Lesson 4

Solving Inequalities

Outline

Part A
Single-Variable Inequalities

- Inequality Symbols and Wording
- Solving Inequalities
- Multiplying Inequalities by Negatives

Part B
Compound Inequalities

- Compound Inequalities with Two Symbols
- Absolute Value Inequalities
- Absolute Value Inequalities with No Solution or All Real Numbers

Targeted Review

Vocabulary

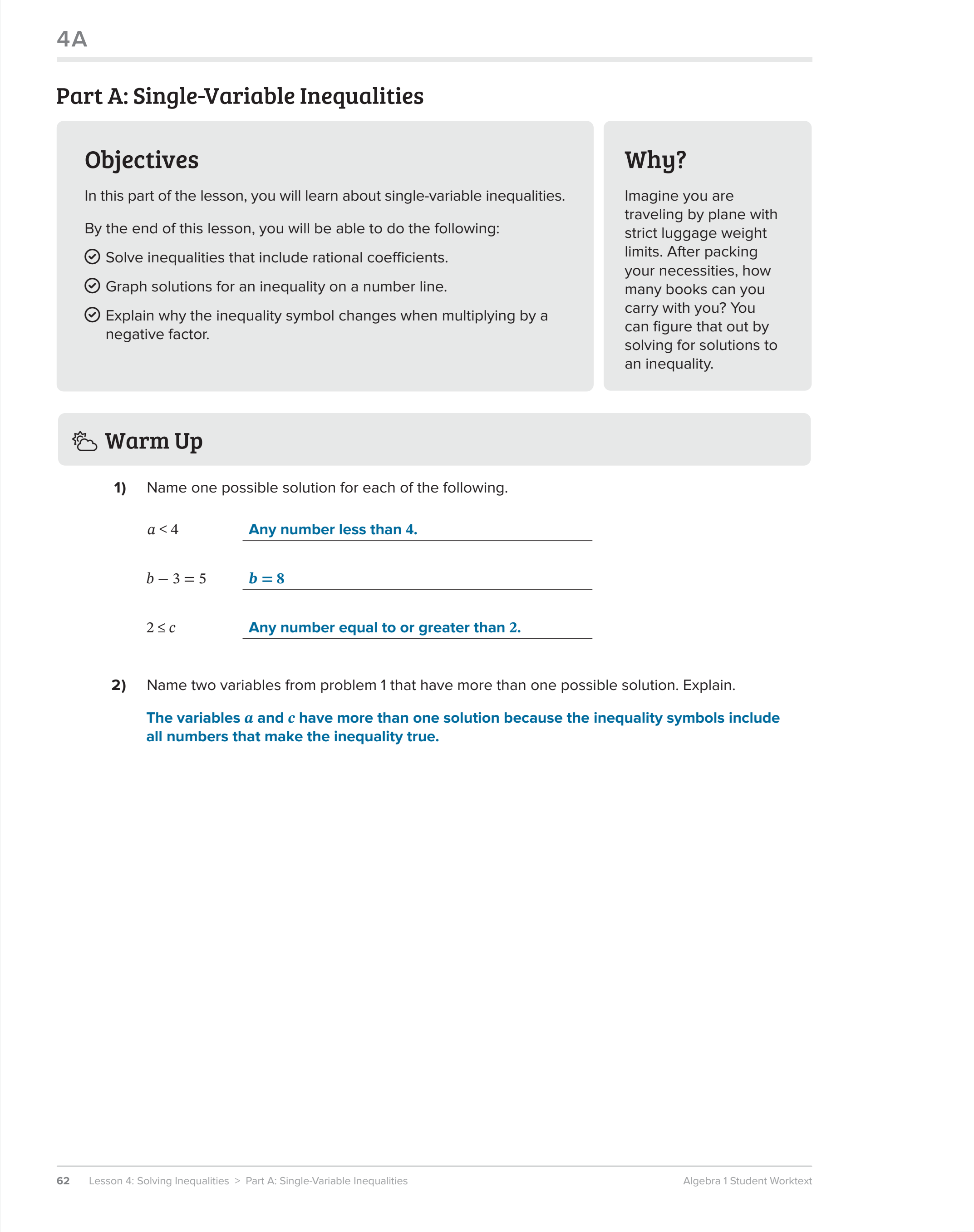
- inequality
- compound inequality
- absolute value inequality

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

2) Ask your student to read the equations and inequalities out loud to check if they say < represents “is less than” and ≤ represents “is less than or equal to.”



Part A: Single-Variable Inequalities

Objectives

In this part of the lesson, you will learn about single-variable inequalities.

By the end of this lesson, you will be able to do the following:

- ✓ Solve inequalities that include rational coefficients.
- ✓ Graph solutions for an inequality on a number line.
- ✓ Explain why the inequality symbol changes when multiplying by a negative factor.

Why?

Imagine you are traveling by plane with strict luggage weight limits. After packing your necessities, how many books can you carry with you? You can figure that out by solving for solutions to an inequality.

Warm Up

1) Name one possible solution for each of the following.

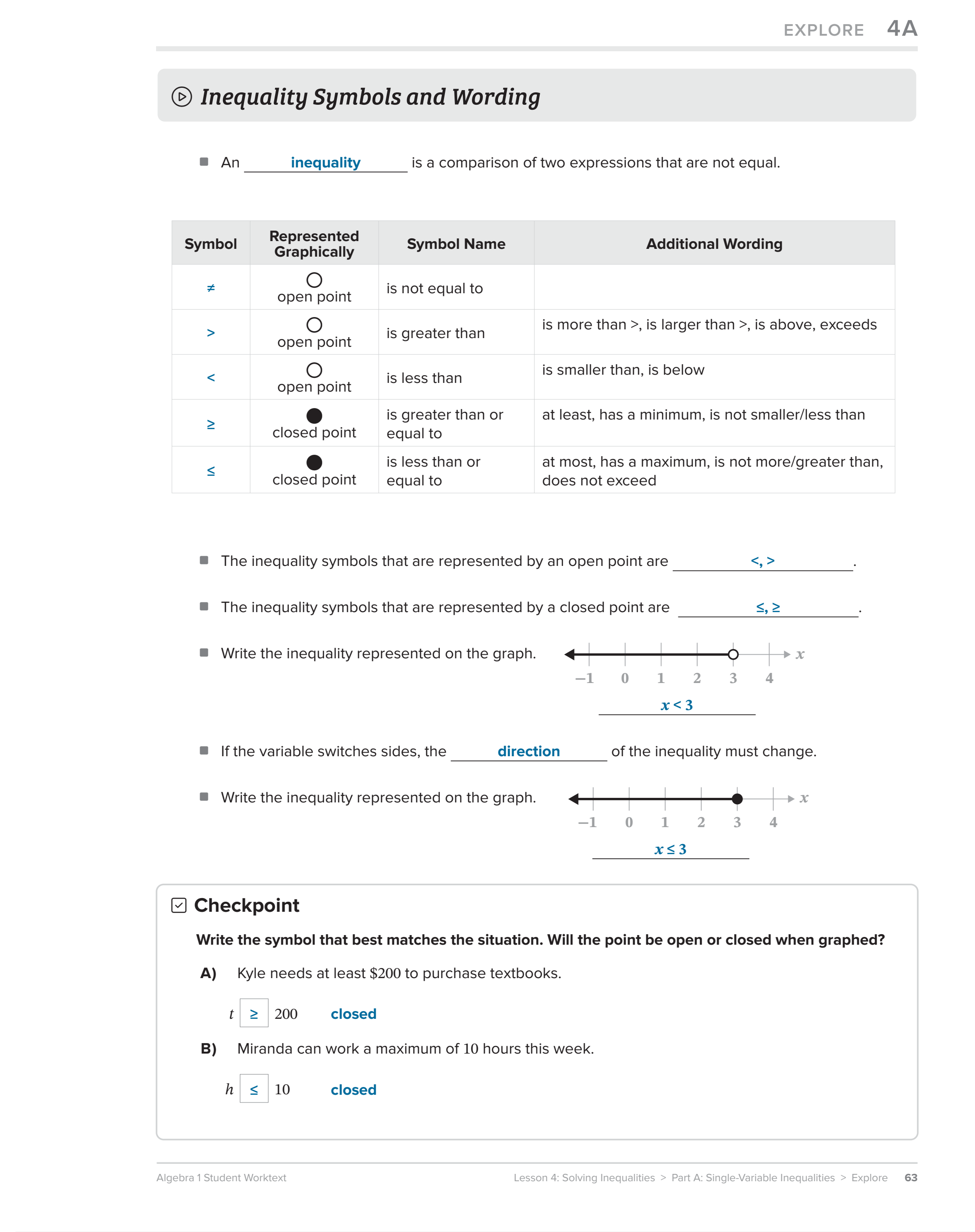
$a < 4$ **Any number less than 4.**

$b - 3 = 5$ $\boldsymbol{b = 8}$

$2 \le c$ **Any number equal to or greater than 2.**

2) Name two variables from problem 1 that have more than one possible solution. Explain.

The variables a and c have more than one solution because the inequality symbols include all numbers that make the inequality true.



Inequality Symbols and Wording

- An inequality is a comparison of two expressions that are not equal.

Symbol	Represented Graphically	Symbol Name	Additional Wording
≠	○ open point	is not equal to	
>	○ open point	is greater than	is more than >, is larger than >, is above, exceeds
<	○ open point	is less than	is smaller than, is below
≥	● closed point	is greater than or equal to	at least, has a minimum, is not smaller/less than
≤	● closed point	is less than or equal to	at most, has a maximum, is not more/greater than, does not exceed

- The inequality symbols that are represented by an open point are <, >.
- The inequality symbols that are represented by a closed point are ≤, ≥.
- Write the inequality represented on the graph.

$x < 3$

- If the variable switches sides, the direction of the inequality must change.
- Write the inequality represented on the graph.

$x \leq 3$

☑ Checkpoint

Write the symbol that best matches the situation. Will the point be open or closed when graphed?

A) Kyle needs at least $200 to purchase textbooks.

t ≥ 200 closed

B) Miranda can work a maximum of 10 hours this week.

h ≤ 10 closed

☑ Checkpoint

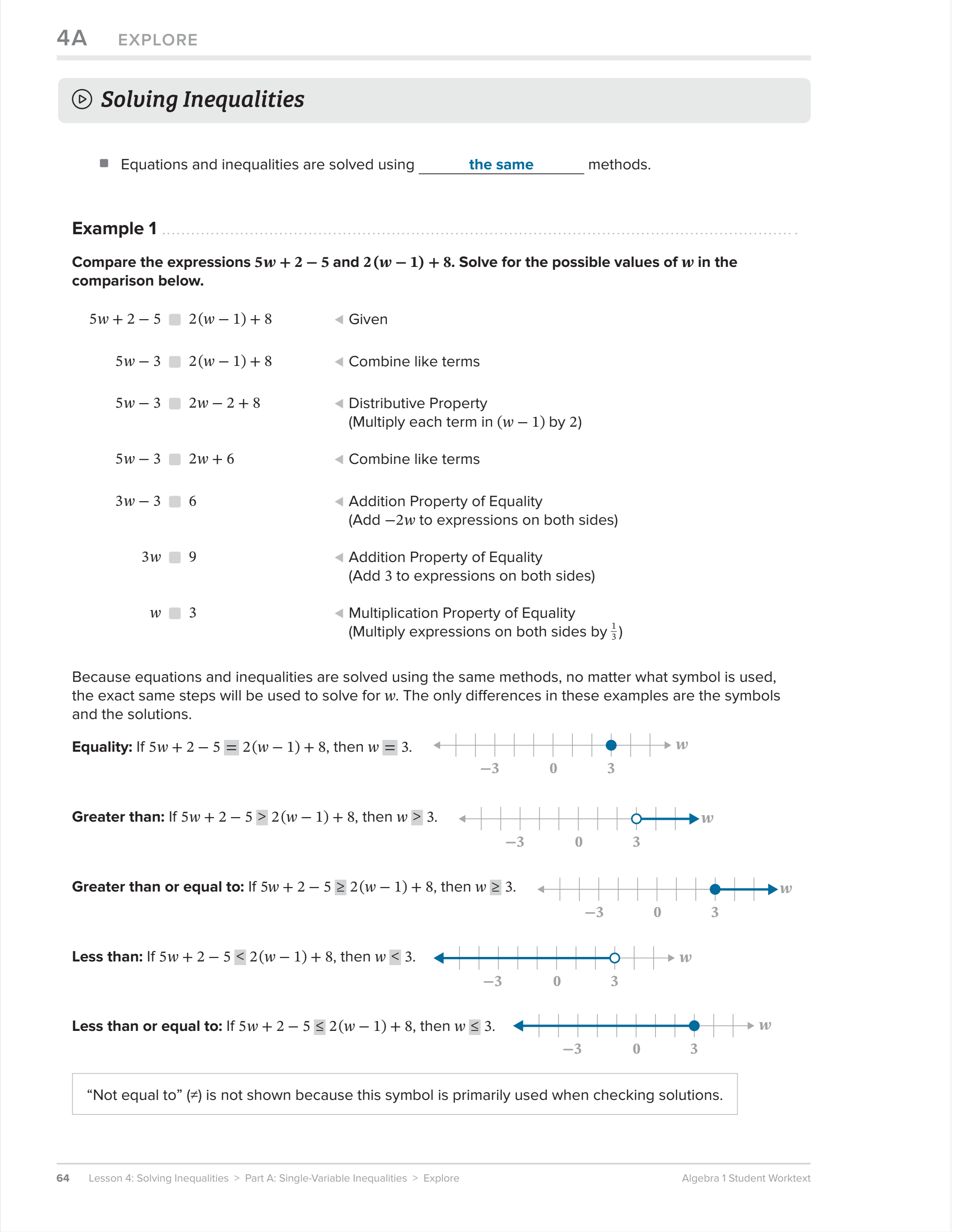
To continue past this checkpoint, students should confidently and correctly answer this problem.

Have your student refer to the table if they are unsure what symbol is represented by the word choice.

If your student is not sure what symbol to use, have your student talk out the problem before writing an inequality.

Example 1

The only difference in these five examples is the symbol and the resulting solutions. However, each example is solved in the exact same way.



Solving Inequalities

- Equations and inequalities are solved using the same methods.

Example 1

Compare the expressions $5w + 2 - 5$ and $2(w - 1) + 8$. Solve for the possible values of w in the comparison below.

$5w + 2 - 5 \ \square\ 2(w - 1) + 8$	Given
$5w - 3 \ \square\ 2(w - 1) + 8$	Combine like terms
$5w - 3 \ \square\ 2w - 2 + 8$	Distributive Property (Multiply each term in $(w - 1)$ by 2)
$5w - 3 \ \square\ 2w + 6$	Combine like terms
$3w - 3 \ \square\ 6$	Addition Property of Equality (Add $-2w$ to expressions on both sides)
$3w \ \square\ 9$	Addition Property of Equality (Add 3 to expressions on both sides)
$w \ \square\ 3$	Multiplication Property of Equality (Multiply expressions on both sides by $\frac{1}{3}$)

Because equations and inequalities are solved using the same methods, no matter what symbol is used, the exact same steps will be used to solve for w. The only differences in these examples are the symbols and the solutions.

Equality: If $5w + 2 - 5 = 2(w - 1) + 8$, then $w = 3$.

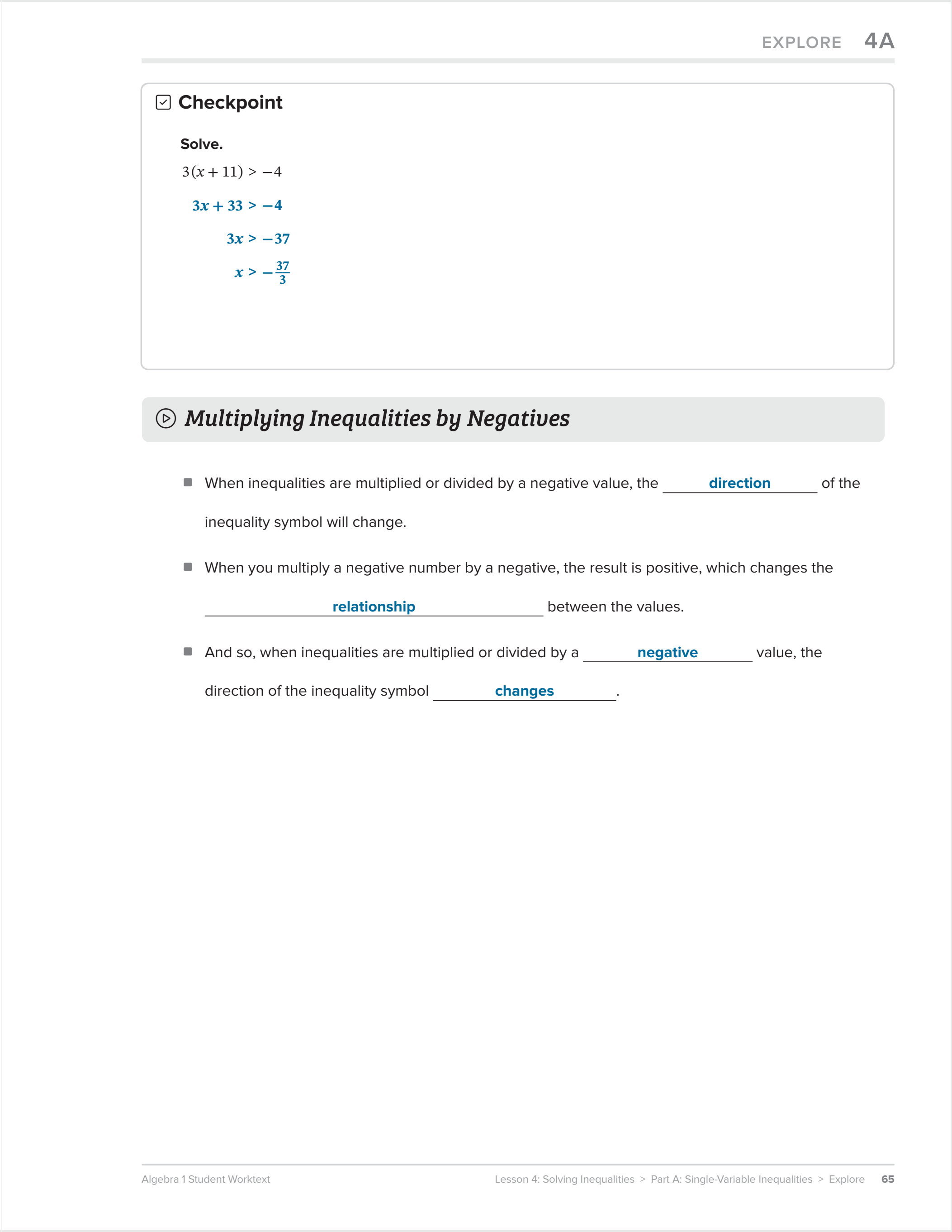
Greater than: If $5w + 2 - 5 > 2(w - 1) + 8$, then $w > 3$.

Greater than or equal to: If $5w + 2 - 5 \geq 2(w - 1) + 8$, then $w \geq 3$.

Less than: If $5w + 2 - 5 < 2(w - 1) + 8$, then $w < 3$.

Less than or equal to: If $5w + 2 - 5 \leq 2(w - 1) + 8$, then $w \leq 3$.

"Not equal to" (≠) is not shown because this symbol is primarily used when checking solutions.



EXPLORE 4A

☑ **Checkpoint**

Solve.

$3(x + 11) > -4$

$3x + 33 > -4$

$3x > -37$

$x > -\frac{37}{3}$

Multiplying Inequalities by Negatives

- When inequalities are multiplied or divided by a negative value, the **direction** of the inequality symbol will change.
- When you multiply a negative number by a negative, the result is positive, which changes the **relationship** between the values.
- And so, when inequalities are multiplied or divided by a **negative** value, the direction of the inequality symbol **changes**.

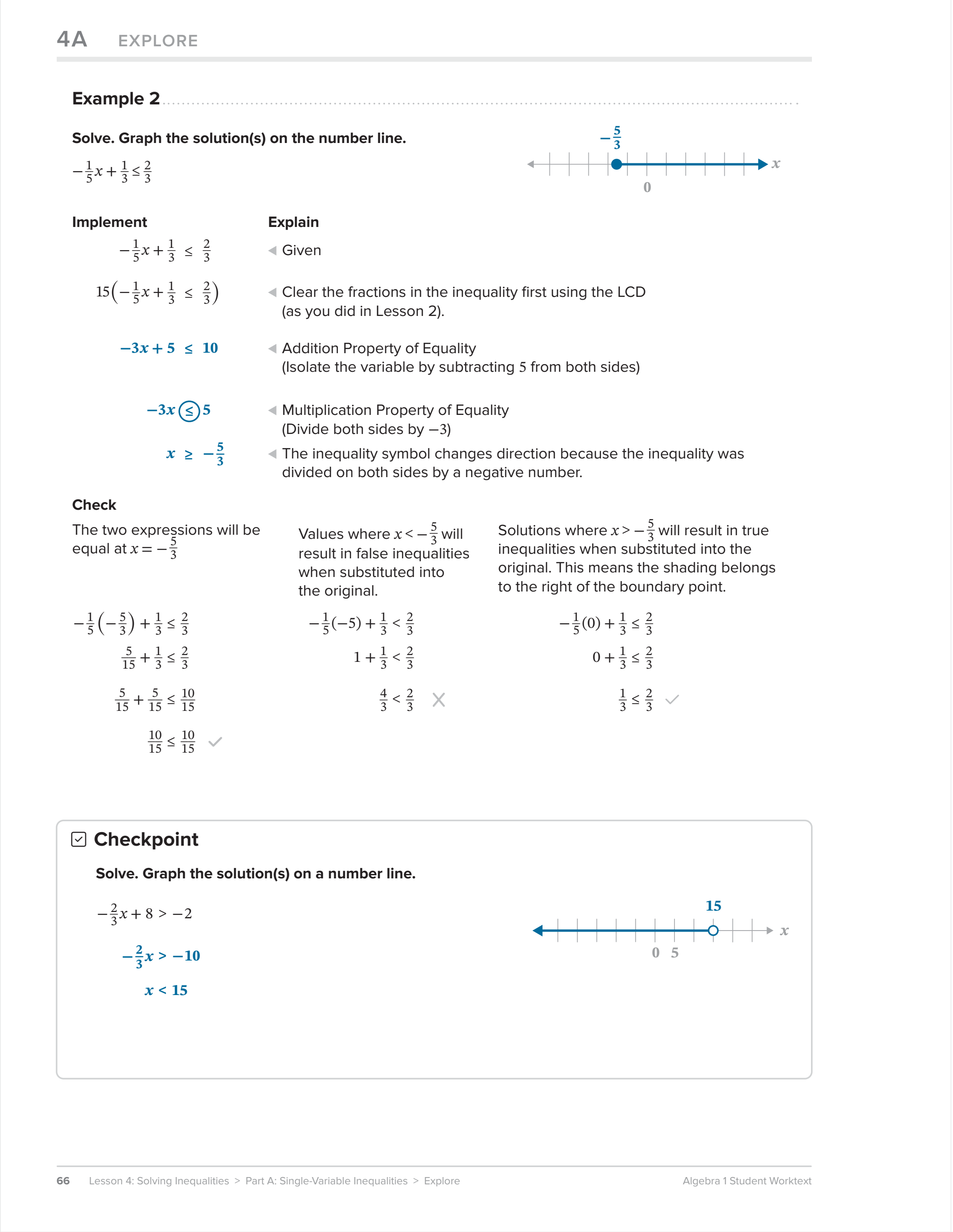
Algebra 1 Student Worktext

Lesson 4: Solving Inequalities > Part A: Single-Variable Inequalities > Explore 65

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Using zero (0) as a value for x is helpful when your student is checking to determine if their answer makes sense.



4A EXPLORE

Example 2

Solve. Graph the solution(s) on the number line.

$-\frac{1}{5}x+\frac{1}{3}\le\frac{2}{3}$

Implement	**Explain**
$-\frac{1}{5}x+\frac{1}{3} \le \frac{2}{3}$	◀ Given
$15\left(-\frac{1}{5}x+\frac{1}{3} \le \frac{2}{3}\right)$	◀ Clear the fractions in the inequality first using the LCD (as you did in Lesson 2).
$-3x+5 \le 10$	◀ Addition Property of Equality (Isolate the variable by subtracting 5 from both sides)
$-3x \textcircled{\le} 5$	◀ Multiplication Property of Equality (Divide both sides by −3)
$x \ge -\frac{5}{3}$	◀ The inequality symbol changes direction because the inequality was divided on both sides by a negative number.

Check

The two expressions will be equal at $x=-\frac{5}{3}$	Values where $x<-\frac{5}{3}$ will result in false inequalities when substituted into the original.	Solutions where $x>-\frac{5}{3}$ will result in true inequalities when substituted into the original. This means the shading belongs to the right of the boundary point.
$-\frac{1}{5}\left(-\frac{5}{3}\right)+\frac{1}{3}\le\frac{2}{3}$	$-\frac{1}{5}(-5)+\frac{1}{3}<\frac{2}{3}$	$-\frac{1}{5}(0)+\frac{1}{3}\le\frac{2}{3}$
$\frac{5}{15}+\frac{1}{3}\le\frac{2}{3}$	$1+\frac{1}{3}<\frac{2}{3}$	$0+\frac{1}{3}\le\frac{2}{3}$
$\frac{5}{15}+\frac{5}{15}\le\frac{10}{15}$	$\frac{4}{3}<\frac{2}{3}$ ✗	$\frac{1}{3}\le\frac{2}{3}$ ✓
$\frac{10}{15}\le\frac{10}{15}$ ✓		

☑ Checkpoint

Solve. Graph the solution(s) on a number line.

$-\frac{2}{3}x+8>-2$

$-\frac{2}{3}x>-10$

$x<15$

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☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

If your student does not remember to change the direction of the inequality symbol, have them use $x=0$ as a substituted value for x. This will demonstrate that their solution is correct when $0<15$ or incorrect when $0>15$.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

2) Q: Why can inequalities have more than one numerical answer?

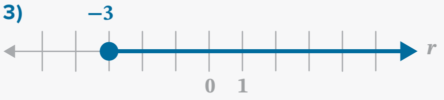
A: *Because the inequality represents all of the numbers above or below the given point.*

3–4)

Q: When should you use a closed point? What about an open point?

A: *A closed point is used when there is an equal to bar under the inequality symbol and the value is included. An open point is used when the value is not included in the solution set.*

3)



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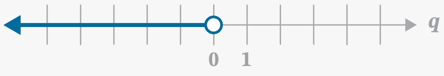
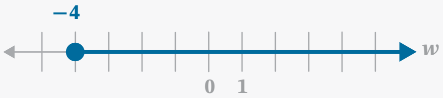
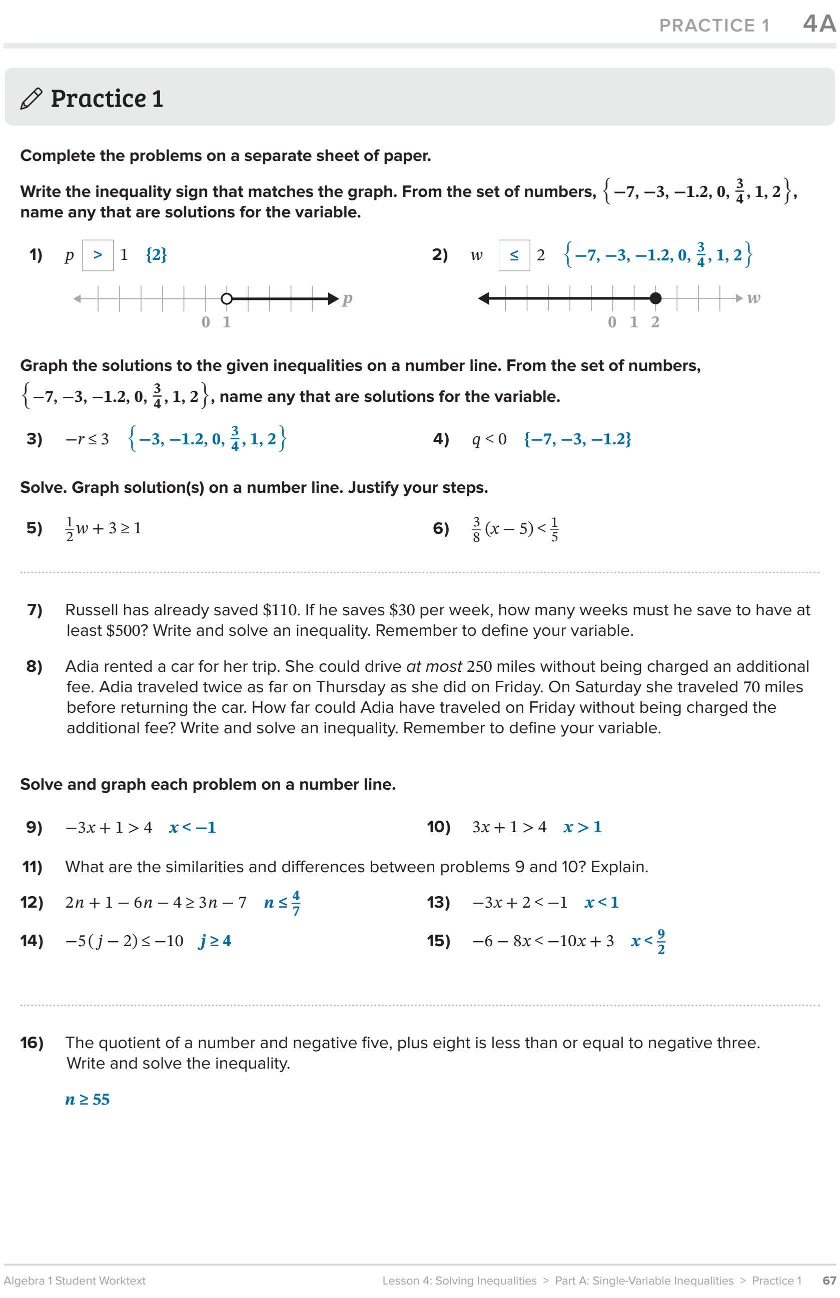


5) $\frac{1}{2}w+3\ge1$ ◀ Given

$\frac{1}{2}w\ge-2$ ◀ Addition Property of Equality
Add −3 to expressions on both sides

$w\ge-4$ ◀ Multiplication Property of Equality
Multiply expressions on both sides by 2





PRACTICE 1 4A

Practice 1

Complete the problems on a separate sheet of paper.

Write the inequality sign that matches the graph. From the set of numbers, $\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$, name any that are solutions for the variable.

1) p $\boxed{>}$ 1 $\{2\}$

2) w $\boxed{\le}$ 2 $\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$

Graph the solutions to the given inequalities on a number line. From the set of numbers, $\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$, name any that are solutions for the variable.

3) $-r \le 3$ $\left\{-3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$

4) $q < 0$ $\{-7, -3, -1.2\}$

Solve. Graph solution(s) on a number line. Justify your steps.

5) $\frac{1}{2}w + 3 \ge 1$

6) $\frac{3}{8}(x - 5) < \frac{1}{5}$

7) Russell has already saved $110. If he saves $30 per week, how many weeks must he save to have at least $500? Write and solve an inequality. Remember to define your variable.

8) Adia rented a car for her trip. She could drive *at most* 250 miles without being charged an additional fee. Adia traveled twice as far on Thursday as she did on Friday. On Saturday she traveled 70 miles before returning the car. How far could Adia have traveled on Friday without being charged the additional fee? Write and solve an inequality. Remember to define your variable.

Solve and graph each problem on a number line.

9) $-3x + 1 > 4$ $x < -1$

10) $3x + 1 > 4$ $x > 1$

11) What are the similarities and differences between problems 9 and 10? Explain.

12) $2n + 1 - 6n - 4 \ge 3n - 7$ $n \le \frac{4}{7}$

13) $-3x + 2 < -1$ $x < 1$

14) $-5(j - 2) \le -10$ $j \ge 4$

15) $-6 - 8x < -10x + 3$ $x < \frac{9}{2}$

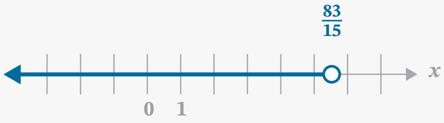
16) The quotient of a number and negative five, plus eight is less than or equal to negative three. Write and solve the inequality.

$n \ge 55$

Algebra 1 Student Worktext   Lesson 4: Solving Inequalities > Part A: Single-Variable Inequalities > Practice 1   67

6)

$\frac{3}{8}(x-5)<\frac{1}{5}$	Given
$40\left(\frac{3}{8}(x-5)\right)<(40)\left(\frac{1}{5}\right)$	Multiply both sides by the LCD
$15(x-5)<8$	
$15x-75<8$	Distribute
$15x<83$	Addition Property of Equality Add 75 to expressions on both sides
$x<\frac{83}{15}$	Multiplication Property of Equality Multiply both sides by $\frac{1}{15}$



If your student clears the fractions make sure they do not distribute across the parentheses.

Q: What is the LCD?

A: *40*

Q: What term(s) will you multiply the LCD by?

A: $\frac{3}{8}$ *and* $\frac{1}{5}$

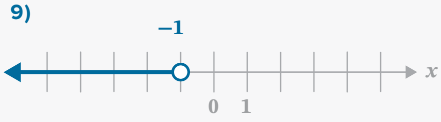
7) $w \ge 13$ Russell must save for at least 13 weeks to have at least $500 saved.

Q: Can you give an example of a number that is at least 500?

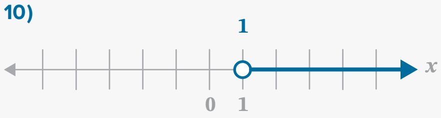
A: *Any number over 500.*

8) $d \le 60$ If Adia traveled 60 miles or less on Friday, then she should not have been charged the additional fee.

9)



10)



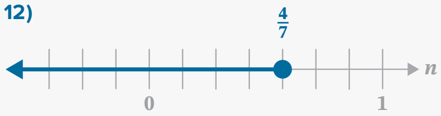
11) Sample:

Both problems are solved by first adding −1 and then multiplying by the reciprocal of the coefficient. The coefficient of the first is negative and the coefficient of the second is positive. The boundaries are on opposite sides of zero, and the solutions are on opposite sides of the number line because when multiplying by a negative, like $-\frac{1}{3}$, ALL signs change.

Q: What whole number would be false for both inequalities?

A: *0*

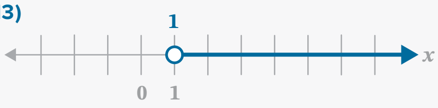
12)



Q: When you combine terms on the left side of the inequality, what operations should you follow? Explain.

A: *You should follow the given operations, not the inverse, since they are on the same side of the inequality.*

13)

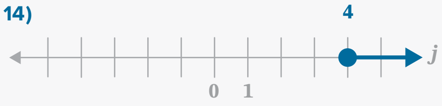


If your student does not remember to change the direction of the inequality symbol, you can use this question to have them solve in another way.

Q: If you do not want to divide or multiply by a negative number, what can you do to avoid this?

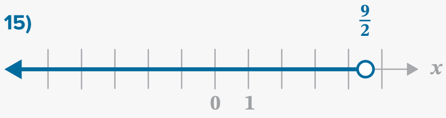
A: *Add the variable to both sides so that it is now a positive term on the other side of the inequality.*

14)



This inequality can be solved by distributing first if your student prefers. This is true for equations and inequalities.

15)



16) Q: What does the word quotient mean?

A: *Divide or to use division.*

Mastery Check

Show What You Know

A) Q: When equations have no solution, what happens to the variable?

A: *It simplifies out of the problem.*

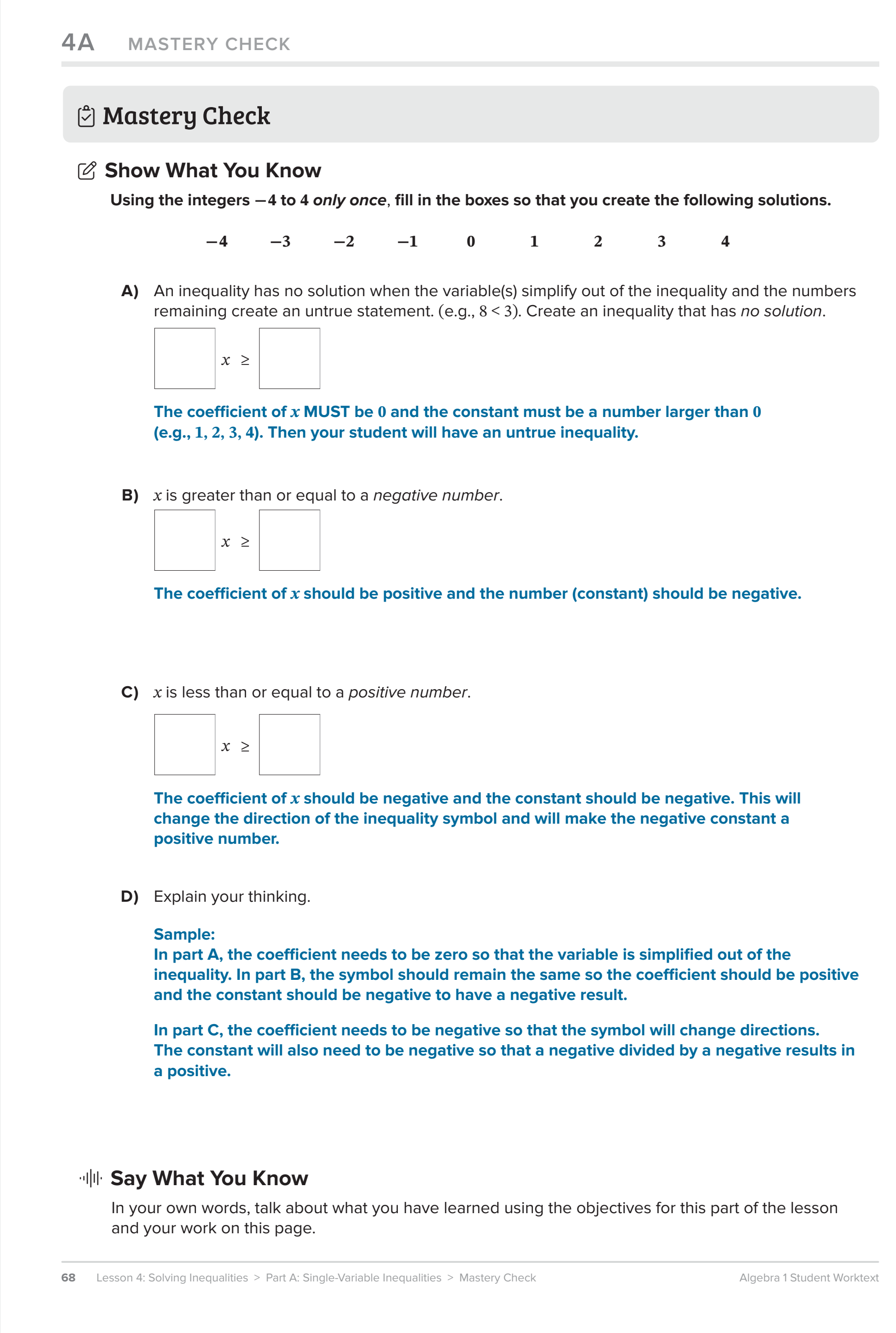
B) Q: When does the inequality symbol change direction?

A: *When you multiply or divide by a negative number.*

D) This teach-back will help your student demonstrate their understanding. You can also have them graph their solution(s) to provide a visual to their inequalities.

Encourage your student to make multiple attempts (at least 3 for each part). This will give them a better idea of whether their understanding is true for all cases. Students are expected to have attempts that are true as well as false so they can make connections to how the values relate to one another.

Using a pen may be helpful so that your student cannot erase attempts made and can see their progression of thinking.



Mastery Check

Show What You Know

Using the integers −4 to 4 *only once*, fill in the boxes so that you create the following solutions.

$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

A) An inequality has no solution when the variable(s) simplify out of the inequality and the numbers remaining create an untrue statement. (e.g., $8 < 3$). Create an inequality that has *no solution*.

$\square\, x \geq \square$

The coefficient of x MUST be 0 and the constant must be a number larger than 0 (e.g., 1, 2, 3, 4). Then your student will have an untrue inequality.

B) x is greater than or equal to a *negative number*.

$\square\, x \geq \square$

The coefficient of x should be positive and the number (constant) should be negative.

C) x is less than or equal to a *positive number*.

$\square\, x \geq \square$

The coefficient of x should be negative and the constant should be negative. This will change the direction of the inequality symbol and will make the negative constant a positive number.

D) Explain your thinking.

Sample:
In part A, the coefficient needs to be zero so that the variable is simplified out of the inequality. In part B, the symbol should remain the same so the coefficient should be positive and the constant should be negative to have a negative result.

In part C, the coefficient needs to be negative so that the symbol will change directions. The constant will also need to be negative so that a negative divided by a negative results in a positive.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Say What You Know

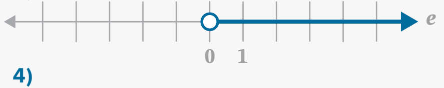
Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- ✓ You will be able to solve inequalities that include rational coefficients.
- ✓ You will be able to graph solutions for an inequality on a number line.
- ✓ You will be able to explain why the inequality symbol changes when multiplying by a negative factor.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

3)



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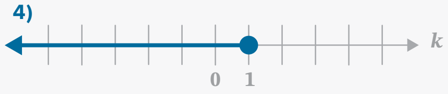
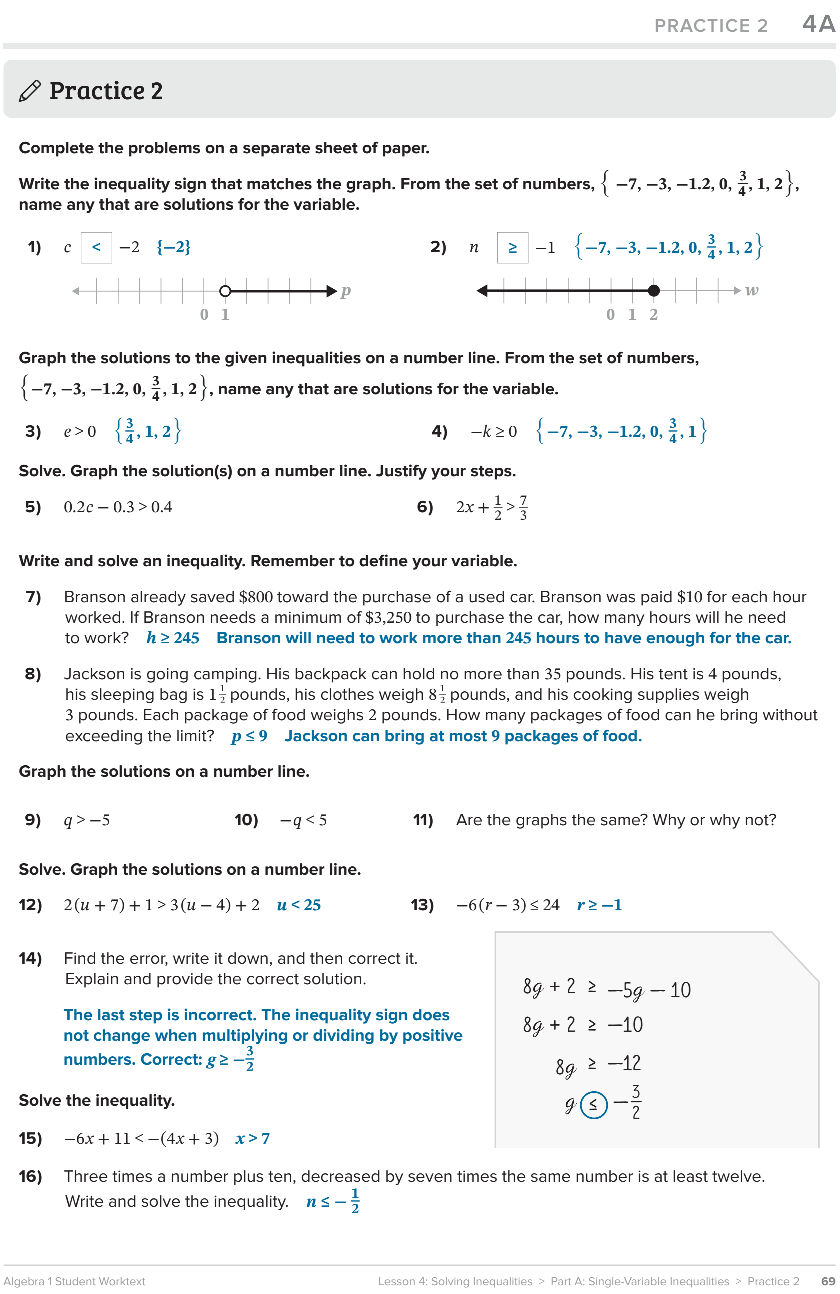


5)

$0.2c - 0.3 > 0.4$	Given
$10(0.2c - 0.3) > 10(0.4)$	Multiplication Property of Equality Multiply expressions on both sides by 10 [LCD]
$2c - 3 > 4$	Distributive Property Distribute 10 through parentheses
$2c > 7$	Addition Property of Equality Add 3 to expressions on both sides
$c > \frac{7}{2}$	Multiplication Property of Equality Multiply expressions on both sides by $\frac{1}{2}$



PRACTICE 2 4A

Practice 2

Complete the problems on a separate sheet of paper.

Write the inequality sign that matches the graph. From the set of numbers, $\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$, name any that are solutions for the variable.

1) c $\boxed{<}$ -2 **$\{-2\}$**

2) n $\boxed{\geq}$ -1 **$\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$**

Graph the solutions to the given inequalities on a number line. From the set of numbers, $\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1, 2\right\}$, name any that are solutions for the variable.

3) $e > 0$ **$\left\{\frac{3}{4}, 1, 2\right\}$**

4) $-k \geq 0$ **$\left\{-7, -3, -1.2, 0, \frac{3}{4}, 1\right\}$**

Solve. Graph the solution(s) on a number line. Justify your steps.

5) $0.2c - 0.3 > 0.4$

6) $2x + \frac{1}{2} > \frac{7}{3}$

Write and solve an inequality. Remember to define your variable.

7) Branson already saved $800 toward the purchase of a used car. Branson was paid $10 for each hour worked. If Branson needs a minimum of $3,250 to purchase the car, how many hours will he need to work? **$h \geq 245$ Branson will need to work more than 245 hours to have enough for the car.**

8) Jackson is going camping. His backpack can hold no more than 35 pounds. His tent is 4 pounds, his sleeping bag is $1\frac{1}{2}$ pounds, his clothes weigh $8\frac{1}{2}$ pounds, and his cooking supplies weigh 3 pounds. Each package of food weighs 2 pounds. How many packages of food can he bring without exceeding the limit? **$p \leq 9$ Jackson can bring at most 9 packages of food.**

Graph the solutions on a number line.

9) $q > -5$

10) $-q < 5$

11) Are the graphs the same? Why or why not?

Solve. Graph the solutions on a number line.

12) $2(u + 7) + 1 > 3(u - 4) + 2$ **$u < 25$**

13) $-6(r - 3) \leq 24$ **$r \geq -1$**

14) Find the error, write it down, and then correct it. Explain and provide the correct solution.

The last step is incorrect. The inequality sign does not change when multiplying or dividing by positive numbers. Correct: $g \geq -\frac{3}{2}$

$$8g + 2 \geq -5g - 10$$
$$8g + 2 \geq -10$$
$$8g \geq -12$$
$$g \; \textcircled{\leq} \; -\frac{3}{2}$$

Solve the inequality.

15) $-6x + 11 < -(4x + 3)$ **$x > 7$**

16) Three times a number plus ten, decreased by seven times the same number is at least twelve. Write and solve the inequality. **$n \leq -\frac{1}{2}$**

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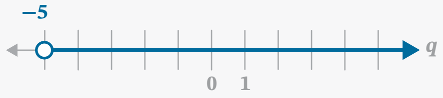
9)



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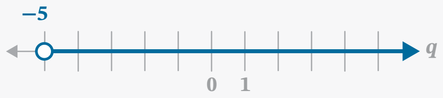
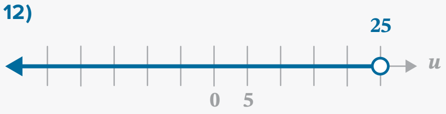
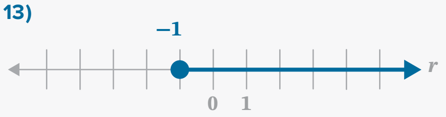


11) Sample:
Yes, the graphs are the same because when solving for q in the second inequality, the relationship between $-q$ and 5 changed, so the symbol changed as well. These two inequalities, $-q < 5$ and $q > -5$, are equivalent.

12)



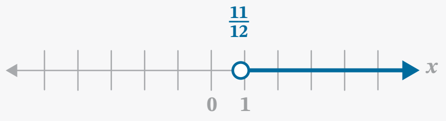
13)



If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

6)

$2x + \frac{1}{2} > \frac{7}{3}$	◂ Given
$6\left(2x + \frac{1}{2}\right) > (6)\left(\frac{7}{3}\right)$	◂ Multiplication Property of Equality Multiply expressions on both sides by 6 [LCD]
$12x + 3 > 14$	◂ Distributive Property Distribute 10 through parentheses
$12x > 11$	◂ Addition Property of Equality Subtract 3 from both sides
$x > \frac{11}{12}$	◂ Multiplication Property of Equality Multiply expressions on both sides by $\frac{1}{12}$



Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Q: Do you need to solve every equation to determine the number of solutions? Explain.

A: *No; you can use process of elimination without solving (since each letter can only be used one time).*

4B EXPLORE

Part B: Compound Inequalities

Objectives

In this part of the lesson, you will learn about compound inequalities.

By the end of this lesson, you will be able to do the following:

- Solve and graph solutions for single-variable compound inequalities.
- Solve and graph solutions for single-variable inequalities that contain absolute value.
- Identify inequalities containing absolute value as having no solution or as true for all real numbers.

Why?

Imagine you are given an assignment for government studies that requires you to poll 95 people but can vary by up to 12. Knowing how to solve compound inequalities would give you the range of the number of people you need to poll to complete the assignment.

Warm Up

Fill in the letter from the list provided that matches the equation with the correct number of solutions.

A) no solutions **B)** one solution **C)** two solutions **D)** all real numbers

B **1)** $x + 4 = 6$

C **2)** $|x - 5| = 7$

D **3)** $x = x$

A **4)** $|x + 8| = -2$

Compound Inequalities with Two Symbols

- Compound inequalities use AND or OR to combine the solutions of two inequalities for graphing on one number line.
- Inequalities that use the word OR have solutions that are true for one or more of the inequalities.
- Solutions to inequalities that use the word AND must be true for both inequalities.
  - When there is no number that makes both inequalities true for an AND problem, no solution exists.
- The solutions to a compound inequality should be graphed on one number line.

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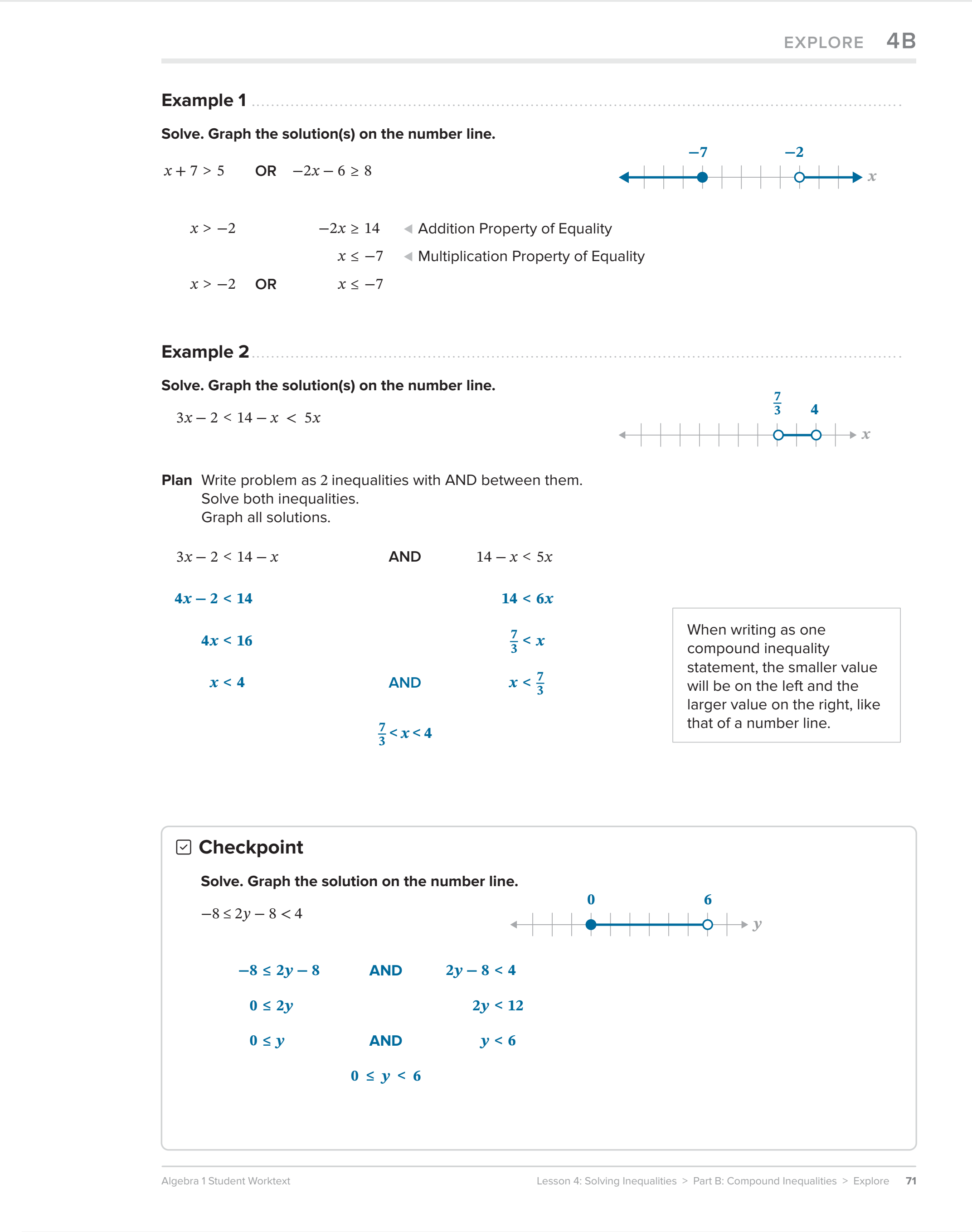


Example 1

Solve. Graph the solution(s) on the number line.

$x + 7 > 5$ OR $-2x - 6 \ge 8$

$x > -2$ $\quad -2x \ge 14$ ◄ Addition Property of Equality

$x \le -7$ ◄ Multiplication Property of Equality

$x > -2$ OR $x \le -7$

Example 2

Solve. Graph the solution(s) on the number line.

$3x - 2 < 14 - x < 5x$

Plan Write problem as 2 inequalities with AND between them.
Solve both inequalities.
Graph all solutions.

$3x - 2 < 14 - x$ AND $14 - x < 5x$

$4x - 2 < 14$ $\quad 14 < 6x$

$4x < 16$ $\quad \frac{7}{3} < x$

$x < 4$ AND $x < \frac{7}{3}$

$\frac{7}{3} < x < 4$

When writing as one compound inequality statement, the smaller value will be on the left and the larger value on the right, like that of a number line.

☑ Checkpoint

Solve. Graph the solution on the number line.

$-8 \le 2y - 8 < 4$

$-8 \le 2y - 8$ AND $2y - 8 < 4$

$0 \le 2y$ $\quad 2y < 12$

$0 \le y$ AND $y < 6$

$0 \le y < 6$

Example 1

Left side inequality: Add −7 to both sides.
Right side inequality: Add 6 to both sides.

Multiply both expressions by $-\frac{1}{2}$

☑ Checkpoint

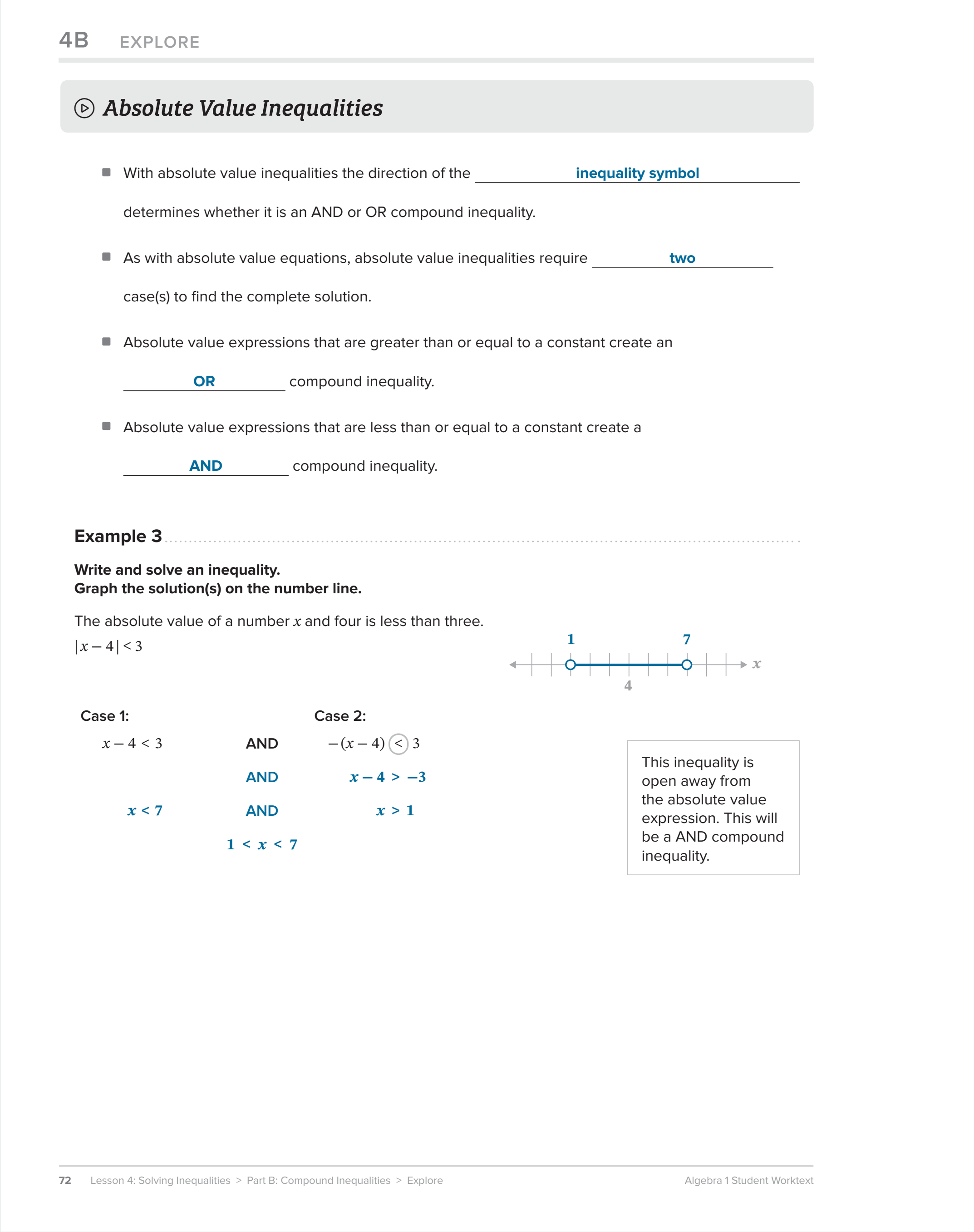
To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What should you do before solving the compound inequality?

A: *Write the compound inequality as two individual inequalities with the word AND between them.*

Q: How can you determine what type of compound inequality this is without the word AND or OR?

A: *This is an AND inequality. OR inequalities use the word OR. AND inequalities do not always have the word AND.*



Absolute Value Inequalities

- With absolute value inequalities the direction of the **inequality symbol** determines whether it is an AND or OR compound inequality.
- As with absolute value equations, absolute value inequalities require **two** case(s) to find the complete solution.
- Absolute value expressions that are greater than or equal to a constant create an **OR** compound inequality.
- Absolute value expressions that are less than or equal to a constant create a **AND** compound inequality.

Example 3

Write and solve an inequality.
Graph the solution(s) on the number line.

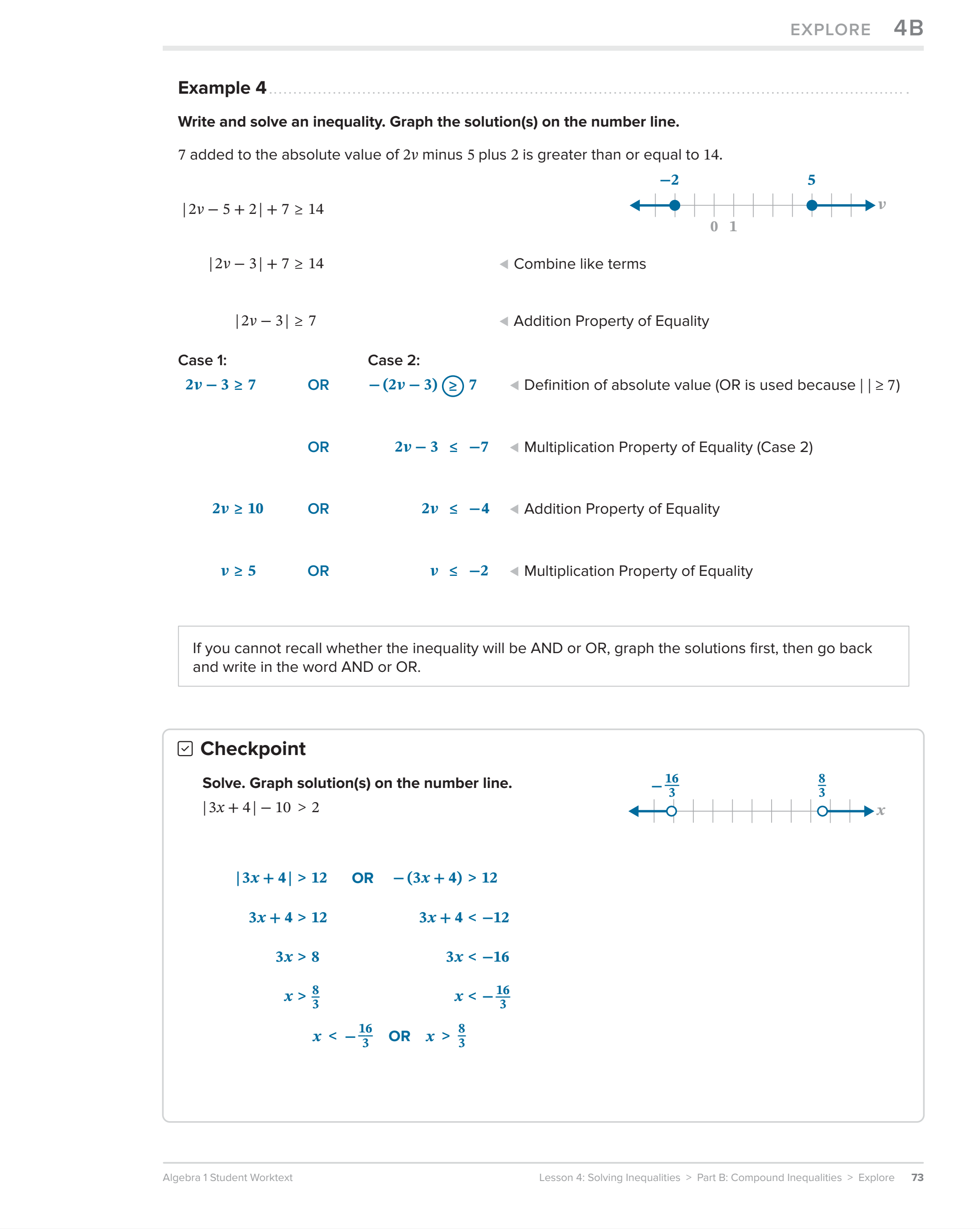
The absolute value of a number x and four is less than three.
$|x-4| < 3$

Case 1:		**Case 2:**
$x-4<3$	AND	$-(x-4) < 3$
	AND	$\mathbf{x-4 > -3}$
$\mathbf{x < 7}$	**AND**	$\mathbf{x > 1}$
	$\mathbf{1 < x < 7}$	

This inequality is open away from the absolute value expression. This will be a AND compound inequality.

Example 3

Remember that the symbol changes direction when you multiply both sides by −1 in Case 2.



Example 4

Write and solve an inequality. Graph the solution(s) on the number line.

7 added to the absolute value of $2v$ minus 5 plus 2 is greater than or equal to 14.

$|2v - 5 + 2| + 7 \geq 14$

$|2v - 3| + 7 \geq 14$ ◀ Combine like terms

$|2v - 3| \geq 7$ ◀ Addition Property of Equality

Case 1: Case 2:

$2v - 3 \geq 7$ OR $-(2v - 3) \, \boxed{\geq} \, 7$ ◀ Definition of absolute value (OR is used because $| \, | \geq 7$)

OR $2v - 3 \leq -7$ ◀ Multiplication Property of Equality (Case 2)

$2v \geq 10$ OR $2v \leq -4$ ◀ Addition Property of Equality

$v \geq 5$ OR $v \leq -2$ ◀ Multiplication Property of Equality

If you cannot recall whether the inequality will be AND or OR, graph the solutions first, then go back and write in the word AND or OR.

☑ Checkpoint

Solve. Graph solution(s) on the number line.

$|3x + 4| - 10 > 2$

$|3x + 4| > 12$ OR $-(3x + 4) > 12$

$3x + 4 > 12$ $3x + 4 < -12$

$3x > 8$ $3x < -16$

$x > \frac{8}{3}$ $x < -\frac{16}{3}$

$x < -\frac{16}{3}$ OR $x > \frac{8}{3}$

Example 4

This inequality is open towards the absolute value expression. This will be an OR compound inequality.

Add −7 to expressions on both sides

Multiply expressions on both sides by −1 [ALL signs change]

Add 3 to expressions on both sides

Multiply expressions on both sides by $\frac{1}{2}$

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How can you determine if the absolute value inequality is an AND or an OR inequality?

A: *OR is open tOwaRds the absolute value expression, AND is open Away from the absolute value expression.*

(Optional)

A: *You can solve and then graph the solutions to determine if the word AND or OR is correct.*

Example 6

Q: What would the number line look like for $|x + 8| > 0$?

A: *An unshaded number line with the words no solution written underneath since there is no solution.*

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

A) Q: Use the words always, sometimes, or never to complete the sentence. The absolute value of a number will ________ be less than zero.

A: *Never*

B) Q: Use the words always, sometimes, or never to complete the sentence. The absolute value of a number will ________ be greater than a negative number.

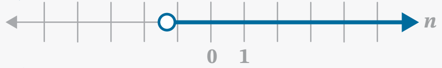
A: *Always*

If your student is unsure how to answer the Q&A or the problem, have them pick a positive number, a negative number, and zero for x and see if the inequality is true. This would be helpful for both questions.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1)



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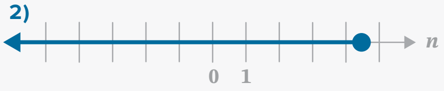
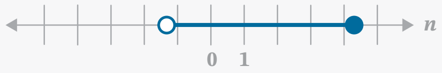
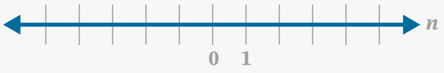


3) $\left\{-\frac{3}{4}, 0, 0.5, \frac{10}{3}\right\}$



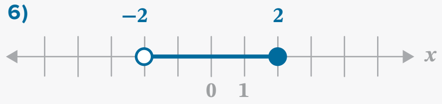
4) $\left\{-8, -4, -1.2, -\frac{3}{4}, 0, 0.5, \frac{10}{3}, 5, 10\right\}$



Remind your student to graph before selecting the true values.

5) Make sure that your student is using the words AND or OR when writing compound inequalities. This must be part of an OR inequality for the answer to be correct.

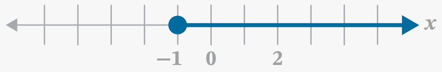
6)

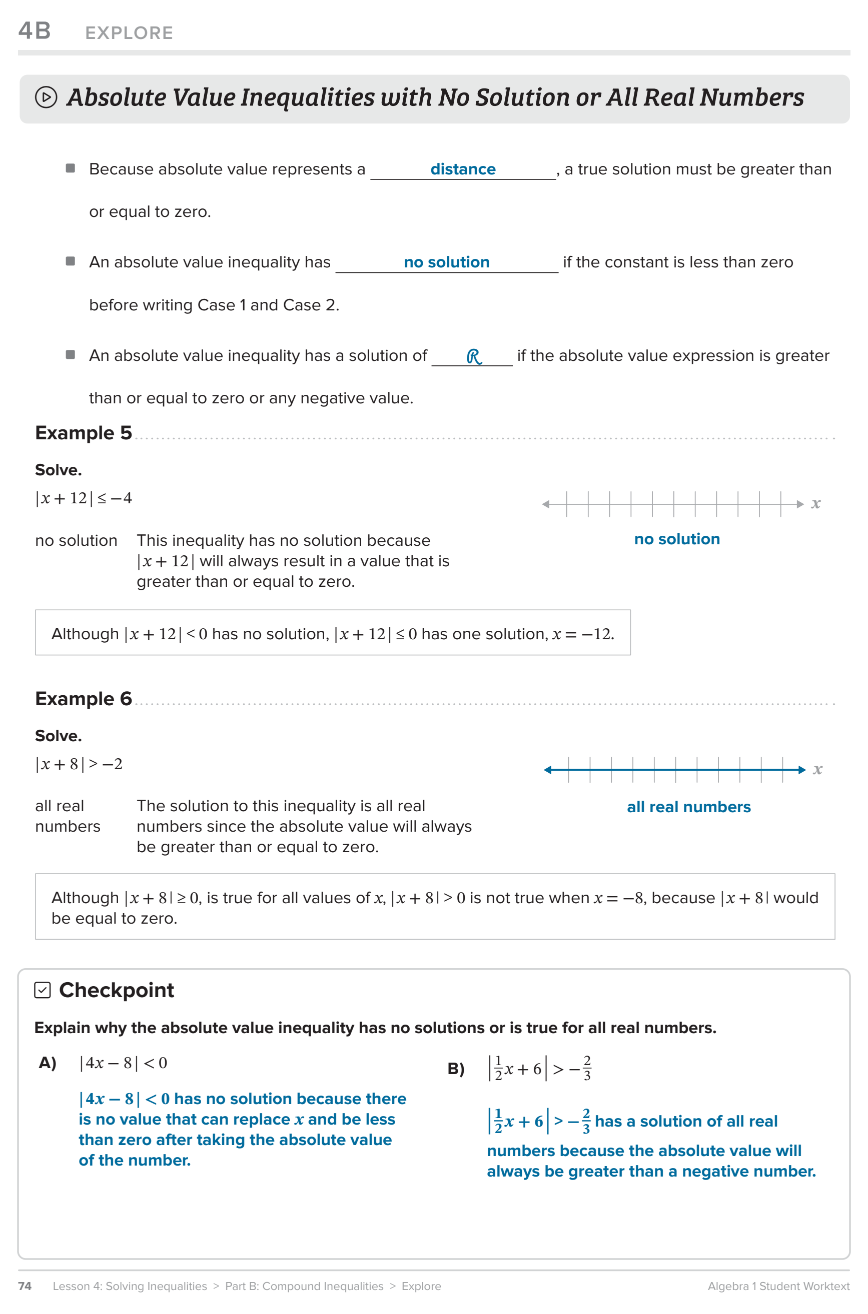


Q: How do you determine if the point will be open or closed?

A: *Look at the inequality symbol. If there is an equal to bar under the symbol, the point will be closed.*

7) $x \ge -1$ OR $x > 2$, which means all solutions can be rewritten as $x \ge -1$.





4B EXPLORE

▷ Absolute Value Inequalities with No Solution or All Real Numbers

- Because absolute value represents a ____distance____, a true solution must be greater than or equal to zero.
- An absolute value inequality has ____no solution____ if the constant is less than zero before writing Case 1 and Case 2.
- An absolute value inequality has a solution of __ℝ__ if the absolute value expression is greater than or equal to zero or any negative value.

Example 5

Solve.

$|x + 12| \le -4$

no solution — This inequality has no solution because $|x + 12|$ will always result in a value that is greater than or equal to zero.

x

no solution

Although $|x + 12| < 0$ has no solution, $|x + 12| \le 0$ has one solution, $x = -12$.

Example 6

Solve.

$|x + 8| > -2$

all real numbers — The solution to this inequality is all real numbers since the absolute value will always be greater than or equal to zero.

x

all real numbers

Although $|x + 8| \ge 0$, is true for all values of x, $|x + 8| > 0$ is not true when $x = -8$, because $|x + 8|$ would be equal to zero.

☑ Checkpoint

Explain why the absolute value inequality has no solutions or is true for all real numbers.

A) $|4x - 8| < 0$

$|4x - 8| < 0$ has no solution because there is no value that can replace x and be less than zero after taking the absolute value of the number.

B) $\left|\frac{1}{2}x + 6\right| > -\frac{2}{3}$

$\left|\frac{1}{2}x + 6\right| > -\frac{2}{3}$ has a solution of all real numbers because the absolute value will always be greater than a negative number.

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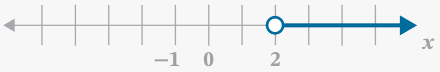
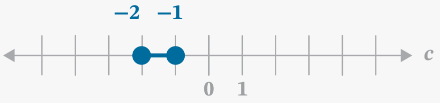
8)



Your student may need a reminder that AND means all values must be true for both inequalities. They can use $x = 0$ to substitute into both inequalities to show that all values must be greater than 2, since 0 is not true for $-3x + 2 < -4$.

9) $-2 \le c \le -1$

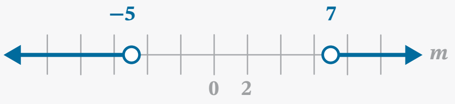


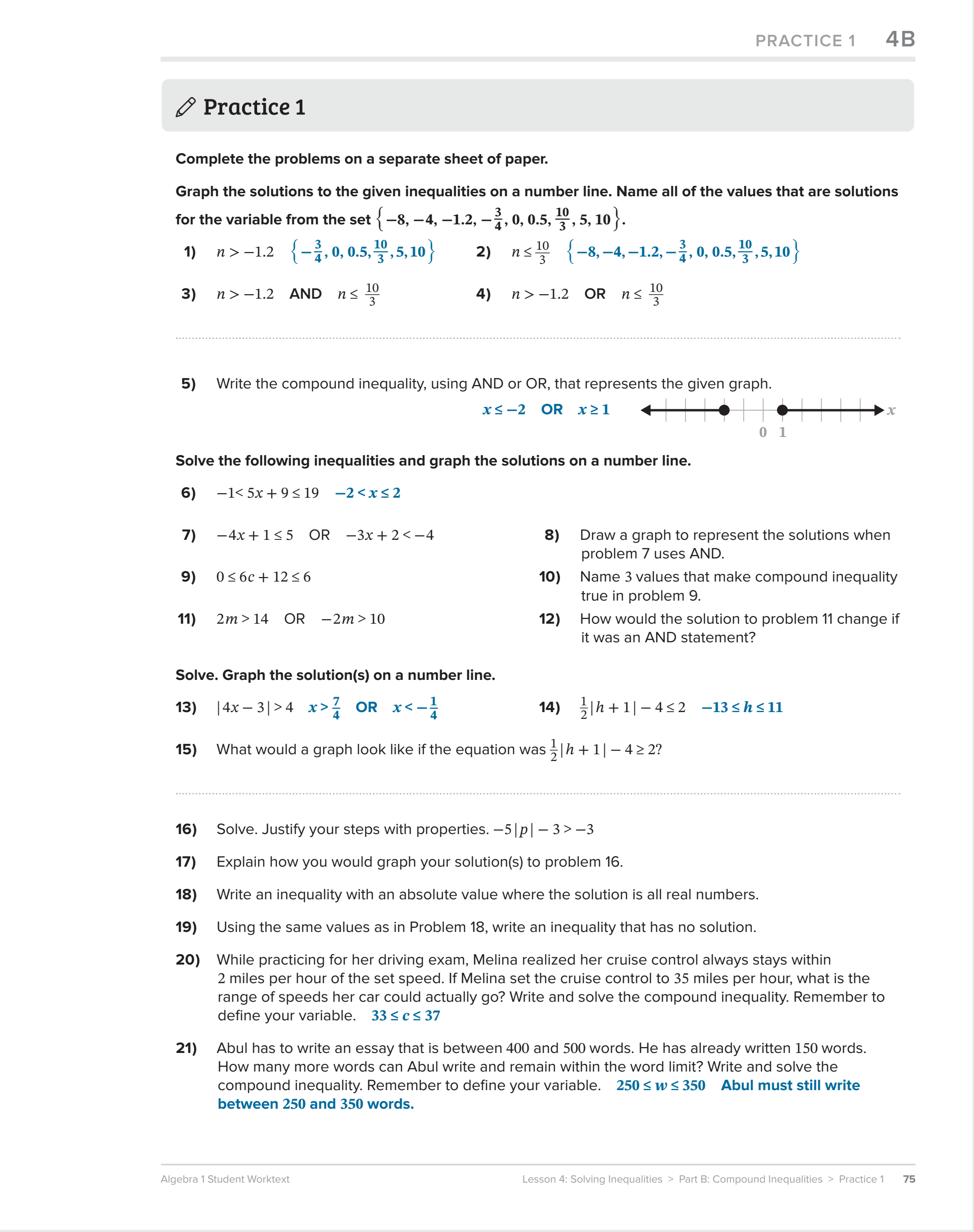
10) Samples:
$c = -2$, $c = -\frac{3}{2}$, $c = -1.25$; (All values between -2 and -1 and including -2 or -1.)

Q: How many numbers are there between your boundary points?

A: *An infinite number of numbers exist between any two points.*

11) $m < -5$ OR $m > 7$. This solution cannot be rewritten as a single inequality.





Practice 1

Complete the problems on a separate sheet of paper.

Graph the solutions to the given inequalities on a number line. Name all of the values that are solutions for the variable from the set $\left\{-8, -4, -1.2, -\frac{3}{4}, 0, 0.5, \frac{10}{3}, 5, 10\right\}$.

1) $n > -1.2$ $\left\{-\frac{3}{4}, 0, 0.5, \frac{10}{3}, 5, 10\right\}$

2) $n \le \frac{10}{3}$ $\left\{-8, -4, -1.2, -\frac{3}{4}, 0, 0.5, \frac{10}{3}, 5, 10\right\}$

3) $n > -1.2$ AND $n \le \frac{10}{3}$

4) $n > -1.2$ OR $n \le \frac{10}{3}$

5) Write the compound inequality, using AND or OR, that represents the given graph. **$x \le -2$ OR $x \ge 1$**

Solve the following inequalities and graph the solutions on a number line.

6) $-1 < 5x + 9 \le 19$ **$-2 < x \le 2$**

7) $-4x + 1 \le 5$ OR $-3x + 2 < -4$

8) Draw a graph to represent the solutions when problem 7 uses AND.

9) $0 \le 6c + 12 \le 6$

10) Name 3 values that make compound inequality true in problem 9.

11) $2m > 14$ OR $-2m > 10$

12) How would the solution to problem 11 change if it was an AND statement?

Solve. Graph the solution(s) on a number line.

13) $|4x - 3| > 4$ **$x > \frac{7}{4}$ OR $x < -\frac{1}{4}$**

14) $\frac{1}{2}|h + 1| - 4 \le 2$ **$-13 \le h \le 11$**

15) What would a graph look like if the equation was $\frac{1}{2}|h + 1| - 4 \ge 2$?

16) Solve. Justify your steps with properties. $-5|p| - 3 > -3$

17) Explain how you would graph your solution(s) to problem 16.

18) Write an inequality with an absolute value where the solution is all real numbers.

19) Using the same values as in Problem 18, write an inequality that has no solution.

20) While practicing for her driving exam, Melina realized her cruise control always stays within 2 miles per hour of the set speed. If Melina set the cruise control to 35 miles per hour, what is the range of speeds her car could actually go? Write and solve the compound inequality. Remember to define your variable. **$33 \le c \le 37$**

21) Abul has to write an essay that is between 400 and 500 words. He has already written 150 words. How many more words can Abul write and remain within the word limit? Write and solve the compound inequality. Remember to define your variable. **$250 \le w \le 350$ Abul must still write between 250 and 350 words.**

12) It would have no solution, because there are no common points between these two inequalities.

Have your student substitute values into the inequalities if they are uncertain of the outcome.

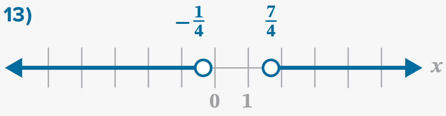
Q: Can a number be greater than 7 AND less than –5 at the same time?

A: *No*

Q: Can there be any solutions when the compound inequality is AND?

A: *No*

13)



13) Your student may need to be reminded that absolute value inequalities have two cases. You can ask:

Q: How can you determine if the absolute value inequality is AND or OR?

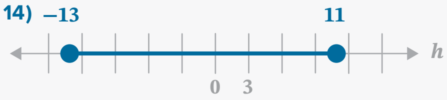
A: *Look at the direction of the inequality symbol. OR: >, AND: <*

Your student may need to be reminded to change the direction of the inequality symbol for Case 2. If they have the correct value but the symbol direction is wrong, you can ask:

Q: What happens when an inequality is multiplied or divided by a negative number?

A: *The direction of the symbol changes.*

14)



15)

Q: What should you do before writing the two cases?

A: *Make sure the absolute value expression is isolated on one side of the inequality.*

16)

$-5\|p\| - 3 > -3$	◀ Given
$-5\|p\| > 0$	◀ Addition Property of Equality Add 3 to expressions on both sides
$\|p\| < 0$	◀ Multiplication Property of Equality Multiply expressions on both sides by $-\frac{1}{5}$ [All signs change]
no solution	◀ Distance must be non-negative.

17) Sample:
This inequality has no solution. Nothing would be marked on the number line since no number will make this true.

Q: What could you change to make the solution all real numbers?

A: *If the question was $|p| > 0$, then the solution would be all real numbers.*

18) Sample 1: $|b| \ge 0$

Sample 2: $|n| > -3$

The absolute value must be greater than or equal to (≥) zero, or the absolute value must be greater than (>) or greater than or equal to (≥) a negative number.

Q: What do you know about absolute value inequalities and negative numbers?

A: *The absolute value must be greater than a negative number since distance cannot be a negative number.*

19) Sample 1: $|b| < 0$

Sample 2: $|n| \le -3$

The absolute value must be less than (<) zero, or the absolute value must be less than (<) or less than or equal to (≤) a negative number.

20) Your student may need to look back at Lesson 3 word problems. You can use the Mastery Check for Lesson 3 to help them complete this problem.

Q: Recall the formula using the midpoint and distance for absolute value equations in Lesson 3. What is the distance for this problem?

A: *2*

Q: What is the midpoint?

A: *35*

Q: Why is this an AND inequality rather than an OR?

A: *When the solutions can be between two numbers (or a range of values), this represents an AND inequality.*

Mastery Check

Show What You Know

A) It is possible that your student will have a number other than 24 hours, this is why their explanation is so important to this question.

B) Q: If Dan never trains for more than 8 hours a day, is it possible that he trains for exactly 8 hours in one day?

A: *Yes*

Q: What symbol is needed if the boundary number is included?

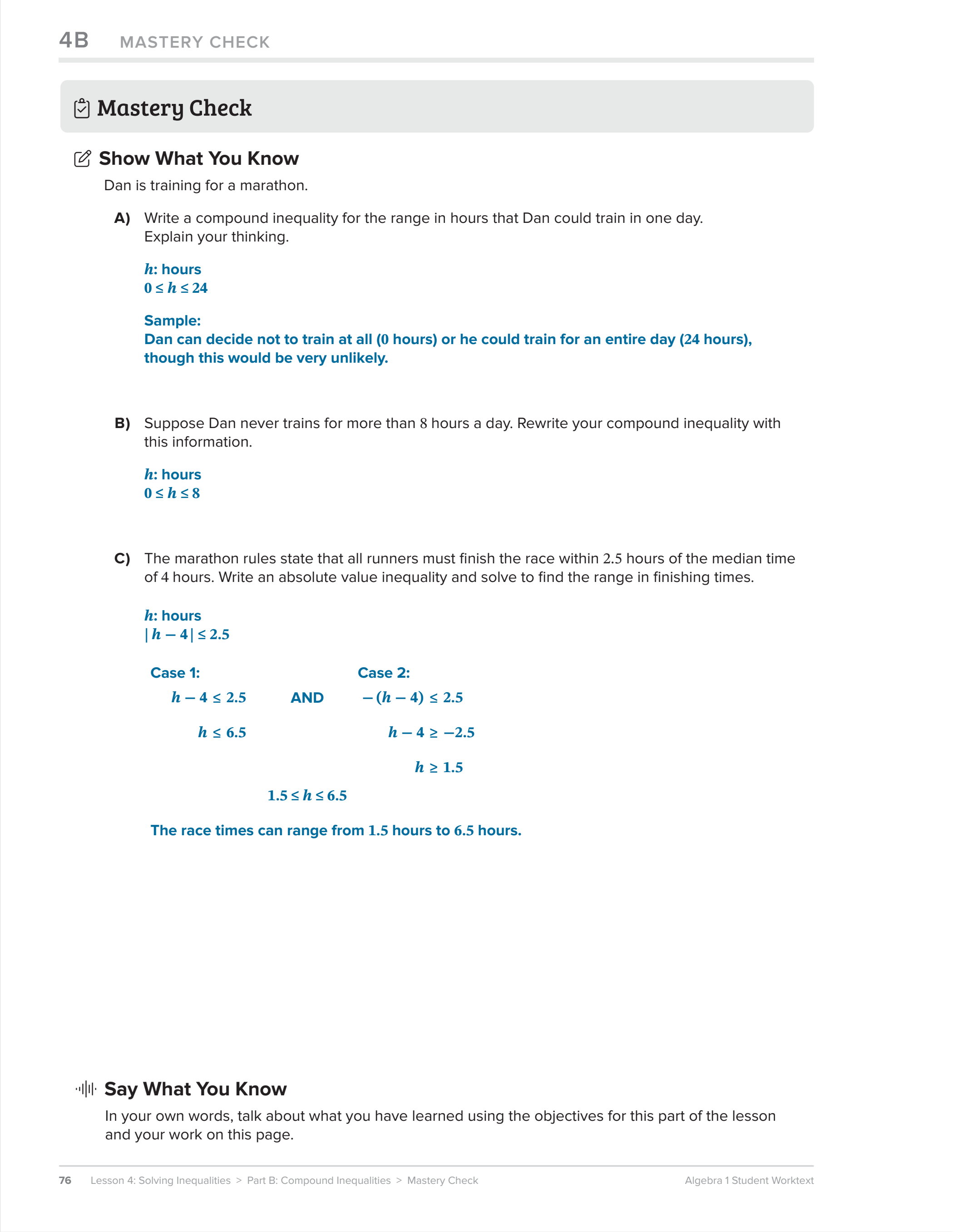
A: *less than or equal to*

C) Q: What is the midpoint?

A: *4*

Q: What is the distance on either side of the midpoint?

A: *2.5*



Mastery Check

Show What You Know

Dan is training for a marathon.

A) Write a compound inequality for the range in hours that Dan could train in one day. Explain your thinking.

h: hours
$0 \leq h \leq 24$

Sample:
Dan can decide not to train at all (0 hours) or he could train for an entire day (24 hours), though this would be very unlikely.

B) Suppose Dan never trains for more than 8 hours a day. Rewrite your compound inequality with this information.

h: hours
$0 \leq h \leq 8$

C) The marathon rules state that all runners must finish the race within 2.5 hours of the median time of 4 hours. Write an absolute value inequality and solve to find the range in finishing times.

h: hours
$|h - 4| \leq 2.5$

Case 1:		Case 2:
$h - 4 \leq 2.5$	AND	$-(h - 4) \leq 2.5$
$h \leq 6.5$		$h - 4 \geq -2.5$
		$h \geq 1.5$

$1.5 \leq h \leq 6.5$

The race times can range from 1.5 hours to 6.5 hours.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- You will be able to solve and graph solutions for single-variable compound inequalities.
- You will be able to solve and graph solutions for single-variable inequalities that contain absolute value.
- You will be able to identify inequalities containing absolute value as having no solution or as true for all real numbers.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

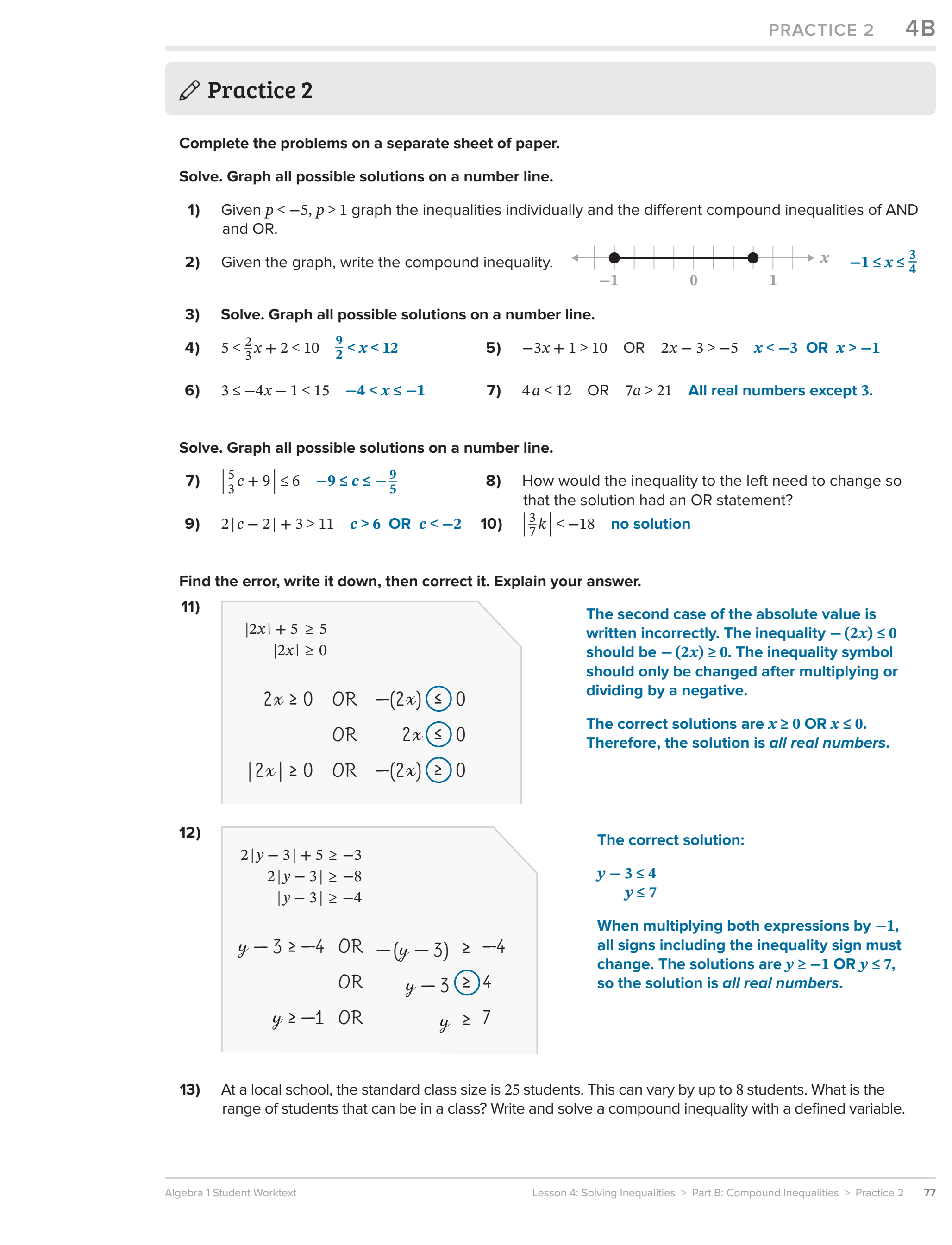
YES

If your student can answer “yes” to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer “yes” to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.



PRACTICE 2 4B

Practice 2

Complete the problems on a separate sheet of paper.

Solve. Graph all possible solutions on a number line.

1) Given $p < -5$, $p > 1$ graph the inequalities individually and the different compound inequalities of AND and OR.

2) Given the graph, write the compound inequality. $-1 \le x \le \frac{3}{4}$

3) **Solve. Graph all possible solutions on a number line.**

4) $5 < \frac{2}{3}x + 2 < 10$ $\frac{9}{2} < x < 12$

5) $-3x + 1 > 10$ OR $2x - 3 > -5$ $x < -3$ **OR** $x > -1$

6) $3 \le -4x - 1 < 15$ $-4 < x \le -1$

7) $4a < 12$ OR $7a > 21$ **All real numbers except 3.**

Solve. Graph all possible solutions on a number line.

7) $\left|\frac{5}{3}c + 9\right| \le 6$ $-9 \le c \le -\frac{9}{5}$

8) How would the inequality to the left need to change so that the solution had an OR statement?

9) $2|c - 2| + 3 > 11$ $c > 6$ **OR** $c < -2$

10) $\left|\frac{3}{7}k\right| < -18$ **no solution**

Find the error, write it down, then correct it. Explain your answer.

11)

$|2x| + 5 \ge 5$
$|2x| \ge 0$

$2x \ge 0$ OR $-(2x) \enclose{circle}{\le} 0$
OR $2x \enclose{circle}{\le} 0$
$|2x| \ge 0$ OR $-(2x) \enclose{circle}{\ge} 0$

The second case of the absolute value is written incorrectly. The inequality $-(2x) \le 0$ should be $-(2x) \ge 0$. The inequality symbol should only be changed after multiplying or dividing by a negative.

The correct solutions are $x \ge 0$ OR $x \le 0$. Therefore, the solution is *all real numbers*.

12)

$2|y - 3| + 5 \ge -3$
$2|y - 3| \ge -8$
$|y - 3| \ge -4$

$y - 3 \ge -4$ OR $-(y - 3) \ge -4$
OR $y - 3 \enclose{circle}{\ge} 4$
$y \ge -1$ OR $y \ge 7$

The correct solution:

$y - 3 \le 4$
$y \le 7$

When multiplying both expressions by -1, all signs including the inequality sign must change. The solutions are $y \ge -1$ OR $y \le 7$, so the solution is *all real numbers*.

13) At a local school, the standard class size is 25 students. This can vary by up to 8 students. What is the range of students that can be in a class? Write and solve a compound inequality with a defined variable.

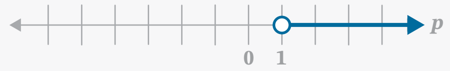
Algebra 1 Student Worktext Lesson 4: Solving Inequalities > Part B: Compound Inequalities > Practice 2 77

Practice 2

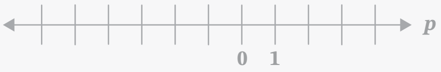
Worked solutions for these problems are located in the Digital Pack.

1) $p < -5$

$p > 1$



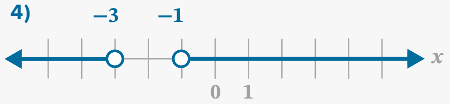
$p < -5$ AND $p > 1$ no solutions



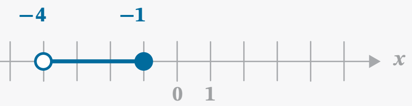
$p < -5$ OR $p > 1$

3)

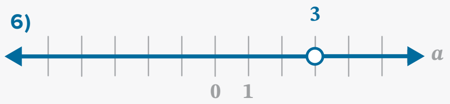
4)



5)



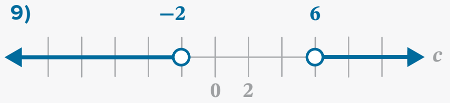
6)



7)

8) The inequality symbol would need to change from ≤ to ≥ for the solution to have an OR statement.

9)



13) $|s - 25| \le 8$

The solution is $s \ge 17$ AND $s \le 33$, which can be rewritten as $17 \le s \le 33$. The class size can range from 17 students to 33 students.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

3) This is very important to understand, especially if your student is using a calculator to check their answers. It may be helpful to do this problem with a few numbers (do not use 0, 1, 2) so that your student can see/be reminded of the order of operations.

4) Sample:
Order of operations says to work with numbers inside parentheses first. On the left, you multiply negative eight by itself. On the right, you multiply 8 by itself and then the product by -1.

5–6)
Remind your student to use their Formula Sheet.

5) Reflexive Property

6) Inverse Property (of Multiplication)

7) -20 (starting balance) -40 (fee) $= -\$60$

$b = 97.50$

Ida's account now has a balance of $\$97.50$

There are many ways to organize the information. Your student may choose to solve without writing an equation.

8) $\frac{1}{2}n + 7 = 3(n - 6)$

$n = 10$

9) 2.54 cm = 1 in

10) $y = mx + b$

11) Distractor Rationale:

A) This makes the midpoint 6 rather than 2.

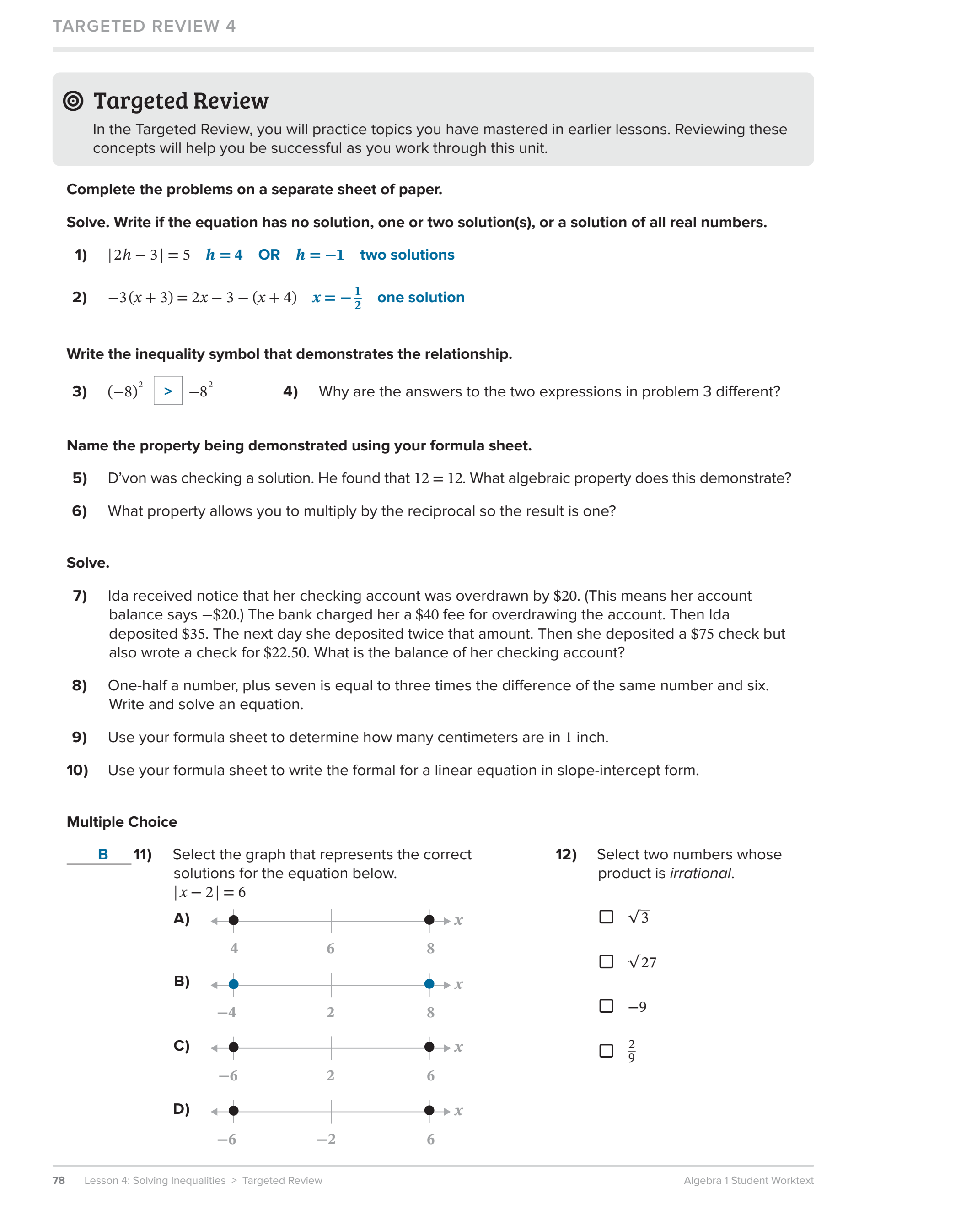
C) This treats "2" as 0 on the number line. The distance from -6 to $2 = 8$ spaces, from 2 to $6 = 4$ spaces.

D) This makes -2 the midpoint but ignores equal distances. The distance from -6 to $-2 = 4$ spaces, from -2 to $6 = 8$ spaces.

12) Have your student start this problem by identifying the numbers that are rational ($\mathbb{Q}$) or irrational ($\mathbb{I}$). Recall the mastery check in Lesson 1 that says the product of an irrational number and a rational number is always irrational.

Students will only answer with one of these possible combinations. All of the combinations shown here are correct.

■ $\sqrt{3}$	■ $\sqrt{3}$	☐ $\sqrt{3}$	☐ $\sqrt{3}$
☐ $\sqrt{27}$	☐ $\sqrt{27}$	■ $\sqrt{27}$	■ $\sqrt{27}$
■ -9	☐ -9	■ -9	☐ -9
☐ $\frac{2}{9}$	■ $\frac{2}{9}$	☐ $\frac{2}{9}$	■ $\frac{2}{9}$



TARGETED REVIEW 4

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Solve. Write if the equation has no solution, one or two solution(s), or a solution of all real numbers.

1) $|2h - 3| = 5$ **$h = 4$ OR $h = -1$ two solutions**

2) $-3(x + 3) = 2x - 3 - (x + 4)$ **$x = -\frac{1}{2}$ one solution**

Write the inequality symbol that demonstrates the relationship.

3) $(-8)^2$ $\boxed{>}$ -8^2

4) Why are the answers to the two expressions in problem 3 different?

Name the property being demonstrated using your formula sheet.

5) D'von was checking a solution. He found that $12 = 12$. What algebraic property does this demonstrate?

6) What property allows you to multiply by the reciprocal so the result is one?

Solve.

7) Ida received notice that her checking account was overdrawn by \$20. (This means her account balance says $-\$20$.) The bank charged her a \$40 fee for overdrawing the account. Then Ida deposited \$35. The next day she deposited twice that amount. Then she deposited a \$75 check but also wrote a check for \$22.50. What is the balance of her checking account?

8) One-half a number, plus seven is equal to three times the difference of the same number and six. Write and solve an equation.

9) Use your formula sheet to determine how many centimeters are in 1 inch.

10) Use your formula sheet to write the formal for a linear equation in slope-intercept form.

Multiple Choice

B **11)** Select the graph that represents the correct solutions for the equation below.
$|x - 2| = 6$

12) Select two numbers whose product is *irrational*.

☐ $\sqrt{3}$

☐ $\sqrt{27}$

☐ -9

☐ $\frac{2}{9}$

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The square root of 3 multiplied by -9 OR $\frac{2}{9}$ is irrational.

The square root of 27 multiplied by -9 OR $\frac{2}{9}$ is irrational.

The only two numbers that result in a rational number are $\sqrt{3} \cdot \sqrt{27} = \sqrt{81} = 9$.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	2–3	2–3	PA	PA	1	1	2	2	FS	FS	3	1