{3}$

13) $f(2) = \frac{8}{3},\ f(-1) = -\frac{4}{3},\ f(6) = 8$
The ordered pair $(-3, -4)$ is a solution because it follows the function rule.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

1) $2(n-7)=\frac{3}{4}n+\frac{7}{2}$
$n=14$
*Multiplying through by the LCD is not required but can be helpful.

2)

5) Point A: $(2, 3)$; QI
Point B: $(-3, 2)$; QII
Point C: $(1, -1)$; QIV
Point D: $(1, -1)$; QIV
Point E: $(4, -1)$; QIV
Point F: $(4, -1)$; QIV

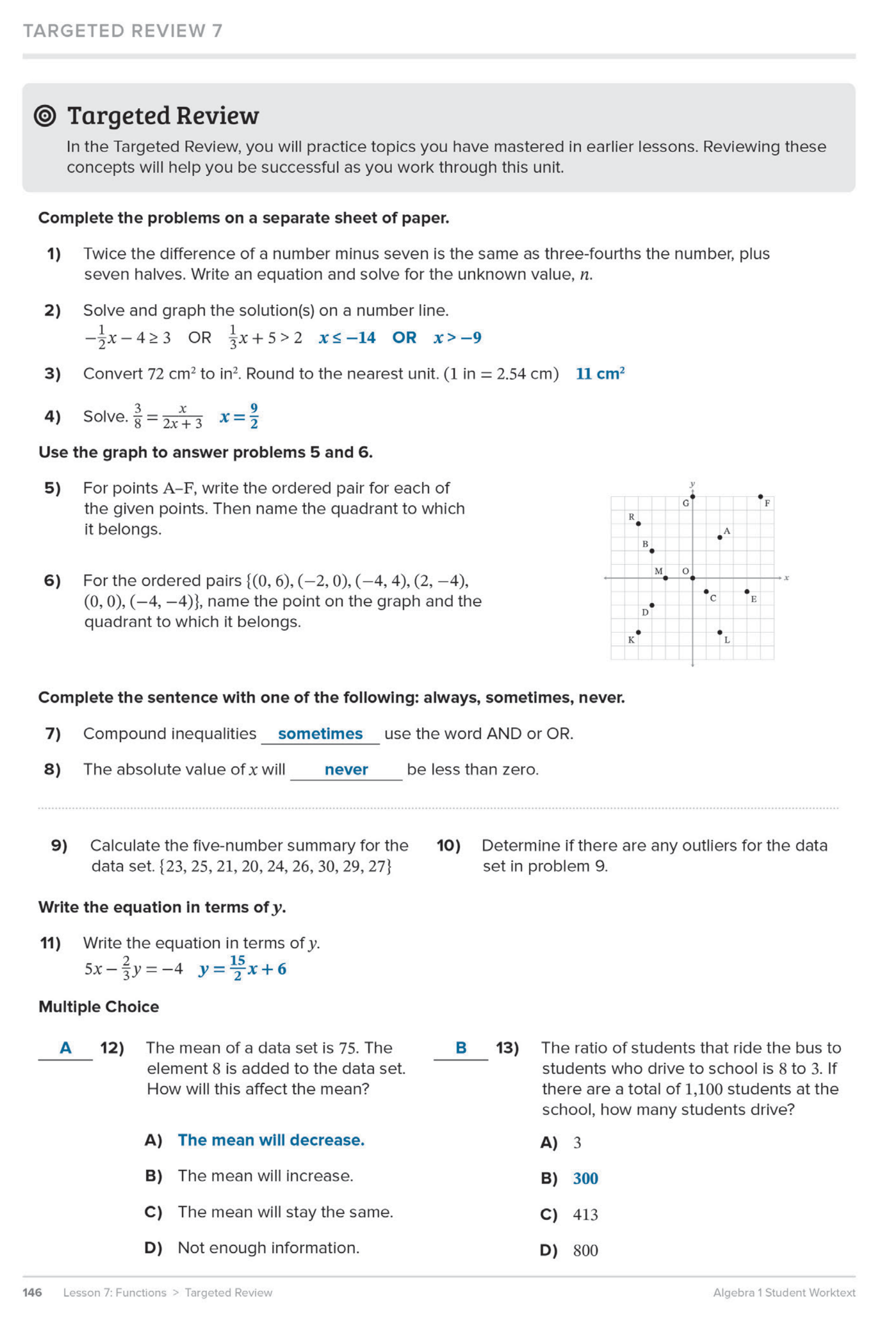
6) $(0, 6)$: G; on the y-axis
$(-2, 0)$: M; on the x-axis
$(-4, 4)$: R; QII
$(2, -4)$: L; QIV
$(0, 0)$: O; the origin
$(-4, -4)$: K; QIII

9) Min: 20
Q1: 22
Med: 25
Q3: 28
Max: 30

10) lower < 13
upper > 37
This data set contains no outliers.

12) Distractor Rationale:
- B) The mean cannot increase when the new element is less than the current mean.
- C) The mean would stay the same if the new element was the same as the current mean.

13) Distractor Rationale:
- A) This is the number of drivers in the ratio without using any other information in the problem.
- C) This is the solution if you use $\frac{8}{3}=\frac{1{,}100}{x}$ rather than $\frac{11}{3}=\frac{1{,}100}{x}$
- D) This is the number of bus riders rather than drivers.



TARGETED REVIEW 7

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

1) Twice the difference of a number minus seven is the same as three-fourths the number, plus seven halves. Write an equation and solve for the unknown value, n.

2) Solve and graph the solution(s) on a number line.
$-\frac{1}{2}x-4\geq 3$ OR $\frac{1}{3}x+5>2$ $\boldsymbol{x\leq -14}$ **OR** $\boldsymbol{x>-9}$

3) Convert 72 cm^2 to in^2. Round to the nearest unit. (1 in = 2.54 cm) **11 cm^2**

4) Solve. $\frac{3}{8}=\frac{x}{2x+3}$ $\boldsymbol{x=\frac{9}{2}}$

Use the graph to answer problems 5 and 6.

5) For points A–F, write the ordered pair for each of the given points. Then name the quadrant to which it belongs.

6) For the ordered pairs $\{(0, 6), (-2, 0), (-4, 4), (2, -4), (0, 0), (-4, -4)\}$, name the point on the graph and the quadrant to which it belongs.

Complete the sentence with one of the following: always, sometimes, never.

7) Compound inequalities **sometimes** use the word AND or OR.

8) The absolute value of x will **never** be less than zero.

9) Calculate the five-number summary for the data set. $\{23, 25, 21, 20, 24, 26, 30, 29, 27\}$

10) Determine if there are any outliers for the data set in problem 9.

Write the equation in terms of y.

11) Write the equation in terms of y.
$5x-\frac{2}{3}y=-4$ $\boldsymbol{y=\frac{15}{2}x+6}$

Multiple Choice

A **12)** The mean of a data set is 75. The element 8 is added to the data set. How will this affect the mean?
- **A)** **The mean will decrease.**
- **B)** The mean will increase.
- **C)** The mean will stay the same.
- **D)** Not enough information.

B **13)** The ratio of students that ride the bus to students who drive to school is 8 to 3. If there are a total of 1,100 students at the school, how many students drive?
- **A)** 3
- **B)** **300**
- **C)** 413
- **D)** 800

146 Lesson 7: Functions > Targeted Review Algebra 1 Student Worktext

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13
Lesson Origin	2	4	5	5	PA	PA	6	6	6	4	2	6	5