Lesson 3
Solving Absolute Value Equations

Outline

Part A
Absolute Value Equations

- One- and Two-Step Absolute Value Equations
- Graphing Solutions Using the Midpoint
- Writing Absolute Value Equations

Part B
Multi-Step Absolute Value Equations

- Multi-step Absolute Value Equations
- Absolute Value Equations with No Solution and All Real Numbers

Targeted Review

Vocabulary

- absolute value

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

1) Q: Why is the distance from Dee's house to Nancy's house 12 blocks instead of 0 blocks?

A: *Distance cannot be negative. So if a person walked from Dee's house to Nancy's house they would walk 12 total blocks.*

Part A: Absolute Value Equations

Objectives

In this part of the lesson, you will learn about absolute value equations.

By the end of this lesson, you will be able to do the following:

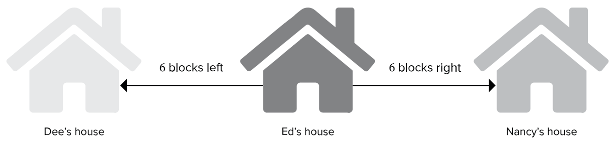
- Solve one- and two-step absolute value equations.
- Graph one- and two-step absolute value equation solutions on a number line.
- Explain why an absolute value equation can have two solutions.
- Write and solve an absolute value equation from a given problem.

Why?

Solving and graphing absolute value equations shows that distance cannot be negative; however, the two solutions are equal distance from a midpoint on a number line in either direction.

Warm Up

Use the figure to answer problems 1–3.



1) Ed walked to Dee's house. How far did he travel?

6 blocks (left of his house)

2) Ed walked from Dee's house to Nancy's house. How far did he travel?

12 blocks
(6 blocks back to his house, 6 blocks to the right of his house)

3) If Ed's house was at zero on the number line where is Dee's house? Nancy's house?

Dee's house is at −6 on the number line. Nancy's house is at 6 on the number line.

One- and Two-Step Absolute Value Equations

- The magnitude of a number is called **absolute value**.
- Absolute value ($|\,|$) can be thought of as **distance** without direction.

$|4| =$ **4** $\qquad |-4| =$ **4**

EXPLORE 3A

- The absolute value of a single term is the distance between that term and **zero**.
- Since opposite terms have the same absolute value, the absolute value of an unknown has **two** possible solutions.
- When graphing absolute values on a number line, you must move both directions from the **midpoint** to find both possible solutions.
- There are two **cases** to consider when solving absolute value equations algebraically.

Example 1

Solve.
$|x - 5| = 7$

Plan Solve both cases of the absolute value equation.

Implement

$|x - 5| = 7$ ◀ Given

Case 1:		Case 2:	Explain
$x - 5 = 7$	OR	$-(x - 5) = 7$	◀ Definition of Absolute Value
		$x - 5 = -7$	◀ Multiplication Property of Equality (Case 2 only)
$x = 12$	OR	$x = -2$	◀ Addition Property of Equality **(Add 5 to expressions on both sides)**

Check

Case 1:	Case 2:
$x = 12$	$x = -2$
$\|(12) - 5\| = 7$	$\|(-2) - 5\| = 7$
$\|7\| = 7$	$\|-7\| = 7$
$7 = 7$ ✓	$7 = 7$ ✓

Algebra 1 Student Worktext Lesson 3: Solving AbsoluteValue Equations > Part A: Absolute Value Equations > Explore 45

Example 1

Notice that some steps from Lesson 2 have been omitted. For example, the Additive Identity Property is not included, because $-5 + 5 = 0$ is understood. Since zero does not change the value, you do not need to write this step. Instead, you can move from $x - 5 = 7$ to $x = 12$, giving the reason Addition Property of Equality. The examples will provide models to help you determine which steps may be omitted.

Example 2

At this step, $|x - 3| = 4$, all of the information is provided for finding the solutions for this equation. The midpoint is 3, and the solutions are a distance of 4 units on either side of 3. The solutions can be found by adding 4 to 3 $(3 + 4 = 7)$ and subtracting 4 from 3 $(3 - 4 = -1)$.

Visually this would be: $|x - 3| = 4$.
"The distance between x and 3 is 4."

The same steps used to solve equations in Lesson 2 are used to solve absolute value equations. The difference is that you must include both cases so that all possible solutions are represented.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How many solutions will your equation have? Explain.

A: *Two, because absolute value equations have two solutions.*

Q: What is the midpoint of the equation?

A: *3*

3A EXPLORE

Example 2

Solve.

$2|x - 3| = 8$

This means that twice the distance between x and 3 is 8.

Plan **Inside absolute value, subtract 3 (or add −3)**
Absolute value.
Multiply by 2.

Implement			Explain
$2\|x - 3\| = 8$			Given
$\|x - 3\| = 4$			Multiplication Property of Equality
Case 1:		Case 2:	
$x - 3 = 4$	OR	$-(x - 3) = 4$	Definition of Absolute Value
	OR	$x - 3 = -4$	Multiplication Property of Equality (Case 2 only)
$x = 7$	OR	$x = -1$	Addition Property of Equality

Checkpoint

Solve.

$2|x - 3| = 10$

$|x - 3| = 5$

Case 1:

$|x - 3| = 5$

$x = 8$

Case 2:

$-(x - 3) = 5$

$x - 3 = 5$

$x = -2$

Graphing Solutions Using the Midpoint

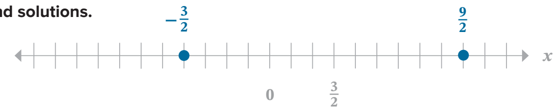
- When absolute value equations are graphed, the **two solutions** and the **midpoint of the solutions** should be included on the number line.
- After you find the two solutions, determine the midpoint by finding the **middle or average** of the two solutions.

EXPLORE 3A

Example 3

Solve. Graph the midpoint and solutions.

$|2x - 3| = 6$



Plan Before solving for the two cases, first isolate the absolute value.
$\cdot 2$ (Multiply by 2)
(-3) (Inside absolute value, subtract 3 (or add -3)
$|\ |$ (absolute value)

Implement			**Explain**
$\lvert 2x - 3\rvert = 6$			◂ Given
Case 1:		**Case 2:**	
$2x - 3 = 6$	OR	$-(2x - 3) = 6$	◂ Definition of Absolute Value
	OR	$2x - 3 = -6$	◂ Multiplication Property of Equality (Case 2 only)
$\mathbf{2x = 9}$	**OR**	$\mathbf{2x = -3}$	◂ Addition Property of Equality
$\mathbf{x = \frac{9}{2}}$		$\mathbf{x = -\frac{3}{2}}$	◂ Multiplication Property of Equality

To determine the midpoint find the mean of the two solutions.

$\left(\frac{9}{2} + \left(-\frac{3}{2}\right)\right) \div 2$

$\frac{6}{2} \cdot \frac{1}{2} = \frac{3}{2}$

The midpoint of the solutions is $\frac{3}{2}$.

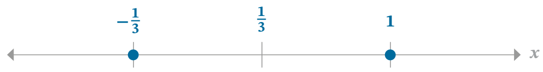
> Not all problems will require that you find the midpoint. However, the midpoint helps you see how absolute value solutions are the same distance from a common point.

☑ Checkpoint

Given the equation and solutions, determine the midpoint for the equation. Name the midpoint and graph the solutions.

$|6x - 2| + 1 = 5$

$x = 1$ *or* $x = -\frac{1}{3}$



Midpoint:

$\frac{\left(1 + -\frac{1}{3}\right)}{2}$ $\quad$ $\frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$

Example 3

Not all problems will require the midpoint but the midpoint helps you see how absolute value solutions are the same distance from a common point that is not always zero.

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: How do you find the midpoint after you have both solutions?

A: *Find the average of the two solutions.*

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

If your student needs help getting started have them think about the answers first and then work backwards.

Q: What is 4 added to 510?
A: *514*

Q: What is 4 subtracted from 510?
A: *506*

Q: How can you set up your equation so that you solve for both of those values?
A: *absolute value = distance*

Your student may notice that the first step in Case 2 could be eliminated using mental math and they will begin at step 2. This is correct as long as they understand how the negative symbol moved from the left to the right side of the equation.

Writing Absolute Value Equations

- $|x -$ (midpoint)$|$ = (distance)
- When writing an absolute value equation, it is helpful to know the **midpoint**.
- In an absolute value equation, the midpoint is the number inside the **absolute value** bars (when the coefficient of the variable is 1).
- In an absolute value equation, the **distance** is what the absolute value equation is equal to.

Example 4

Jim and Cory have a difference of three Liberty nickels. If Jim collected 5 Liberty nickels, then how many Liberty nickels did Cory collect?

Plan Define variables.
Write and solve an equation.

Solve the equation.
Let c = number of Liberty nickels Cory collected.

Implement			Explain
$\lvert c-5\rvert = 3$			
$c-5=3$	OR	$-(c-5)=3$	Definition of Absolute Value
		$c-5=-3$	Multiplication Property of Equality (Case 2 only)
$c=8$	OR	$c=2$	Addition Property of Equality

Explain
These solutions mean that Cory either has **8 Liberty nickels or 2 Liberty nickels**.

Checkpoint

A package of granola can have a difference of 4 grams from the listed weight on the box. If the weight on the granola box says 510 grams, what is the lowest allowed weight of the box? What is the highest allowed weight?

Write an equation and solve.

Plan g: granola
$|g-510|=4$

Case 1:
$g-510=4$
$g=514$

Case 2:
$-(g-510)=4$
$g-510=-4$
$g=506$

The lowest allowed weight of the granola will be 506 grams.
The highest allowed weight of the granola will be 514 grams.

PRACTICE 1 3A

Practice 1

Complete the problems on a separate sheet of paper.

Solve. Remember to check your work.

1) $\left|\frac{3}{4}x\right| = 5$ $\quad x = -\frac{20}{3}, \frac{20}{3}$

2) $|5x + 7| = 22$ $\quad x = -\frac{29}{5}, 3$

3) $|3x - 6| = 4$ $\quad x = \frac{2}{3}, \frac{10}{3}$

4) $4 = 5|v - 5|$ $\quad v = \frac{21}{5}, \frac{29}{5}$

5) $2|k - 2| = 8$ $\quad k = -2, 6$

6) $\left|\frac{2}{3}x - 1\right| = 10$ $\quad x = -\frac{27}{2}, \frac{33}{2}$

Graph the solutions and note the midpoint on a number line.

7) $3 = |x + 10|$

8) $\left|\frac{3}{2}k\right| = 9$

9) $4|w + 6| = 8$ $\quad w = -8, -4$ Midpoint $= -6$

10) $|2x + 6| = 9$ $\quad x = -\frac{15}{2}, \frac{3}{2}$ Midpoint $= -3$

Solve. Remember to check your work.

11) The weight of a dozen large eggs must be within 1.5 oz of 25.5 oz. Write and solve an equation to represent the maximum and minimum weight allowed per dozen large eggs.

12) Malek scored an average of 9.5 points per game last season. This season, Malek has been 1.5 points above and below this average. Write and solve an equation to represent the fewest and the most points Malek has had this season.

p: points per game

$|p - 9.5| = 1.5$
$p = 8, 11$

Malek scored 8 points or 11 points in a game this season.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1–6) Remind your student to set up both cases before solving. Otherwise they may only solve for 1 solution in each equation.

2) Encourage your student to check both solutions. After problem 1, they may be tempted to write the same number with a positive and negative sign rather than solving for Case 2 correctly.

4) If your student prefers the variable on the left side of the equation, remind them that the symmetry property allows them to rewrite their equation in this way. Your student also may need to be reminded that the equation should be split into two cases after the absolute value expression is isolated on one side of the equation.

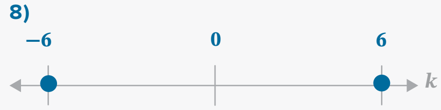
7)

The midpoint means that the number should be included on the number line but not marked with a point since it is not one of the solutions.

Q: Why is the midpoint a negative number?

A: *Because the formula has a subtraction sign and* $-(-10)$ *is* $+10$.

8)

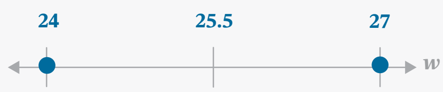


9)

10)

11) Let w = weight of the eggs in ounces
$|w - 25.5| = 1.5$
$w = 24, 27$

The maximum weight allowed is 27 oz.
The minimum weight allowed is 24 oz.



Remind your student to think about what the midpoint is and what number will be added/subtracted from that number.

Mastery Check

Show What You Know

There is extraneous information in the question. Your student may need help understanding that not all numbers in the word problem will be used to write an equation.

A) If your student needs help, have them draw a number line and decide which number will be the midpoint and what number represents the distance from the midpoint.

B) Writing or saying the answer in a sentence will help your student acknowledge if their answer makes sense.

C) If your student is having trouble with this part of the problem, refer them to the Guided Notes Video: *Writing Absolute Value Equations.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- You will be able to solve one- and two-step absolute value equations.
- You will be able to graph one- and two-step absolute value equation solutions on a number line.
- You will be able to explain why an absolute value equation can have two solutions.
- You will be able to write and solve an absolute value equation from a given problem.

Mastery Check

Show What You Know

Reilly's Tackle and Fishing Rentals was holding a rockfish fishing contest. The first rule stated that the rockfish caught must be 12.25 pounds. The second rule of the contest was that the rockfish must be 15.75 inches (plus or minus 1.25 inches).

Write an equation to show the shortest length and the longest length of the rockfish. Define your variable.

A) Write an equation to show the shortest length and the longest length of fish. Define your variable.

r: length of the rockfish

$|r - 15.75| = 1.25$

B) Solve your equation from Part A.

$|r - 15.75| = 1.25$

$r - 15.75 = 1.25$ **OR** $-(r - 15.75) = 1.25$

$r = 17$ $\quad$ $r - 15.75 = -1.25$

$r = 14.50$

The smallest rockfish will be 14.50 inches.
The largest rockfish will be 17 inches.

C) Write a formula that will work for any length of rockfish but still uses plus or minus 1.25.

$|r - (\text{length})| = 1.25$

r: length of the rockfish

length: the length determined by Reilly's Tackle and Fishing Rentals

1.25: the distance between the midpoint and highest or lowest length of the rockfish

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

PRACTICE 2 3A

Practice 2

Complete the problems on a separate sheet of paper.

Solve. Remember to check your work.

1) $|-d| = 2$ $\mathbf{d = -2, 2}$

2) $\left|\frac{1}{2}x + 6\right| = 3$ $\mathbf{x = -18, -6}$

3) $|2r - 7| = 3$ $\mathbf{r = 2, 5}$

4) $3|x - 6| = 4$ $\mathbf{x = \frac{14}{3}, \frac{22}{3}}$

5) $4|z + 5| = 16$ $\mathbf{z = -9, -1}$

6) $\left|\frac{5}{4}x + 3\right| = 0$ $\mathbf{x = -\frac{12}{5}}$

Graph the solutions and note the midpoint on a number line.

7) $6 = \left|\frac{3}{4}x\right|$

8) $|x - (-1)| = 1$

9) $|-2x - 1| = 5$ $\mathbf{x = -3, 2}$ **Midpoint =** $\mathbf{-\frac{1}{2}}$

10) $|4r + 3| = 7$ $\mathbf{r = -\frac{5}{2}, 1}$ **Midpoint =** $\mathbf{-\frac{3}{4}}$

Solve. Remember to check your work.

11) A group of anglers were catching fish. The weight of the fish must be within 2 pounds of the standard weight to be sold at market. If the standard weight is 23 pounds, what is the minimum and maximum the fish can weigh and still be sold? $\mathbf{w = 21, 25}$ **The minimum weight is 21 pounds and the maximum weight is 25 pounds.**

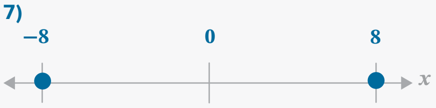
12) Susanna was playing in a golf tournament. Par for the course is 65. She was 7 strokes from par. What were Susanna's possible scores? $\mathbf{g = 58, 72}$ **Susanna's score was either 58 or 72.**

Practice 2

Worked solutions for these problems are located in the Digital Pack.

6) The equation has one solution because the opposite of 0 is 0.

7)



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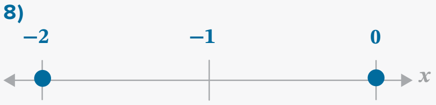

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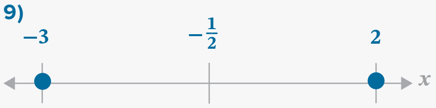

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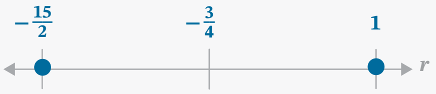


If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Because absolute value equations can also have no solutions or all real numbers it is important your student remembers what this means before starting the lesson. (From Lesson 2)

Part B: Multi-Step Absolute Value Equations

Objectives

In this part of the lesson, you will learn about multi-step absolute value equations.

By the end of this lesson, you will be able to do the following:

- Solve multi-step absolute value equations.
- Graph multi-step absolute value equation solutions on a number line.
- Determine when an absolute value equation has no solution or is an identity.

Why?

Understanding the different types of solutions to multi-step absolute value equations is important to determining if your solutions make sense.

Warm Up

1) What does it mean when the solution to an equation is all real numbers?

Sample:
This means that any real number will make the equation true.

2) What does it mean for an equation to have no solutions?

Sample:
An equation with no solutions has no answer OR no number will make the equation true.

Multi-Step Absolute Value Equations

- Solving multi-step absolute value equations is similar to solving equations without **absolute values (| |)**.
- Solving multi-step absolute value equations involves isolating the absolute value bars and then the **variable**.

EXPLORE 3B

Example 1

Solve for x.
$3|2x-4|=12$

Plan Determine the operations occurring to the variable.

$\cdot 2$ (inside)
-4 (inside)
$|\ |$ (absolute value)
$\cdot 3$ (outside)

To solve for x, follow the operations in backwards order using the inverse operations.

Implement	Explain
$3\|2x-4\|=12$	◀ Given
$\|2x-4\|=4$	◀ Multiplication Property of Equality

Case 1:		Case 2:	
$2x-4=4$	OR	$-(2x-4)=4$	◀ Definition of Absolute Value
	OR	$2x-4=-4$	◀ Multiplication Property of Equality (Case 2 only)
$2x=8$	OR	$2x=0$	◀ Addition Property of Equality
$x=4$	OR	$x=0$	◀ Multiplication Property of Equality

Check

Case 1:	Case 2:
$x=4$	$x=0$
$3\|2(4)-4\|=12$	$3\|2(0)-4\|=12$
$3\|8-4\|=12$	$3\|0-4\|=12$
$3\|4\|=12$	$3\|-4\|=12$
$3(4)=12$	$3(4)=12$
$12=12$ ✓	$12=12$ ✓

Example 2

Multiply both sides by $-\frac{1}{2}$.

Subtract 6 from both sides.

3B EXPLORE

Example 2

Solve.

$11 - 2|x + 6| = -51$

It is important to remember when solving that the absolute value bars must be alone on one side of the equation before the problem can be split into Case 1 and Case 2.

Plan Determine what is happening to the variable. Then work backwards.

+6 (Inside the absolute value bars, 6 is added.)
| | (The absolute value of x is taken.)
$\cdot(-2)$ (The absolute value of x is multiplied by -2.)
+11 (Outside the absolute value bars, 11 is added.)

Implement	Explain
$11 - 2\|x + 6\| = -51$	Given
$-2\|x + 6\| = -62$	Addition Property of Equality
$\|x + 6\| = 31$	Multiplication Property of Equality

Case 1:		Case 2:	
$x + 6 = 31$	OR	$-(x + 6) = 31$	
		$x + 6 = -31$	Multiplication Property of Equality
$x = 25$	OR	$x = -37$	Addition Property of Equality

Check

$x = 25$	$x = -37$
$11 - 2\|(25) + 6\| = -51$	$11 - 2\|(-37) + 6\| = -51$
$11 - 2\|31\| = -51$	$11 - 2\|-31\| = -51$
$11 - 2(31) = -51$	$11 - 2(31) = -51$
$11 - 62 = -51$	$11 - 62 = -51$
$-51 = -51$ ✓	$-51 = -51$ ✓

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Having your student think about all of the operations that are happening to the variable can help them develop more efficient problem solving approaches. It is helpful to write this out (shorthand is fine) so the inverse operation can be performed.

Q: What is the goal before writing the two cases?

A: *The goal is to isolate the absolute value expression on one side of the equation.*

☑ Checkpoint

Say or write the steps occurring to the variable. Solve.

$5\left|\frac{1}{5}x - 2\right| + 3 = 23$

Plan

$\cdot\frac{1}{5}$ (inside)
-2 (inside)
| | (absolute value)
$\cdot 5$ (outside)
$+3$ (outside)

Implement

$5\left|\frac{1}{5}x - 2\right| + 3 = 23$

$5\left|\frac{1}{5}x - 2\right| = 20$

$\frac{1}{5}x - 2 = 4$ OR $-\left(\frac{1}{5}x - 2\right) = 4$

$\frac{1}{5}x = 6$ $\quad$ $\frac{1}{5}x - 2 = -4$

$x = 30$ $\quad$ $\frac{1}{5}x = -2$

$x = -10$

EXPLORE 3B

Absolute Value Equations with No Solution and All Real Numbers

- The absolute value of a number will always be greater than or equal to zero because **distance** is a **positive** number. It is not possible to travel a negative number of units.
- It is important to make sure the absolute value expressions are equal to a **non-negative** number before solving for the two cases.
- Absolute value expressions that are set equal to a negative number have **no solution**.

Example 3

Solve.

$-3|x+4| = 15$

$\frac{-3|x+4|}{-3} = \frac{15}{-3}$ ◂ Multiplication Property of Equality

$|x+4| = -5$

no solution **There is no value of x that will make the absolute value expression equal to –5.**

- When any value will make an absolute value equation true, the solution is $\mathbb{R}$.

Example 4

Solve. Graph the solution(s) on the number line.

$|3x - 3x| = 5(2-2)$

Implement	Explain
$\lvert 3x - 3x\rvert = 5(2-2)$	◂ Given
$\lvert 0\rvert = 5(0)$	◂ Simplify expressions
$0 = 0$	◂ This absolute value equation does not need to be separated into Case 1 and Case 2 because zero is the opposite of zero.
all real numbers, $\mathbb{R}$	

Checkpoint

Solve.

$\frac{1}{2}|x-1| + 5 = 1$

$\frac{1}{2}|x-1| = -4$

$|x-1| = -8$

no solution **The absolute value expression cannot be equal to a negative value.**

Example 4

The graph of this equation is a number line with every point marked on the number line. Again, labeling values on the number line is optional when the solution is all real numbers.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Why does an absolute value expression need to be set equal to a positive number or positive constant?

A: *Because the constant represents distance and distance cannot be negative.*

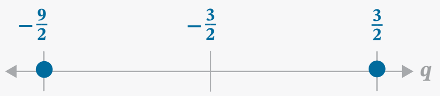
Practice 1

Worked solutions for these problems are located in the Digital Pack.

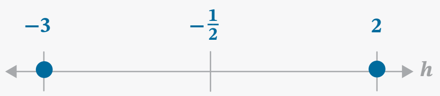
1–3)
Remind your student that the absolute value expression should be isolated on one side of the equation before the two cases are written.

4) Remind your student to note the midpoint but not to mark this on the graph since it is not one of the solutions.

5)



6)



Your student should write the height using the same unit (inches preferable). The equation is difficult to work with when the measurements are a combination of feet and inches. Encourage the use of the formula sheet if your student needs to recall the number of inches in a foot.

7–13)
These could be graphed on a number line to give a visual to the number of possible solutions. In 7) $x = 0$ has one solution because zero is neither positive nor negative.

Practice 1

Complete the problems on a separate sheet of paper.

Solve.

1) $|3x-7|+5=11$ $\quad x=\frac{1}{3}, \frac{13}{3}$

2) $-\left|\frac{4}{5}x+8\right|=-1$ $\quad x=-\frac{45}{4}, -\frac{35}{4}$

3) $\frac{1}{2}|3x-4|+5=6$ $\quad x=\frac{2}{3}, 2$

Graph the solutions and note the midpoint on a number line.

4) $-2\left|\frac{3}{2}x-5-\frac{1}{2}x\right|=-8$ $\quad x=1, 9$ Midpoint $=5$

5) $-\frac{1}{2}|q+8+3q-2|=-6$ $\quad q=-\frac{9}{2}, \frac{3}{2}$

6) The average height of men in the Coes family is 5 feet 11 inches. The Coes have two sons within 3 inches of the family average for the men. What is the shortest possible height of the sons? The tallest possible height? Write an equation, solve, and graph. $h = 68, 74$ **The shortest the sons can be is 68 inches or 5 feet 8 inches. The tallest is 74 inches or 6 feet 2 inches.**

Determine if the variables in the equations below have no solution, some (one or two) solutions, or is an identity.

7) $|x|=0$ **one solution**

8) $|2x|=|-2x|$ **identity**

9) $|x|=-1$ **no solution**

10) $4|z+5|+2=18$ **two solutions**

11) $12=-5|y|+7$ **no solution**

12) $-|x+15|=26$ **no solution**

13) $3|7-2x+2x-3|-1=11$ **identity or all real numbers, $\mathbb{R}$**

Explain why the given equation is an identity.

14) $|2x+4-2x|=4$

The variable simplifies out of the absolute value bars (additive inverse) making the equation $|4|=4$, which is always true regardless of the value of the variable.

Mastery Check

Show What You Know

Complete each part of the problem.

A) Solve.

$\frac{4}{3}(2x-6)+16=0$

$\frac{4}{3}(2x-6)=-16$

$2x-6=-12$

$2x=-6$

$x=-3$

B) Solve.

$\frac{4}{3}|2x-6|+16=0$

$\frac{4}{3}|2x-6|=-16$

$|2x-6|=-12$

no solution

C) How does changing the grouping symbols in Part A ( ) to absolute bars | | in Part B affect the solution(s)? Explain.

Part A has one solution, $x=-3$. Part B has no solution.

When the parentheses change to absolute value bars, there are more restrictions on solving. Absolute value expressions cannot be set equal to a negative number because distance cannot be negative.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

C) Part C is a critical comparison. This should help your student understand that the absolute value bars can significantly change the solution(s).

If your student ignores the negative number and proceeds to solve the two cases, have them substitute their values in to check their work. They should see that none of the values will be true for the absolute value equation.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- You will be able to solve multi-step absolute value equations.
- You will be able to graph multi-step absolute value equation solutions on a number line.
- You will be able to determine when an absolute value equation has no solution or is an identity.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer “yes” to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

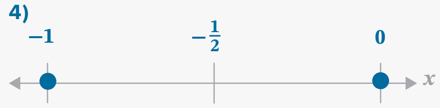
If your student cannot answer “yes” to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

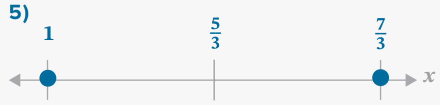
Practice 2

Worked solutions for these problems are located in the Digital Pack.

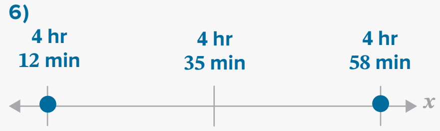
4)



5)



6)



If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Practice 2

Complete the problems on a separate sheet of paper.

Solve.

1) $-\frac{2}{3}\left|\frac{1}{4}x+2\right|=-2$ **$x=-20, 4$**

2) $-|8x-3|=-13$ **$x=-\frac{5}{4}, 2$**

3) $\frac{1}{10}|9-2x|-3=0$ **$x=-\frac{21}{2}, \frac{39}{2}$**

Solve. Draw a graph that includes the solutions and the midpoint.

4) $|2x-3-6x+1|=2$ **Midpoint $-\frac{1}{2}$**

5) $-\frac{3}{2}|5-3x|+1=-2$ **Midpoint $=\frac{5}{3}$**

6) The average time to read a 300-page novel in Mr. Webber's class is 4 hours and 35 minutes. If all of the students are within 23 minutes of the average, what is the shortest time to read the novel? The longest time? Write an equation, solve, and graph on a number line.

m = 23 min + 4 hr 35 min = 4 hr 58 min OR m = 4 hr 12 min

Determine if the variables in the equations below have no solution, some (one or two) solutions, or is an identity.

7) $|2x-2x|=0$ **identity**

8) $-\left|\frac{1}{3}x\right|=-2$ **two solutions**

9) $|x-x-12|=11$ **no solution**

10) $9|2q-2q|-4=-4$ **identity or all real numbers**

11) $3|2x+3-x|+4=3-2$ **no solution**

12) $|2r-7|=5$ **two solutions**

13) $-\frac{1}{2}|q+6+3q|+3=6$ **no solution**

Explain why the given equation has no solution.

14) $|5x+3|=-12$

The equation has no solution because an absolute value equation cannot equal a negative value since distance is always positive.

TARGETED REVIEW 3

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Solve and write if the equation has one solution, no solution, or a solution of all real numbers.

1) $25 = 9 - 8(x - 9)$ $x = 7$ **one solution**

2) $\frac{2}{3}(x + 9) = 3x - (2x - 6)$ $x = 0$ **one solution**

3) $\frac{1}{2}(12x - 8) = 6x + 3$ $-4 = 3$ **no solution**

4) $\frac{1}{4}x + 5 = \frac{2}{5}x - 1$ $x = 40$ **one solution**

5) Solve for the indicated variable.
$V = \frac{lwh}{3}; w$ $w = \frac{3V}{lh}$

6) Briley bought three bottles of water and a tube of lip balm. The lip balm cost \$2.00. If Briley paid \$9.05 for the water and lip balm, how much did each bottle of water (w) cost? Write and solve the equation.

$w = 2.35$ **Each bottle of water cost \$2.35.**

Evaluate. Then name all sets of numbers to which the value belongs.

7) $\frac{0.3}{0.05}$ 6; $\{\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

8) $-\sqrt{16}$ -4; $\{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

9) Write the values from the set $\{-2, 0, 2, 15\}$ that are solutions for the inequality $x \le 2$. $-2, 0, 2$

10) Use your formula sheet to find the value of the expression 3^0. 1

Multiple Choice

A **11)** Which irrational number when squared will still be an irrational number?

A) π

B) $\sqrt{5}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{7}}{\sqrt{3}}$

D **12)** Determine the number of solutions to the given equation.
$8x - 6x + 3 = \frac{1}{4}(8x + 12)$

A) no solution

B) one solution

C) two solutions

D) **all real numbers**

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

5) This is the volume formula for a rectangular pyramid.

6) If your student does not use the variable w, they can still solve for the correct answer. Encourage them to read the problem carefully so they use the suggested variable.

7) 6 is natural, whole, integer, rational, real or $\{\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

Evaluate means find the value (or the number).

(Hint: Try multiplying the numerator and denominator by 100 to eliminate the decimals.)

8) -4 is integer, rational, real or $\{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

10) A base raised to the power of 0 is equal to 1.

11) Distractor Rationale:

B) The square root of 5 squared is equal to 5.

C) This number squared is $\frac{3}{4}$.

D) This number squared is $\frac{7}{3}$.

12) Distractor Rationale:

A) This happens when an equation has no variables and unequal numbers.

B) The coefficients of the variables are equal on both sides so one solution is not possible.

C) This is not possible for the given equation.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Lesson Origin	2	2	2	2	2	2	1	1	PA	PA	1	2