Lesson 2
Solving Equations

Outline

Part A
Multi-Step Equations

- Solving Multi-Step Equations
- Variables on Both Sides of the Equation
- Defining Variables in Word Problems
- Solving Equations with More than One Variable

Part B
Equations with Special Cases

- Rewriting Equations with Integer Coefficients
- One Solution, No Solution, or All Real Numbers

Targeted Review

Vocabulary

- no solution
- identity

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Inverse operations are the foundation of solving equations and inequalities.

1) Q: Instead of dividing you can also multiply by the ____________.

A: *reciprocal*

Q: What property is this?

A: *inverse*

Part A: Multi-Step Equations

Objectives

In this part of the lesson, you will learn about multi-step equations.

By the end of this lesson, you will be able to do the following:

- Solve a multi-step equation.
- Use substitution to prove your solutions are correct.
- Write a single-variable equation when given a problem with defined variables.
- Solve for a specific variable when given an equation that has more than one variable.

Why?

Solving for a variable in multi-step equations is a foundation for algebra. Mastering multi-step equations is an important skill for success in secondary math.

Warm Up

1) Label the equation with the words coefficient, expression, and variable.

coefficient (4) **variable** (n)

$$4n + 1 = 7$$

expression ($4n + 1$)

2) Name the inverse operation.

A) The inverse of division is **multiplication**.

B) The inverse of addition is **subtraction**.

Did you remember all the parts of an equation? What about inverse operations?

Having a solid foundation of basic algebraic language will help as you build on that knowledge.

Solving Multi-Step Equations

- When you solve problems in Algebra 1, **Plan** is how you will approach the problem.
- During the Implement stage of the problem solving method in Algebra 1, you will **implement** your plan and **check** your work.
- When solving a problem, you should be able to **explain** why your answer makes sense.

EXPLORE 2A

- **Inverse operations** are used to solve equations by rearranging terms on both sides of the equals sign (=).
- When solving for a variable in an equation, the goal is to **isolate** the variable.

Example 1

Use the Plan, Implement, Explain method to solve the equation. Check your solution.

$-5x+8=12$

Plan $\cdot(-5)$, $+(8)$ — Move backwards through the plan, using inverse operations.

Implement	Explain	Check
$-5x+8=12$		$-5x+8=12$
$-8 \quad -8$	Addition Property of Equality	$-5\left(-\frac{4}{5}\right)+8=12$
$-5x=4$		$4+8=12$
$\left(-\frac{1}{5}\right)(-5x)=\left(-\frac{1}{5}\right)(4)$	Multiplication Property of Equality	$12=12$ ✓
$x=-\frac{4}{5}$		

Example 2

Solve. The Plan portion is completed for you.

$\frac{1}{3}(2x+7)=-4$

Plan $\cdot(2)$, $+(7)$, $\cdot\left(\frac{1}{3}\right)$ — Work backwards through the steps to isolate x.

Implement	Explain	Check
$\frac{1}{3}(2x+7)=-4$		$x=-\frac{19}{2}$
$(3)\left(\frac{1}{3}\right)(2x+7)=(3)(-4)$	Multiplication Property of Equality	$\frac{1}{3}\left(2\left(-\frac{19}{2}\right)+7\right)=-4$
$2x+7=-12$		$\frac{1}{3}(-19+7)=-4$
$-7 \quad -7$	Addition Property of Equality	$\frac{1}{3}(-12)=-4$
$2x=-19$		$-4=-4$ ✓
$\left(\frac{1}{2}\right)(2x)=\left(\frac{1}{2}\right)(-19)$	Multiplication Property of Equality	
$x=-\frac{19}{2}$		

Example 1

If your student explains their problem out loud they would say, “I subtracted 8 from both sides using the Addition Property of Equality. Then I multiplied both sides by the reciprocal negative one-fifth using the Multiplication Property of Equality.” In this way, your student tells you what is happening and why.

Example 2

Example 2 shows a method to solve an equation without distributing. Sometimes distributing a fraction can make the equation more challenging to solve because more terms will end up containing a fraction. If you work backwards, then the fraction is cleared from the equation in the first step.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Your student may distribute and then solve the equation. This approach is also correct. However, this would also result in more terms having fractions.

Q: What can you do first to prevent distributing the fraction?

A: *Multiply both sides by the reciprocal.*

2A EXPLORE

Example 3

Solve.

$\frac{2}{5}(1-3q)=6$

Plan

$\cdot(-3)$
$+(1)$
$\cdot\left(\frac{2}{5}\right)$

Implement / **Explain**

$\frac{2}{5}(1-3q) = 6$

$\left(\frac{5}{2}\right)\left(\frac{2}{5}\right)(1-3q) = (6)\left(\frac{5}{2}\right)$ ◂ Multiplication Property of Equality

$1-3q = 15$

$-1 \quad -1$ ◂ Addition Property of Equality

$-3q = 14$

$\left(-\frac{1}{3}\right)(-3q) = (14)\left(-\frac{1}{3}\right)$ ◂ Multiplication Property of Equality

$q = -\frac{14}{3}$

Checkpoint

Plan, Implement, and Explain to solve.

$\frac{4}{5}(2x+3)=12$

Plan

$\cdot 2$
$+3$
$\cdot\frac{4}{5}$

Move backwards through the plan, using inverse operations.

Implement and Explain

$\frac{4}{5}(2x+3) = 12$

$\frac{5}{4}\left(\frac{4}{5}\right)(2x+3) = \left(\frac{5}{4}\right)(12)$ ◂ Multiplication Property of Equality

$2x+3 = 15$

$2x = 12$ ◂ Addition Property of Equality

$\left(\frac{1}{2}\right)(2x) = \left(\frac{1}{2}\right)(12)$ ◂ Multiplication Property of Equality

$x = 6$

EXPLORE 2A

Variables on Both Sides of the Equation

- As you become more confident using Plan, Implement, Explain to solve multi-step equations, you will likely no longer need to write out the **plan** stage.
- Equations can be solved in more than one way, but the **solution** will be the same.
- If a variable is found on both sides of an equation, first determine the side of the **equation** on which you want to keep the variable.

Example 4

Solve.

$8(2x-1)+6=11x+7-4$

Implement	Explain
$8(2x-1)+6=11x+7-4$	
$16x-8+6=11x+3$	Distribute and simplify terms
$16x-2=\ \ 11x+3$	
$-11x \qquad -11x$	Addition Property of Equality
$5x-2=\ \ 3$	
$+2 \quad +2$	Addition Property of Equality
$\left(\frac{1}{5}\right)(5x)=\left(\frac{1}{5}\right)(5)$	Multiplication Property of Equality
$x=1$	

Check

$8(2(1)-1)+6=11(1)+7-4$

$8(2-1)+6=11+7-4$

$8+6=18-4$

$14=14$ ✓

Variables on Both Sides of the Equation

Explain can also sometimes be done in the student's head or verbally rather than being written out. However, the student should always be prepared to explain their answers.

Example 5

Since $5x$ is a larger positive coefficient, solve in terms of the variable on the right. In this case, the variables need to be moved to one side of the equation before the equation can be solved.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the first step in this equation?

A: *Distribute −2*

If your student says 2, remind them that the sign in front of the term determines if it is positive or negative. If their answer is not $x = 7$, most likely they did not distribute −2 to both terms in the parentheses.

2A EXPLORE

Example 5

Solve.

$3x + 8 = 5x + 4$

Implement	Explain	Check
$3x + 8 = 5x + 4$		$3x + 8 = 5x + 4$, when $x = 2$
$-3x \quad -3x$	Addition Property of Equality	$3(2) + 8 = 5(2) + 4$
$8 = 2x + 4$		$6 + 8 = 10 + 4$
$-4 \quad -4$	Addition Property of Equality	$14 = 14$ ✓
$4 = 2x$		
$\left(\frac{1}{2}\right)(4) = \left(\frac{1}{2}\right)(2x)$	Multiplication Property of Equality	
$2 = x$		
$x = 2$	Reflexive Property	

Checkpoint

Solve.

$11x - 2(x + 6) = 51$

$$11x - 2(x+6) = 51$$
$$11x - 2x - 12 = 51$$
$$9x - 12 = 51$$
$$+12 \quad +12$$
$$9x = 63$$
$$\left(\frac{1}{9}\right)(9x) = \left(\frac{1}{9}\right)(63)$$
$$x = 7$$

Defining Variables in Word Problems

- **Variables** allow formulas to be written to generalize the way that numbers relate to one another.
- When solving word problems that do not have a standard formula, the **person** who is solving the problem **defines** them.

EXPLORE 2A

Example 6

Define a variable, then write and solve your equation.

To make sure a program was correct, the first scientist ran 48 initial experiments. After that, the first scientist ran three experiments every day. A second scientist ran 15 initial experiments of a program. After that, the second scientist ran four experiments per day. How many days will it take for both scientists to run the same number of experiments?

d: days experiments are running.

$$48 + 3d = 15 + 4d$$
$$-3d \quad -3d$$
$$48 = 15 + d$$
$$-15 \quad -15$$
$$33 = d$$

Check

$$48 + 3(33) = 15 + 4(33)$$
$$48 + 99 = 15 + 132$$
$$147 = 147 \checkmark$$

Explain

It will take 33 days until the experiments are equal.

For word problems, Explain is more about the meaning of the answer and less about justifying using the algebraic properties. By answering with a sentence that resolves the problem, you complete the Explain step.

Example 7

Write and solve an equation to find the three integers.

The sum of three consecutive integers is −57.
Write and solve an equation to find the three integers.

In this problem, consecutive means three numbers in order, for example 1, 2, 3.

n: first number
$n + 1$: second number
$n + 2$: third number
$n + (n + 1) + (n + 2) = -57$

$$3n + 3 = -57$$
$$3n + 3 - 3 = -57 - 3$$
$$3n = -60$$
$$\left(\frac{1}{3}\right)(3n) = \left(\frac{1}{3}\right)(-60)$$
$$n = -20$$

$n = -20$ **This is the value of n, but not the answer to the question. You must apply negative twenty to the original statements about n.**

$$n + 1 = (-20) + 1 = -19$$
$$n + 2 = (-20) + 2 = -18$$

Check

$$-20 + -19 + -18 = -57 \checkmark$$

In this course, answers are usually written from least to greatest.

Example 6

Using the variable s is not the best choice because it can look like a 5. Other variables to avoid if possible include: b, e, i, l, o, s, t. These variables all look like other numbers. Sometimes even b is not clear, as it could be confused with the number 6.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What are some common variables used in Algebra?

A: x, n

Q: What letters might you want to avoid so you do not confuse a number with a variable?

A: b, e, i, l, o, s, t

Example 8

When the directions say "solve in terms of ____" your goal is to isolate that variable.

Checkpoint

Why is it important to define the variable or variables you use when writing an equation?

Sample:
Defining the variables shows what you are solving for without the original problem being there.

Solving Equations with More than One Variable

- When equations have more than one variable, the goal shifts from solving for an exact value to ____**isolating**____ a specific variable.
- Unless specified, when solving equations that have more than one variable in this course, all variables are ____**non-zero**____.
- The directions, "Solve in terms of x," means to isolate, or get x ____**by itself**____ on one side of the equation.
- Another way to represent isolating variables is $3x + 4 = 5y; x$

  The variable after the semicolon is what needs to be isolated.

Example 8

Solve the equation in terms of y.

$Ax + By = C$

Plan $\cdot (B)$
$+ Ax$

Implement	Explain
$Ax + By = C$	
$-Ax \qquad -Ax$	Addition Property of Equality
$By = C - Ax$	
$\left(\frac{1}{B}\right)(By) = \left(\frac{1}{B}\right)(C - Ax)$	Multiplication Property of Equality
$y = \frac{C - Ax}{B}$	
$y = \frac{-Ax + C}{B}$	Commutative Property
OR $y = \frac{-Ax}{B} + \frac{C}{B}$	

EXPLORE 2A

Example 9

Solve the equation in terms of b.

$\frac{m-a+b}{5} = 3c$

Plan $-(a)$

$+(m)$

$\div(5)$

Implement	Explain
$(5)\frac{m-a+b}{5} = (5)(3c)$	◂ Multiplication Property of Equality
$m-a+b = 15c$	
$-m \qquad -m$	◂ Addition Property of Equality
$+a \qquad +a$	◂ Addition Property of Equality
$b = 15c-m+a$	
$b = a+15c-m$	◂ Commutative Property

Example 10

Solve.

$\frac{1}{3}(2R-r)=x; R$

Plan Solve for R

$\cdot(2)$

$-(r)$

$\cdot\left(\frac{1}{3}\right)$

Implement

$\frac{1}{3}(2R-r) = x$	
$\left(\frac{3}{1}\right)\left(\frac{1}{3}\right)(2R-r) = (x)\left(\frac{3}{1}\right)$	◂ Multiplication Property of Equality
$2R-r = 3x$	
$+r \qquad +r$	◂ Addition Property of Equality
$2R = r+3x$	◂ Commutative Property
$\left(\frac{1}{2}\right)(2R) = (3x+r)\left(\frac{1}{2}\right)$	◂ Multiplication Property of Equality
$R = \frac{r+3x}{2}$ OR $R=\frac{r}{2}+\frac{3x}{2}$	

Checkpoint

Max wanted to use the formula for the area of a triangle to find the height when given the area and the base. Write the formula and solve in terms of h.

$A = \frac{1}{2}bh$

$(2)(A) = (2)\left(\frac{1}{2}\right)bh$

$\left(\frac{1}{b}\right)2A = \left(\frac{1}{b}\right)bh$

$h = \frac{2A}{b}$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Encourage your student to reference their Formula Sheet.

Q: Where can you find the formula for the area of a triangle?

A: *The Formula Sheet.*

Q: What operation is being performed in the given formula?

A: *Multiplication.*

Q: What is the inverse of this operation?

A: *Divide or multiply by the reciprocal.*

Practice 1

Worked solutions for these problems are located in the Digital Pack.

Encourage your student to read the question out loud to hear what is happening to the variable. Sometimes hearing the question out loud helps your student determine what steps to take to solve. Using the Formula Sheet will help justify the steps while still focusing on solving the problem rather than remembering all of the properties.

Q: How can you move terms from one side of the equation to the other?

A: *Use inverse operations.*

2) Remember that equations can be solved in more than one way. If the solution is the same, then both methods of solving are correct.

5) Q: What are like terms on the left-side of the equation in problem 5?

A: *$-2b$ and $-b$, as well as 5 and 3.*

Q: All of the variables are on one side. Do you use inverse operations to combine them? Explain.

A: *No, when they are on the same side, you follow the indicated operation.*

7) $x = 2$
width: $x + 4 = (2) + 4 = 6$
length: $2x + 3 = 2(2) + 3 = 4 + 3 = 7$
The width of the rectangle is 6 inches and the length of the rectangle is 7 inches.

Encourage your student to label ALL sides of the figure to make sure they combine terms correctly. They can also use the formula for perimeter of a rectangle, $P = 2l + 2w$.
If they have $3x + 7$, your student ignored the sides that were not labeled.
Your student needs to use the value of x to find the side lengths to answer the question completely.

8) Emily and Reggie each mowed two lawns.

9) The three numbers are $-25, -26, -27$.

10) Your student should solve for the variable noted after the semicolon.

Encouraging the use of an equation rather than guessing and checking is critical for this type of problem. Once your student can set up the equation, they can solve any problem of this type.

If your student struggles to write the equation in terms of x, have them replace some of the variables with numbers and then solve. Then go back and do the same steps with the original equation to see that the steps will remain the same.

12) Encourage your student to use Plan and think about terms rather than individual variables and numbers. There are only 3 terms in this problem, SA, $2\pi r^2$, and $2\pi r$. Your student may further simplify the fraction to $h = \frac{SA}{2\pi r} - r$

Practice 1

Complete the problems on a separate sheet of paper.

Solve. Show Plan, Implement, Explain.

1) $\frac{x}{4} - 8 = -3$ $\quad x = 20$

2) $\frac{1}{4}(3x - 5) = -1$ $\quad x = \frac{1}{3}$

Solve.

3) $2(b - 4) + 5 = 9$ $\quad b = 6$

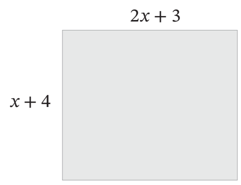
4) $4v = 2v - 7 + 5$ $\quad v = -1$

5) $-2b + 5 - b + 3 = -12$ $\quad b = \frac{20}{3}$

6) $7(q + 2) = 9q + 5$ $\quad q = \frac{9}{2}$

Define a variable and write an equation for the given scenario. Solve your equation.

7) The perimeter of the rectangle equals 26 inches.
What is the length of the rectangle?
What is the width of the rectangle?



8) Emily and Reggie were comparing how much money they had each made from mowing lawns that week. Emily charged \$15 per lawn, and she made \$5 in tips. Reggie charged \$10 per lawn, and made \$15 in tips. After everything was counted, they discovered they made the same amount of money and each mowed the same amount of lawns. How many lawns did they each mow?

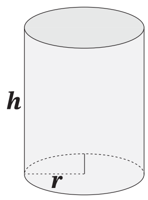
9) Three consecutive integers have a sum of -78. Write and solve an equation to find the three integers.

Solve in terms of the named variable. Remember to write your Plan.

10) $qx - 4 = c; x$ $\quad x = \frac{c+4}{q}$

11) $d = rt; r$ $\quad r = \frac{d}{t}$

12) The formula for the surface area of a cylinder is shown below. Solve the formula in terms of h.



$SA = 2\pi r^2 + 2\pi rh$

$h = \frac{SA - 2\pi r^2}{2\pi r}$

MASTERY CHECK 2A

Mastery Check

Show What You Know

A) Find the largest possible value for x by filling in the boxes using the natural numbers 1 through 9 and then solving for x.

1 2 3 4 5 6 7 8 9

- You can use each number only once in the equation.
- Repeat the process until you have the largest value for x.
- Show all of your work.
- (Hint: Do not erase. Rather, keep a log of all attempts.)

$$\frac{1}{\square}\left(x+\square\right)=\square$$

$$\frac{1}{9}(x+1) = 8$$
$$x = 71$$

$$\frac{1}{8}(x+1) = 9$$
$$x = 71$$

B) Explain why you believe your answer to x is the largest possible value.

Sample:
The denominator of the fraction will multiply the number on the other side of the equation. The two biggest numbers should be multiplied together to get the largest product. Then the smallest number should be in the parentheses (1) so when it is subtracted the value for x only decreases by 1.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Mastery Check

Show What You Know

Here are some samples of solutions that do not result in the largest value for x. Your student will need to use the largest and smallest of the numbers (1, 8, 9) to find the biggest value.

$$\frac{1}{2}(x+9)=8$$
$$x=7$$

$$\frac{1}{5}(x+2)=9$$
$$x=47$$

This problem may be challenging but will help determine some misconceptions your student may have about solving equations and how numbers relate to one another. Your student should be making multiple attempts for this problem. Encourage them to try different approaches to see if their understanding of equations is true. The goal is to see how the terms relate to one another, not to get the right answer on the first try.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- You will be able to solve a multi-step equation.
- You will be able to use substitution to prove your solutions are correct.
- You will be able to write a single-variable equation when given a problem with defined variables.
- You will be able to solve for a specific variable when given an equation that has more than one variable.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

3) In this case your student can write the solution as a decimal because dividing by 10 makes this more efficient than simplifying the fraction. If your student prefers, they can also simplify $\frac{58}{10}$ to $\frac{29}{5}$.

9) Even integers being $n + 2$, $n + 4$

12) Your student may wish to further simplify the solution to $l = \frac{SA}{2s} - \frac{s}{2}$

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Practice 2

Complete the problems on a separate sheet of paper.

Solve. Show Plan, Implement, Explain.

1) $\frac{2}{3}x + 15 = -1$ $\quad \boldsymbol{x = -24}$

2) $\frac{5}{9}(x - 18) - 3 = 2$ $\quad \boldsymbol{x = 27}$

Solve.

3) $5(2x + 3) - 8 = 65$ $\quad \boldsymbol{x = \frac{58}{10} = 5.8}$

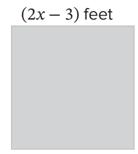
4) $\frac{3}{2}x - 11 = x - 4$ $\quad \boldsymbol{x = 14}$

5) $17 = -12x + 3 - 1 - 3x$ $\quad \boldsymbol{x = -1}$

6) $16(2x - 1) + 4 = -12$ $\quad \boldsymbol{x = 0}$

Define a variable and write an equation for the given scenario. Solve your equation.

7) The perimeter of a square is 96 feet. Write an equation using the square to find the value of x and the length of each side. Write your answers as decimal values. **A side length of the square is 24 feet.**



8) Four-fifths of a number less eight is equal to three-fifths of the same number plus fourteen. Write an equation and solve for the number. $\boldsymbol{n = 110}$

9) Three consecutive even integers have a sum of 312. Write and solve an equation to determine the three integers. **The three numbers are 102, 104, and 106.**

Solve in terms of the named variable.

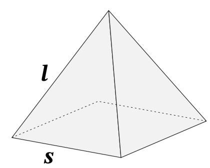
10) $10a - b = c; a$ $\quad \boldsymbol{a = \frac{(b + c)}{10}}$

11) $V = lwh; w$ $\quad \boldsymbol{w = \frac{V}{lh}}$

12) The formula for the surface area of a square pyramid is given below. Write the formula in terms of l.

$SA = s^2 + 2sl$

$\boldsymbol{l = \frac{SA - s^2}{2s}}$



EXPLORE 2B

Part B: Equations with Special Cases

Objectives

In this part of the lesson, you will learn about equations with special cases.

By the end of this lesson, you will be able to do the following:

- Rewrite equations with rational coefficients to integer coefficients before solving.
- Determine that an equation has no solution or is an identity.

Why?

Understanding the different types of solutions to algebraic equations is important to determining if your solutions make sense.

Warm Up

The Least Common Denominator (LCD) is the smallest multiple that fractions have in common.

Name the LCD, then evaluate.

1) $\frac{1}{6}-\frac{3}{4}$

LCD: 12

$\frac{2}{12}-\frac{9}{12}=-\frac{7}{12}$

2) $\frac{5}{7}+\frac{3}{5}-\frac{1}{2}$

LCD: 70

$\frac{50}{70}+\frac{42}{70}-\frac{35}{70}=\frac{57}{70}$

Being able to find the least common denominator is important for solving some algebraic problems.

Rewriting Equations with Integer Coefficients

- It is very helpful to know how to clear fractions and decimals from equations so that only **integer** values remain.
- To clear fractions, you should find the **least common denominator**.
- To clear decimals, you should **multiply** by the greatest place value of the decimals.

Check out **More to Explore** in the Digital Pack to see if there are additional activities for this part of the lesson.

Warm Up

Your student should spend no more than 5 minutes on the Warm Up. This should be a quick review to activate prior knowledge.

Encourage your student to state the LCD and explain what they will multiply each fraction by. This will help them recall that not every fraction will be multiplied by the same factor.

2B EXPLORE

Example 1

Solve.

$\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$

Plan Clear fractions using LCD

Implement	Explain
$\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$	
$12\left(\frac{2}{3}x + \frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$	Multiplication Property of Equality
$8x + 6 = 9$	Distributive Property
$-6 \quad -6$	Addition Property of Equality
$8x = 3$	
$\left(\frac{1}{8}\right)8x = \left(\frac{1}{8}\right)(3)$	Multiplication Property of Equality
$x = \frac{3}{8}$	

Check

$\frac{2}{3}\left(\frac{3}{8}\right) + \frac{1}{2} = \frac{3}{4}$

$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\frac{3}{4} = \frac{3}{4}$ ✓

Example 2

Solve. The first part of this example is completed for you.

$0.15n + 0.8 = 2$

Plan Clear the decimals from this equation by multiplying by the greatest place value of the decimals.

Implement	Explain	Check
$0.15n + 0.8 = 2$		
$100(0.15n + 0.8) = 100(2)$	Multiplication Property of Equality / Distributive Property	
$\mathbf{15n + 80 = 200}$		
$\mathbf{-80 \quad -80}$	**Addition Property of Equality**	$\mathbf{0.15(8) + 0.8 = 2}$
$\mathbf{\left(\frac{1}{15}\right)15n = \left(\frac{1}{15}\right)120}$	**Multiplication Property of Equality**	$\mathbf{1.2 + 0.8 = 2}$
$\mathbf{n = 8}$		$\mathbf{2 = 2}$

EXPLORE 2B

Checkpoint

Solve the equation by writing all numbers as integers. Show Implement.

Use the Distributive Property to solve for x.

$\frac{2}{3}x + \frac{2}{5} = \frac{8}{15}$

$$\begin{aligned} \text{LCD} &= 15 \\ 15\left(\frac{2}{3}x + \frac{2}{5}\right) &= 15\left(\frac{8}{15}\right) \\ 10x + 6 &= 8 \\ -6 \quad & \quad -6 \\ 10x &= 2 \\ \left(\frac{1}{10}\right)(10x) &= \left(\frac{1}{10}\right)(2) \\ x &= \frac{2}{10} = \frac{1}{5} \end{aligned}$$

One Solution, No Solutions, or All Real Numbers

- You can use substitution to determine if your solution to an equation with **one solution** is correct.
- For some equations there is no value, or **no solution** that makes the equation true.

Example 3

Solve.

$3x + 2 = 3(x - 5)$

Implement and Explain

$3x + 2 = 3(x - 5)$

$3x + 2 = 3x - 15$ ◂ Distributive Property

$-3x \quad -3x$ ◂ Addition Property of Equality / Inverse

$2 = -15$

no solution **or ∅ [empty set]**

Check

You can replace the variable with any **real number** and the equation will have a false equality statement.

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: If you do not clear the fractions, what do you need to do before solving?

A: *Make sure that all terms have the same denominator.*

Once your student determines the LCD:

Q: When you multiply $\frac{2}{3}$ and 15, what is the result?

A: *10*

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Will any number make this equation true? Explain.

A: *Yes, all real numbers means that every number in the real number system will be true.*

If your student determines there is no solution, check to see if they distributed correctly on the right hand side of the equation. They may have incorrectly written $-4x + 1$ or $-4x + 4$.

2B EXPLORE

- Another possible solution to an equation is that the equation is true for $\mathbb{R}$.
- An equation with a solution that is all real numbers is also called an **identity**.

Example 4

Solve.

$18 = 6(x + 3) - 6x$

Implement

$18 = 6(x + 3) - 6x$

$18 = 6x + 18 - 6x$

$18 = 18$

Check Use any **number** for the value of x and the sides of the equation will remain **equal**.

$x = 0$	$x = 3$
$18 = 6(0 + 3) - 6(0)$	$18 = 6(3 + 3) - 6(3)$
$18 = 18 - 0$	$18 = 36 - 18$
$18 = 18$	$18 = 18$

Example 5

Solve.

$2x - 16 = 2(x - 8)$

$$\begin{aligned} 2x - 16 &= 2x - 16 \\ -2x \quad &\quad -2x \\ +16 &\quad +16 \\ 0 &= 0 \end{aligned}$$

All terms on both sides are exactly the same, All real numbers, $\mathbb{R}$ can be written at any step.

Checkpoint

Determine if x has no solution, one solution, or is true for all real numbers.

$-4x - 4 = -4(x + 1)$

$-4 = -4$

All Real Numbers, $\mathbb{R}$

PRACTICE 1 2B

Practice 1

Complete the problems on a separate sheet of paper.

Solve.

1) $2g-\frac{2}{5}=\frac{7}{5}$ $g=\frac{9}{10}$

2) $0.5h+0.15=0.43h-0.34$ $h=-7$

3) $\frac{5}{8}x+1=\frac{1}{3}(x-2)$ $x=-\frac{40}{7}$

4) $0.4z+0.24=1.5+0.1z$ $z=\frac{126}{30}=4.2$

5) Two-thirds of a number (n) plus three is the same as one-fourth of the same number minus one. Write and solve your equation. $n=-\frac{48}{5}$

6) Barbara spent $15.51 at a local farmers' market. She spent $5.26 on tomatoes and then bought p pounds of fruit for $4.10 per pound. How many pounds of fruit did Barbara buy?

7) Explain what an equation with no solutions will look like when solved.

Solve. Determine if the equation has one solution, no solutions, or has a solution of all real numbers.

8) $49=1-(x+1)$

9) $5(2x+3)=8x-(-2x+6)$

10) $\frac{1}{4}(12x-8)=3x-2$

11) $0.70k+0.71=0.70k+0.68$

12) Write your own example of an equation with a solution of all real numbers.

Your student should construct an equation that results in the variables being eliminated and the remaining numbers being equal to one another.

Practice 1

Worked solutions for these problems are located in the Digital Pack.

1) If your student is struggling to solve for the correct answer, check that they are distributing the LCD to every term in the equation. Sometimes students will only multiply the terms with fractions rather than all terms.

2) Remind your student to check their work. Some of the solutions are fractions or decimals and they may feel less confident when all of the answers are not integer values.

3) Remind your student that only the term outside of the parentheses is multiplied by the LCD. Otherwise, each individual term is multiplied by the LCD.

In problem 3), when your student multiplies the right-side by the LCD, they should only multiply $\frac{1}{3}$ and 24 , since $\frac{1}{3}(x-2)$ is considered one term.

5) Have your student read each sentence carefully. After each sentence, write down the algebraic symbols that match the sentence. Then put all of the pieces together into one equation.

6) $p=2.5$, Barbara bought 2.5 pounds of fruit.

7) Sample:
An equation with no solutions will have an untrue equation in the last line. The variables will simplify out of the equation and the remaining numbers will not be equal to one another (e.g., $3=8$).

Remind your student that no solution occurs when the variable is eliminated and the contents are not equal. This is different from the solution $x=0$. Zero is a real number and can, therefore, be a solution to some equations.

8) $x=-49$
This equation has one solution.

Your student should distribute -1 in the parentheses on the right side of the equation. If they forget this step, their answer will be one solution rather than no solution.

9) $15=-6$
This equation has no solution, $\varnothing$

10) $-2=-2$, The equation has a solution of all real numbers, $\mathbb{R}$

11) $71=68$, no solution
If your student notices that the variables will simplify out and the constants are not equal they can write no solution without any additional work.

Mastery Check

Show What You Know

Determine the LCD of the fractions. Remind your student that on the left side of the equation only the term outside of the parentheses is multiplied by the LCD but each individual term on the right side is multiplied by the LCD.

B) This question ties in previous learning from Lesson 1 so that your student can see how the lessons relate to one another. Have your student review the vocabulary for rational and irrational numbers and look at the real number system diagram if they are stuck.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- You will be able to rewrite equations with rational coefficients to integer coefficients before solving.
- You will be able to determine that an equation has no solution or is an identity.

2B MASTERY CHECK

Mastery Check

Show What You Know

A) Solve the equation by first writing numbers as integers.
Determine if the equation has one solution, no solutions, or is an identity.

$$\frac{3}{10}(x - 20) = \frac{1}{2}x - \frac{1}{5}x - 6$$

LCD: 10

$$10\left(\frac{3}{10}(x-20)\right) = \left(\frac{1}{2}x - \frac{1}{5}x - 6\right)10$$

$$3(x-20) = 5x - 2x - 60$$

$$3x - 60 = 3x - 60$$

$$-3x \qquad -3x$$

$$-60 = -60$$

all real numbers, $\mathbb{R}$

B) Rian says that *all rational numbers* will make this equation true. Jo says that *all irrational numbers* will make this equation true. Explain how both students are correct.

Rational and irrational numbers are sets of numbers in the real number system. Both rational numbers and irrational numbers are real numbers and all real numbers will make this true. If you take Rian's and Jo's answers together, it will form all real numbers and the complete answer to the equation.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Lesson Test

After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2.
- Review the videos, Guided Notes, and Examples.

PRACTICE 2 2B

Practice 2

Complete the problems on a separate sheet of paper.

Solve the equation by first writing the coefficients as integer values.

1) $\frac{2}{3}x - 3 = \frac{7}{3}x + 8$ $\boldsymbol{x = -\frac{33}{5}}$

2) $0.1h - 0.2 = 1.4$ $\boldsymbol{h = 16}$

3) $-\frac{2}{3}x + \frac{5}{4} = \frac{1}{2}$ $\boldsymbol{x = \frac{9}{8}}$

4) $5.75 - 1.50 - 0.17p = 0$ $\boldsymbol{p = 25}$

5) Five-fourths of a number, less twenty-one divided by two is equal to one-fifth of the same number.

6) Mrs. Trainer and Mr. Rodriguez took a poll comparing food preferences among students in their classes. In Mrs. Trainer's class, 2 more than $\frac{3}{8}$ of the students prefer sweet potatoes over peas. In Mr. Rodriguez's class, 5 more than $\frac{1}{4}$ of the students preferred sweet potatoes. The two teachers have the same amount of students in the class and the same number of students who prefer sweet potatoes. How many students are in each class?

7) Explain what an equation that is an identity will look like when solved.

Solve. Determine if the equation has one solution, no solutions, or is an identity.

8) $2(b - 4) = 2b - 8$

9) $3p + 5 = 5p - 2p - 2$

10) $2n + 7 = 3n - 2$

11) $\frac{6}{7} - \frac{2}{3}v + \frac{1}{7} = \frac{2}{3}v + \frac{3}{7} - 1\frac{1}{3}v + \frac{4}{7}$

12) Write your own equation with no solution.

Your student should construct an equation that results in the variables being eliminated and the remaining numbers being unequal to one another.

Practice 2

Worked solutions for these problems are located in the Digital Pack.

5) $n = 10$

6) $c = 24$, There are 24 students in each class.

7) Sample:
An identity will result in an equation that is always true. The variables will simplify out of the equation and the remaining numbers will be equal to one another (e.g., $5 = 5$).

8) $-8 = -8$, The equation has a solution of all real numbers for b, $\mathbb{R}$

9) $5 = -2$, The equation has no solution, $\varnothing$

10) $n = 9$, This equation has one solution.

11) $3 = 3$, This equation has a solution of all real numbers, $\mathbb{R}$

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Lesson Test

Refer to the Part B Mastery Check instructor note to determine if your student is ready for the test.

Targeted Review

Worked solutions for these problems are located in the Digital Pack.

If your student is going to take the Lesson Test, it is recommended that they do so before beginning the Targeted Review.

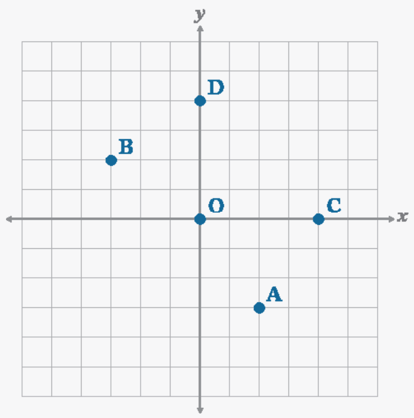
5) Identity Property of Addition

6) Addition Property of Equality

7) Distributive Property, Zero-Product Property

8) Commutative Property, Associative Property

12)



15) Distractor Rationale:

$\sqrt{4}+\sqrt{5}$ $\quad Q + I = I$

$\pi \cdot \pi$ $\quad I \cdot I = I$

16) Distractor Rationale:

A) This is 20% of 90 but not 120% of Samson.

B) This is the percent without the % symbol.

D) This is 90 + 20.

Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Complete the problems on a separate sheet of paper.

Classify the number using ALL of the sets to which it belongs using math shorthand.

1) -3.7 $\{\mathbb{Q}, \mathbb{R}\}$

2) -27 $\{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

Determine if the expression will result in a rational or irrational number.

3) $\sqrt{25}+\frac{1}{\sqrt{9}}$ **rational** $\left(5+\frac{1}{3}\right)$

4) $\sqrt{3}+6$ **irrational**

Name the property being demonstrated.

5) $x+(-5+5)=x+0$

6) $4n+8=7+8$

Name the *two* properties being demonstrated.

7) $(0)(8x+7y-z)=0$

8) $9+(3+1)=(9+1)+3$

Draw a number line to order the given numbers.

9) P: 0 Q: $\frac{2}{3}$ R: $-\frac{1}{4}$ S: $-\frac{\pi}{\pi}$

Simplify the expression when $m = 6, p = -4$.

10) $m+(p-m)-p$ **20**

11) $-(mp)+m$ **−18**

Draw and graph the ordered pairs on a coordinate plane.

12) A $(2,-3)$ B $(-3,2)$ C $(4,0)$ D $(0,4)$ O $(0,0)$

13) The perimeter of a rectangular shaped garden was 18 feet. The length of the garden was 6 feet. What is the width? Use your Formula Sheet to write the necessary formula, define the variables, and then solve. **$w = 3$, The width of the rectangular garden is 3 feet.**

14) The volume of a rectangular box was found to be 24 cubic feet. The height and the width of the box were 2 feet. What is the length of the box? Use your Formula Sheet to write the necessary formula, define the variables, and then solve. **$l = 6$, The length of the box is 6 feet.**

Multiple Choice

15) Select all expressions that will result in a *rational* answer by filling in the appropriate box(es).

- ☐ $\sqrt{4}+\sqrt{5}$
- ■ $\frac{1}{2}+\frac{11}{13}$
- ☐ $\pi \cdot \pi$
- ■ $\frac{8}{\pi} \cdot \pi$

C **16)** Samson reads 90 pages in his book each day. Kelley reads 20% more pages than Samson every day. How many pages does Kelley read daily?

A) 18 pages

B) 20 pages

C) **108 pages**

D) 110 pages

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Lesson Origin	1	1	1	1	1	1	1	1	1	PA	PA	PA	PA	PA	1	1